The 2013 World Congress on
Advances in Structural Engineering and Mechanics (ASEM13)
Jeju, Korea, September 8-12, 2013

Evaluation of wind fragilities of existing multi-story shear frames using maximum a posteriori estimation

Vincent Z. Wang[1),*], John D. Ginger[2)], and David J. Henderson[3)]

[1), 2)] *School of Engineering and Physical Sciences, James Cook University, Townsville, Queensland 4811, Australia*
[3)] *Cyclone Testing Station, James Cook University, Townsville, Queensland 4811, Australia*
[*] *E-mail: vincent.wang@jcu.edu.au*

ABSTRACT

Wind fragility analysis provides a quantitative instrument for delineating the safety performance of civil structures under hazardous wind loading conditions such as cyclones and tornados. It has attracted and would be expected to continue to attract intensive research spotlight particularly in the nowadays worldwide context of adapting to the changing climate. One of the challenges encumbering efficacious assessment of the safety performance of existing civil structures is the possible incompleteness of the structural appraisal data. Addressing the issue of the data missingness, the study presented in this paper forms a first attempt to investigate the feasibility of using the expectation-maximization (EM) algorithm and Bayesian techniques to predict the wind fragilities of existing civil structures. A numerical example of a multi-story shear frame is introduced with the wind loads derived from a widely used power spectral density function. Specifically, the application of the maximum a posteriori estimates of the distribution parameters for the story stiffness is examined.

1. INTRODUCTION

From Cyclone Tracy in Darwin 1974 (Walker 1975) to Cyclone Yasi in North Queensland 2011 (Boughton et al. 2011), from Hurricane Katrina in New Orleans 2005 to the most recent Oklahoma tornado, civil engineering community kept being reminded in a hard way of the necessity of furthering the state of the art of design, construction, and maintenance of structures that are resilient to hazardous wind loads. Among various fast growing techniques, wind fragility analysis provides a quantitative instrument for dealing with the inherent uncertainty, and thus helps mitigate to some extent the risks associated with hazardous wind events (see, for example, Lee and

[1)] Lecturer
[2)] Associate Professor
[3)] Director

Rosowsky 2006, and Rocha et al. 2011).

The performance of civil structures subject to hazardous wind loads can be assessed in a destructive way. Typically, this is achieved by loading corresponding scaled model structures, or full-scale model structures where appropriate, until one or more limit states of interest are reached. Destructive testing, if meticulously designed and conducted, could offer arguably the most straightforward mechanism through which the response of structures in reality can be studied. Alternatively, more and more nondestructive testing procedures including structural health monitoring techniques (Chang 1997-2011; Chan and Thambiratnam 2011) have recently be formulated to carry out the performance assessment in such a way that no substantive damage will be introduced to the structures being tested, and even the interruption to the everyday operations relevant to the structures will be kept to a minimum.

As far as nondestructive testing procedures are concerned, a concomitant issue is that sometimes not all the structural appraisal data that are supposed to be collected actually turn out to be collected (Wang et al. 2013). The reasons for this include data acquisition system breakdowns, signal transmission errors, and vandalism and sabotage activities, to name but a few. Thereby, the challenge here is firstly how wind fragilities of an in-service structure can be evaluated when only incomplete appraisal data for the structure are available, and secondly, in terms of the final wind fragility evaluation results, how to achieve in an incomplete-data scenario a level of accuracy comparable (at least from some practical point of view) to that exhibited in the complete-data scenario.

The paper is organized as below: The second section is from a frequentist perspective, and a multi-story shear frame is used as an example to introduce some expectation-maximization (EM) algorithm (Dempster et al. 1977; Wu 1983; Meng and van Dyk 1997) based wind fragility evaluation procedures. The procedures are then validated by a comparison between the wind fragilities obtained in the incomplete-data scenario and those in the corresponding complete-data scenario. The potential of a Bayesian approach for wind fragility evaluation is then explored in the third section, followed by some concluding remarks.

2. WIND FRAGILITY EVALUATION USING THE EM ALGORITHM: IMPLEMENTATION AND VALIDATION

As the main objective of this study is to evaluate the wind fragilities using incomplete structural appraisal data, an established, representative wind load model in the literature (Soong and Grigoriu 1993; Simiu and Scanlan 1996) is chosen to synthesize the time varying wind loads. For the ease of reference and the completeness of presentation, the formulas and parameters used for the wind load synthesis are reviewed and reproduced below.

The power law is applied to determine the mean wind velocity u_h at the height h, as in Eq. (1) (Simiu and Scanlan 1996):

$$u_h = u_{\mathrm{r}} \left(\frac{h}{h_{\mathrm{r}}} \right)^{\alpha} \tag{1}$$

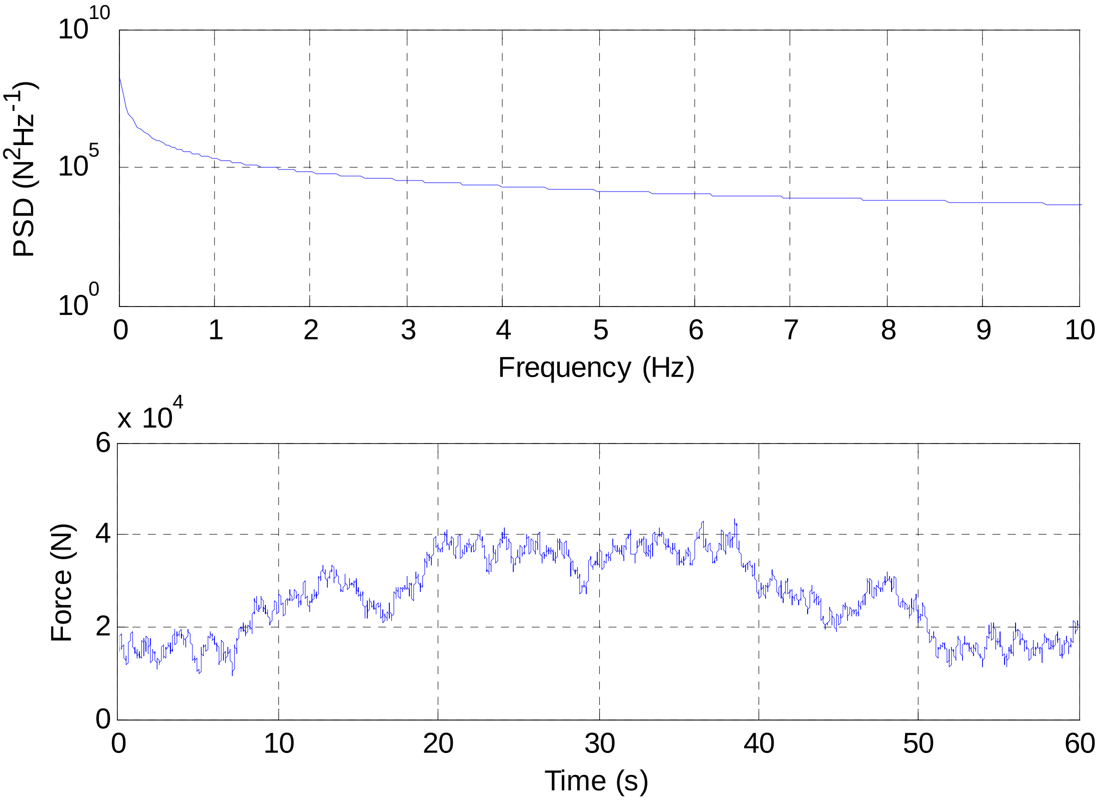


Fig. 1. One-sided PSD (upper subfigure) and time history (lower subfigure) of the wind induced excitation force at the roof level (i.e., $h = 15$ m)

where u_r is the mean wind velocity at the reference height h_r, and α is a constant. Throughout the paper, the reference height h_r is taken to be 10 m with the corresponding mean wind velocity u_r being 50 m/s, and the constant exponent α is assumed to be 1/7. At a given height of h, $G(\omega)$, the one-sided power spectral density (PSD) function of the wind induced excitation force acting at a surface area A normal to the direction of the wind, is shown by Eq. (2) (Soong and Grigoriu 1993; Simiu and Scanlan 1996):

$$G(\omega) = 0.52 A^2 \rho^2 C_d^2 u_h^{\frac{8}{3}} u_* h^{-\frac{2}{3}} \omega^{-\frac{5}{3}} \tag{2}$$

where ω is the angular frequency; ρ is the air density; u_* is the friction velocity; and C_d is the drag coefficient. In this study, $A = 10\ \text{m}^2$, $\rho = 1.25\ \text{kg/m}^3$, $u_* = 3.75\ \text{m/s}$, and $C_d = 1.5$.

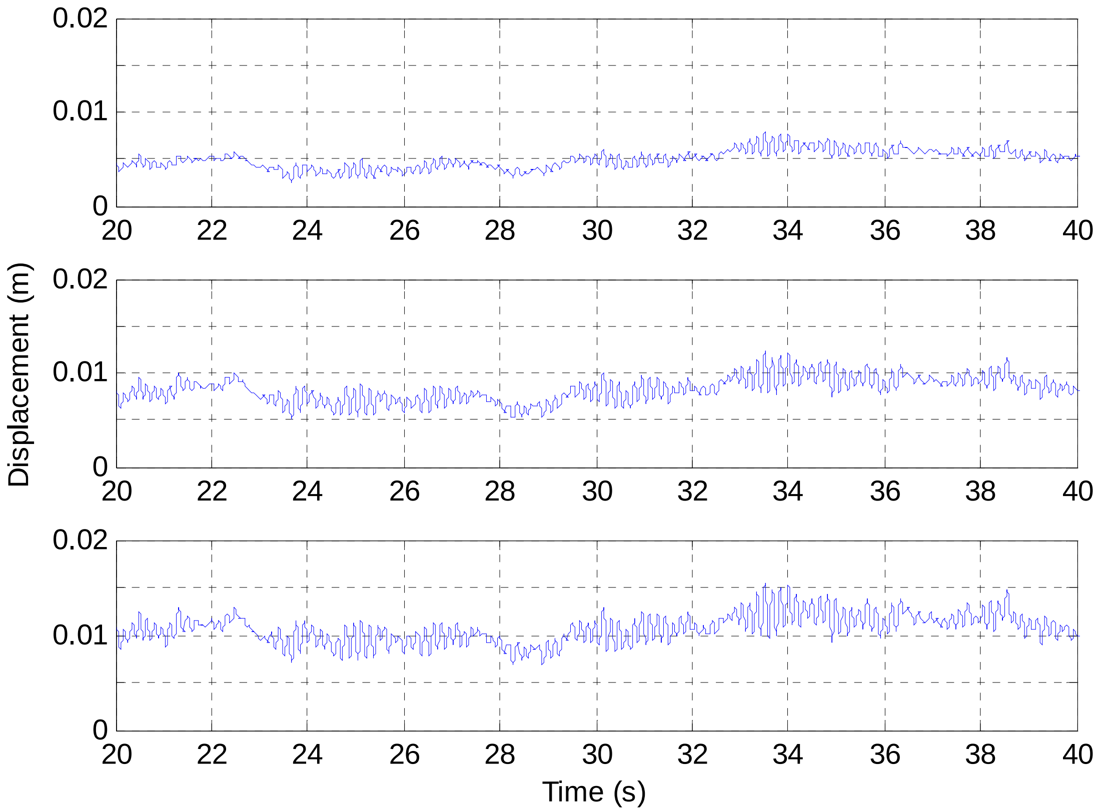
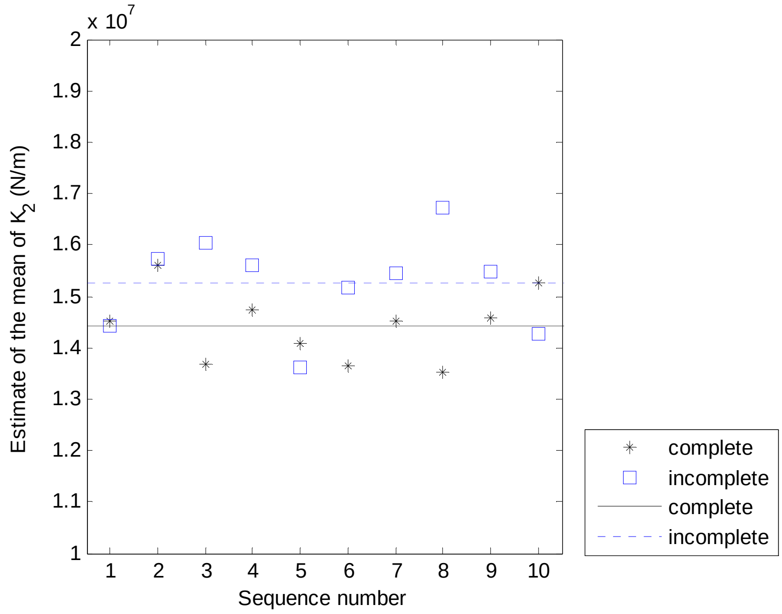


Fig. 2. Examples of the displacement time histories obtained at the first floor level (upper subfigure), the second floor level (middle subfigure), and the roof level (lower subfigure)

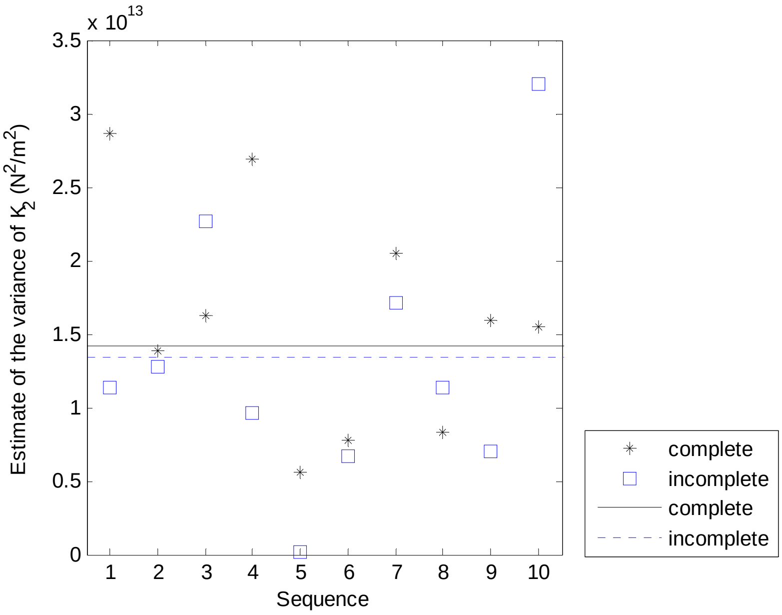
Table 1. An example of the incomplete structural appraisal data

Story ID	Incomplete structural appraisal data for the story stiffness (×10^7 N/m)
1	1.7843; 1.8050; NA; 1.4833; NA; 2.0190; 1.8119; 2.0233; NA; 1.6220; 1.0276; NA; NA; NA; NA; 2.0829; 1.8250; 1.6292; NA; NA; 0.8409; 1.3106; 1.7182; NA; 1.6078; 1.7673; 1.5995; NA; 0.7471; NA.
2	1.2363; 1.6302; 1.2494; 1.8557; 1.0746; 1.1058; NA; NA; 0.7130; NA; 1.6557; 1.4519; 1.4944; 0.9117; 1.7041; 2.1901; 1.3980; 1.4777; 1.4856; 1.0454; NA; 1.6080; NA; 1.4439; NA; 1.7750; NA; 1.8832; 1.4319; NA.
3	NA; 1.7167; 1.0266; 1.7964; 0.9410; 1.5674; 1.8447; 1.4013; 1.2851; 1.7263; 1.2698; 0.7939; 2.1242; 1.9900; 1.7958; NA; 2.0807; NA; 1.0034; 1.8905; NA; 2.0350; NA; 1.8023; NA; 1.7116; 1.6808; NA; 0.7966; NA.

Note: An “NA” denotes a missing data point.



(a)



(b)

Fig. 3. Comparisons of some selected estimates of (a) the mean and (b) variance of K_2 in the complete- and incomplete-data scenarios

Take a three-story shear frame as an example. Suppose that the shear frame can be modeled as a linear three-degree-of-freedom (DOF) system with its mass evenly lumped at the three floor/roof levels. Further assume that the shear frame has a

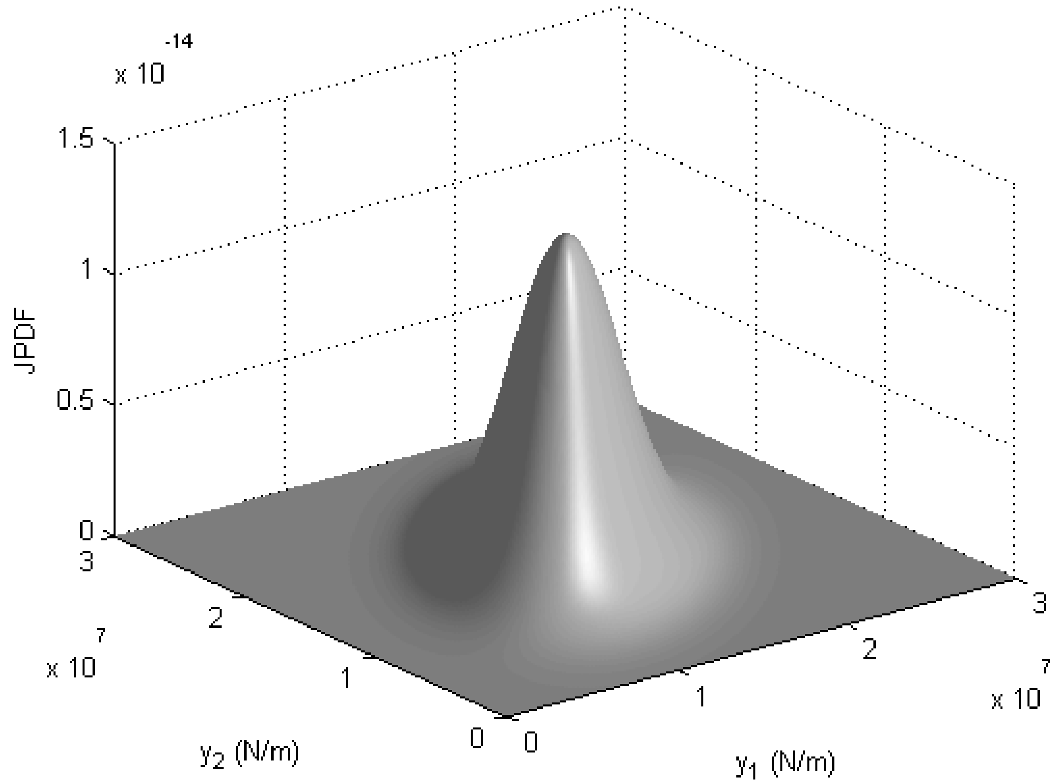


Fig. 4. Estimated JPDF of K_1 and K_2 based on the averages of the relevant maximum a posteriori estimates

Table 2. Estimated wind fragilities in Cases I and II

Case ID	Estimated wind fragilities
I	0.421; 0.500; 0.506; 0.478; 0.585; 0.637; 0.516; 0.573; 0.490; 0.474.
II	0.422; 0.321; 0.451; 0.510; 0.520; 0.380; 0.493; 0.414; 0.430; 0.526.

uniform story height of 5 m. At the roof level, the one-sided PSD function for the wind induced excitation force, together with a corresponding time history, is illustrated in Fig. 1. Each story mass is considered as a deterministic quantity with its value equal to 2,000 kg, while a common trivariate normal distribution with all the three means, three variances, and six covariances respectively taken to be 1.5×10^{7} N/m, 2.025×10^{13} N^2/m^2, and 1.0125×10^{13} N^2/m^2 is used to simulate both the complete and incomplete structural appraisal data for the story stiffnesses. Also, denote the first, second, and third story stiffnesses by K_1, K_2, and K_3, respectively. Fig. 2 shows an example of the displacement time histories obtained at the floor/roof levels. In the incomplete-data scenario, the missingness pattern is assumed to follow the missing-completely-at-random manner (Heitjan and Basu 1996), and a probability of missingness of 0.3 is

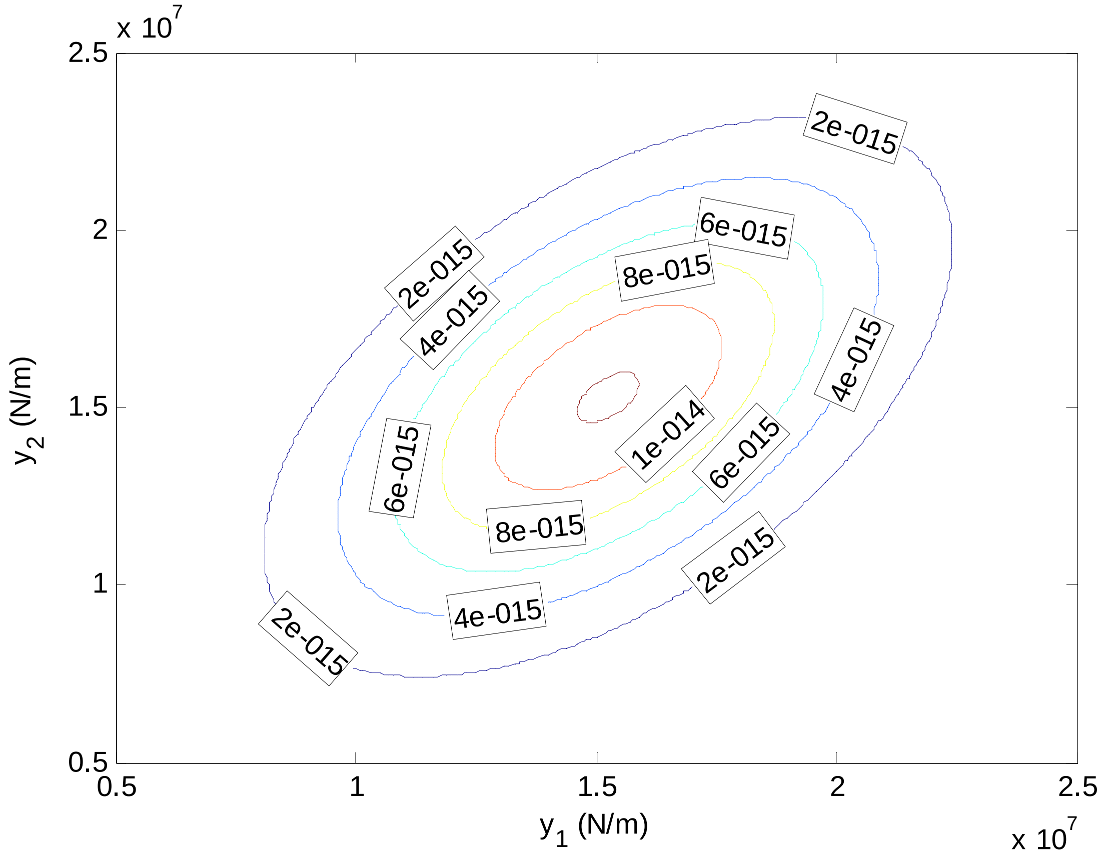


Fig. 5. Contour plot of the estimated JPDF shown in Fig. 4

Table 3. Estimated wind fragilities in Cases III and IV

Case ID	Estimated wind fragilities
III	0.739; 0.684; 0.780; 0.622; 0.676; 0.697; 0.819; 0.725; 0.708; 0.657.
IV	0.297; 0.173; 0.324; 0.193; 0.216; 0.118; 0.286; 0.251; 0.261; 0.183.

assigned to each appraisal data point. As an example, Table 1 lists some appraisal data in the incomplete-data scenario.

For the fragility analysis here, failure of the shear frame is defined as the situation when the maximum inter-story relative displacement exceeds a pre-selected threshold U_m , and the fragility is quantified by the probability of this failure event. In this section, U_m is chosen to be 0.01 m. To deal with the incomplete appraisal data for the story stiffness, the EM algorithm is implemented through the statistical computing environment *R* (R Core Team 2012) and the package *norm* (Novo and Schafer 2012). To validate results obtained in the incomplete-data scenario, the following scheme is designed: In the incomplete-data scenario, the 1,000-time Monte Carlo simulation is independently carried out ten times, yielding a sample containing ten realizations of the fragility. In parallel, another fragility sample is constructed in the complete-data scenario. The null hypothesis that these two samples are from a same population is then tested against the alternative hypothesis that their populations are different from each other. Accordingly, one may expect that the null hypothesis cannot be rejected if the procedures used to deal with the appraisal data missingness are deemed effective. The above validation scheme leads to the estimates illustrated in Fig. 3 and the fragilities shown in Table 2, where Cases I and II respectively correspond to the complete- and incomplete-data scenarios. For the data in Table 2, the null hypothesis cannot be rejected based on the two-sample Kolmogorov-Smirnov test at a commonly used significance level of 0.05.

3. AN ILLUSTRATIVE EXAMPLE FOR WIND FRAGILITY EVALUATION BASED ON BAYESIAN TECHNIQUES

Bayesian techniques along with their application to civil engineering form an intriguing family of procedures thanks to their general capacity of systematically taking into account cognate prior information. As a preliminary attempt to explore the potential application of Bayesian techniques to the analysis of wind fragilities with incomplete structural appraisal data, a normal inverse Wishart distribution, which is a conjugate prior distribution in this context, is selected to model the mean vector and the covariance matrix of the story stiffnesses. Specifically, Figs. 4 and 5 describe the estimated joint probability density function (JPDF) of the story stiffnesses K_1 and K_2. This JPDF results from the averages of the maximum a posteriori estimates of the means, variances, and covariances of the story stiffnesses. With the threshold U_{m} chosen to be 0.008 m and 0.012 m in Cases III and IV respectively, the estimated wind fragilities in the incomplete-data scenario are listed in Table 3. It can be observed that with a more stringent threshold in Case III than in Case IV the resulting wind fragilities increase considerably.

4. CONCLUDING REMARKS

The study presented in this paper lays the groundwork for further investigation into the appropriate incorporation of relevant prior information into the evaluation of the safety performance of civil structures against hazardous wind loads. The prior information under the current circumstances could come from code specifications, design experience, and previously conducted structural appraisal activities, among other sources.

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