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Mechanical behavior of composite gel periodic structures with the pattern transformation

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ABSTRACT

When the periodic cellular structure is loaded or swelling beyond the critical value,the structure may undergo a pattern transformation owing to the local elasticinstabilities, thus leading to structure collapses and the change to anewconfiguration. Based on this deformation-triggered pattern, we have developed thenovel composite gel material. This designed material is a type of architecture materialwhich achieves special mechanical properties. In this paper, the mechanical behavior of the composite gel periodic structurewith pattern transformation has been studied through numerical simulations. When pattern transformation occurs, it can result in a different elastic relationshipcompared with the material at untransformed state. From the obtained nominal stress versusnominal strainbehavior, the Poisson's ratio and corresponding deformed structure patterns, we investigate the effects ofthe uniformly distributed gel inclusions oncomposite material, thereby having a better understanding ofthe characteristics of composite materials. We hope this study can provide future perspectives for thenew composite material.

1. INTRODUCTION

Periodiccellularstructure has been widely investigated for generating novel patterntransformation and special mechanical properties.When it is loaded or swelling beyond the critical value, this type of periodic structurecanexhibit structural instabilities(Mullin et al., 2007), thus leading to structure collapses and the change to a new configuration.The structure with the new configuration can display a special mechanical property. It is imperative to study the influence of instabilities on global material properties. Thus the researchers can design the structure of the material to achieve or expand the special functionality of material. Evans and Aldersonhadmodeledchiral and anti-chiral honeycomb materialstructures, and consideredthe deformation mechanismsresponsible forauxetic functionalityof the two structures(Alderson et al., 2010). Besides the honeycomb structures, the othershapes

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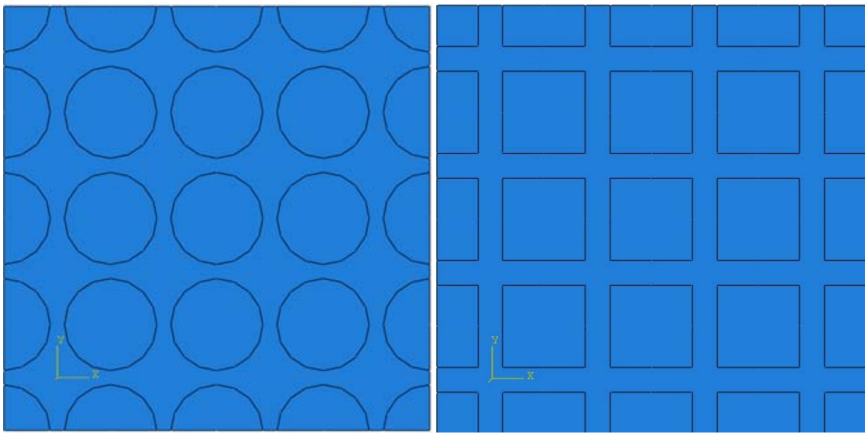
of re-entrant structures(Barnes et al., 2012; Gaspar et al., 2011; Miller et al., 2011)or buckliball(Shim et al., 2012)have also attracted large numbers of researchers to study the pattern transformation of those structural materials, especially for studying the critical value of applied load for different pattern.Meanwhile, Mullin group has studied mechanical properties of the porous elastic material structures through theory, simulations and experiments(Bertoldi et al., 2008; Bertoldi et al., 2010; Mullin et al., 2007; Willshaw and Mullin, 2012). In the application of structural materials, Jang et al. (2009)and Li et al. (2012)combined pattern instability and shape-memory hysteresis for photonic switching.In soft structural materials, many studies ofthe collapse of a void or hole in an elastomerunder swelling and de-swelling were carried out (Cai et al., 2010; Ding et al.,2013; Hong et al., 2009).The similar novel composite material is experimentally studied by combining the silicone rubber samples with jelly filling each of the holes(Mullin et al., 2013). From previous studies, pattern transformation is a result of a local buckling instability in the compressively loaded vertical inter-holeligaments in the case of the arrays.Based on this novel deformation-triggered pattern, we have developed thenovel composite gel structure material, in which the arrays of quadrate andcircular gel particles or rods are filled into the periodic cellular structures. This designed material is a type of architecture materialwhich can achieve special mechanical properties and display a new mechanicalbehavior.

In this paper, the mechanical behavior of the composite gel periodic structurewith pattern transformation has been studied through numerical simulations. From the obtained nominal stress versusnominal straincurves and corresponding deformed structure patterns, the effects ofuniformly distributed gel inclusionson mechanical properties of composite materials are discussed.The study can provide a better understanding ofthe characteristics of thenovel composite gel materials.

2. MODELING OF THE COMPOSITE MATERIAL

Thecomposite soft material that is composed of two components has already been manufactured by Mullin et al. (2013).In their experiment, the square arrays of holes in silicone rubber sheet are filledwith jelly.According to the previous study, we understand that ifthe holes are filled with a more rigid material than that of the matrix,the cellular structurewill not lead to any pattern switch (Michel et al., 2007). Therefore, a much softermaterial needs beplaced in the holesto achieve pattern switch for composite material.Mullin et al. (2013) has discovered that the inclusion l which has a Young's modulus of ≤ 1%of the bulk does not suppress the pattern switch.Based on the Liu's work on the incremental modulus of gel(Liu et al., 2011), we try to plug the gel materials into the porous elastomers and find whetherthe periodic composite material can still undergo the pattern transformation, and how the gel material affects on the critical strain.In the study, the numerical simulations of composite material under compressed load are carried out to study the deformation behavior of the structure material. In the modeling, the specific combinations of two materials aremodeled as illustrated in Fig.1(a and b). The size of Fig.1(a) is39.88 mm × 39.88mm, comprising a microstructure of a 9.97×9.97 mm^2 square arrays ofcircular gel inclusions of 8.67mm diameter with 9.97mm center-to-center spacing in vertically and horizontally; The size of model in Fig.1(b) is 40 mm×40mm with 10×10 mm^2 square arrays of 7.7mm length of squaregel with 10mm center-to-center spacing. The volume fraction of gel in the two types of composite materialsissame value of 0.59.



(a) (b)

Fig.1 Composite material structureswith arrays of gel inclusions:(a) circular gel shape, and (b) square gel shape.The gel fraction in two models is the same.

2.1 Material properties

In the composite gel structural material, two different materials are used to form the composite material, i.e.PMMA(polymethyl methacrylate) for matrix and hydrogel forinclusion. For matrix material, a compressible neo-Hookean model(Michel et al., 2007) is assumed. The strain energy $W(F)$ form for neo-Hookean material in plane strain is:

$$W = \frac{\mu}{2}\left[\left(I - 2\right) - 2\ln J\right] + \frac{\kappa - \mu}{2}\left(J - 1\right)^2, \quad (1)$$

where μ and κ are, respectively the shear and bulk moduli of the solid at zero strain. The material PMMA(polymethyl methacrylate) was modeled as nearly incompressible, characterized by $\kappa/\mu = 50$. From the early studies(Bertoldi et al., 2008; Mullin et al., 2007;Willshaw and Mullin, 2012), the initial Young's modulus was given as 3.25MPa, so that $\mu = 0.55MPa$.

The inclusion material is gel material(Hong et al., 2009), with a strain energy $W(F)$ given by

$$W = \frac{1}{2}NkT(I - 3 - 2\log J) - \frac{kT}{\nu}\left[\left(J - 1\right)\log\frac{J}{J - 1} + \frac{\chi}{J}\right], \quad (2)$$

where $kT = 4 \times 10^{-21}J$ at room temperature. The two dimensionless material parameter $N\nu$ and χ are chosen appropriately in the numerical examples below.We take the values $N\nu = 10^{-2}$ and $\chi = 0.1$.

2.2.Boundary condition

It is realizedthat the results of finite-sized models are necessarily influenced by boundary conditions at both loaded and traction free edges. To eliminate the boundary condition effects, thecomposite material can be represented as a periodic array of representative volume elements (RVEs). Thusthe numerical investigations areperformed on periodic structures.Actually compared with analysis of the full finite structure model, the RVEs results exhibit an earlier switch(Mullin et al., 2007), i.e. boundary effects delay the structure transformation.Therefore,periodic boundary conditions are imposed on all cell boundaries, and the general form of periodic boundary condition can be expressed as (Berger et al., 2005; Xia et al., 2003).

$$u_i = \overline{S}_{ij} x_j + u_i^* \quad (3)$$

where $\overline{S}_{ij}$ are the average strain, u_i^* is the periodic part of thedisplacement components (localfluctuation) on the boundary surfaces, which is dependent on the applied globalloads. The indices *i*and *j*denote the two-dimensional coordinate directions in the range from 1 to 2.

3. PATTERN SWITCHING IN COMPOSITE MATERIAL

When periodic elastomeric cellular solid is compressed, the array of pores undergoesan instabletransformationat a critical point(Bertoldi and Boyce, 2008; Bertoldi and Boyce, 2007; Bertoldi et al., 2008; Bertoldi and Gei, 2011; Bertoldi et al., 2010; Jang et al., 2009; Kang et al., 2013; Mullin et al., 2007; Singamaneni et al., 2009a; Singamaneni et al., 2009b). Similar instabilities also trigger the change to the new configuration in the novel composite gel material.The numerical approach captures the mechanical behavior of composite materialfor exploring the effect of gel materialand gel shape.

3.1Stress versus strain behavior

To obtainthe nominal stress vs. nominal strain response, the total force on bottomedge as shown in Fig.1 is monitored as a function of theapplied displacement. The numerical results for the nominal stress versus nominal strain behavior of the two types of composite materials are shown in Fig.2. Figure 2also provides a directivecomparison betweenthe matrix material without gel inside and the one with gel. The behavior of the composite material is characterizedby an initial almostlinear elastic behavior with a sudden change to a different elastic relationship. The transition point response to the pattern transformationin the composite materials and the patternsat the nominal strain of 0.2 are shown in Fig.3.Atthe turning point to the new configuration, much ofthe macroscopic deformation is observed to be accommodatedby the rotation of the four matrix domains diagonallybridging neighboring inclusions; these domains experience negligiblestrain but undergo large rotations. Then after transformation, due to that gel inclusion suppress the motivation of the matrix domains, the rotation leads to a different composite structure from the initial state. The circular gel inclusions of the novel composite materialbreak their initialshapes andbifurcate into ellipses ofvertical and horizontal directions alternatively. Meanwhile, the linear structure of the composite matrix material with square gel inclusions changes into sinusoidal shapeinthe vertical direction.In addition, the pattern transformation occurs much later in the composite material with the square gel inside than the circular gel inside, though both having the same value of gel fraction.

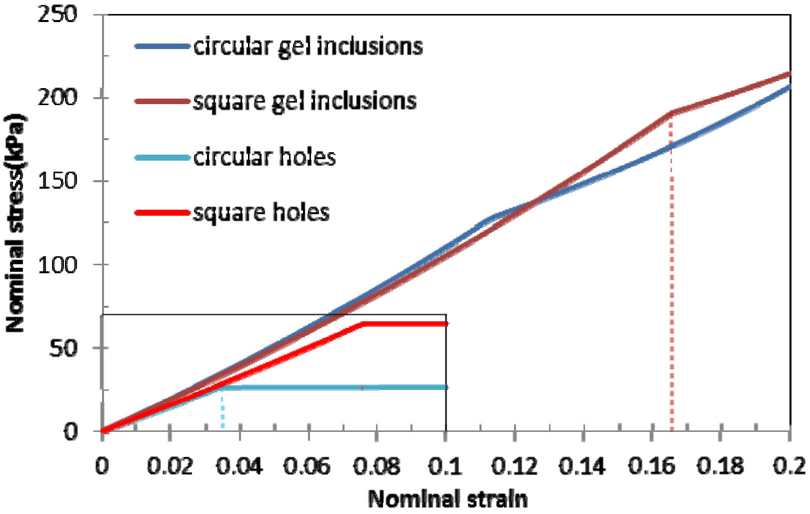
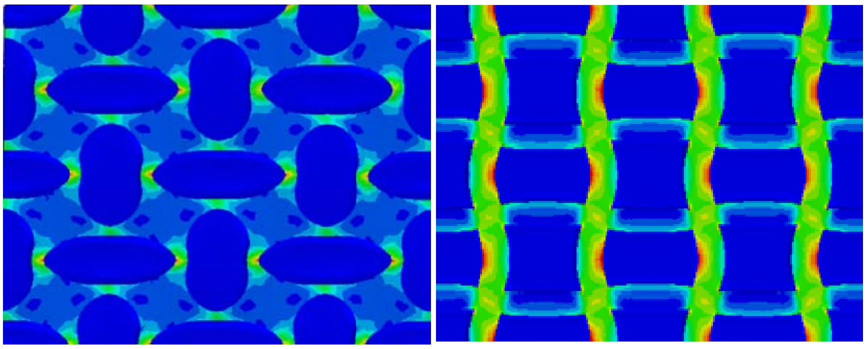


Fig.2 The nominal stress versus nominal strain behavior of the two types of structures with gel inclusions and no gel inside (The red lines represent the square holes or gel inclusions and the blue lines represent the circularholes orgel inclusions).



(a) (b)

Fig.3 The new composite material structure at the nominal strain of 0.2.

Besides the different shapes of gel inclusions,the propertyof gel inclusionalso affects the properties of the composite material. As discussed in Section 2.2, we use gel mono-phase theory to simulate the gel inclusions. The initial state of gel inclusions in these novel composite materials is characterized by the free-swelling stretch λ_0, which is equilibrated in a solvent of chemical potential μ as follows(Hong et al., 2009),

$$N\nu\left(\frac{1}{\lambda_0}-\frac{1}{\lambda_0^3}\right)+\log\left(1-\frac{1}{\lambda_0^3}\right)+\frac{1}{\lambda_0^3}+\frac{\chi}{\lambda_0^6}=\frac{\mu_0}{kT}. \tag{4}$$

When the composite material is compressed, gel inclusions are subjected to constraint de-swelling, which is similar to the example case of a 1-D rod of a gel equilibrated in a solvent of chemical potential μ, and subjected to a uniaxial stresss_1 along the longitudinal direction.From the theoretical calculation of Liu et al.(2011), the reduced incremental modulus or tangent stiffnessof gel is a function of current deformation and stretch(current chemical potential), as is illustrated in Fig.4.According to the definition of incremental modulus of hydrogel(Liu et al., 2011), the free-swelling states or the initial states are hence at the intersection points of the curves for different chemical potentials, which means that the modulus of gel at free swelling state is almost same.The value of the incremental modulus of gel increases as the stretch value decreases or the initial chemical potential increases.

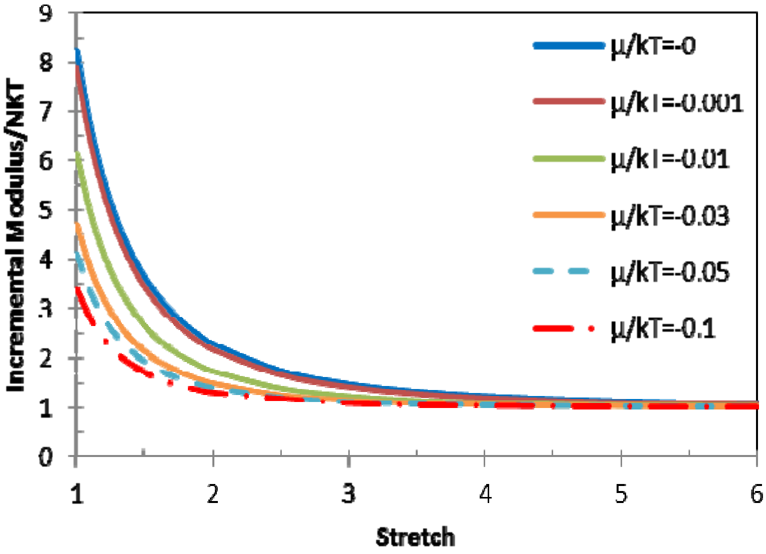


Fig.4 Incremental modulus of gel varying with stretch for various initialchemical potentials.

Now we exploitgel at free-swelling state subjected to different chemical potential, to fill the holes of porous elastic material to develop this novel composite material. Fig.5 shows stress versus strain behavior using the RVE model with array of square gel inclusions in an elastomeric matrix various forinitial chemical potentials.

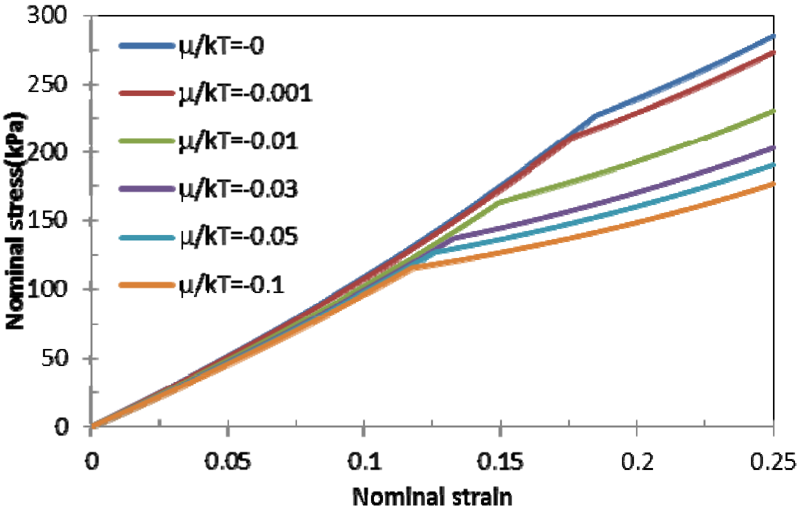


Fig.5 Stress vs. strain behaviors using the RVE model with array of square gel inclusions in an elastomeric matrix various forinitial chemical potentials.

From the plots, we find out several interesting properties of the new material. Firstly, instead of perfectly linear elastic before the pattern switching, the material shows the increasing modulus because the gel inclusions becomes harderas the water in gel inclusions comes out with the applied load;so does the material after transformation.Secondly,the larger the initial chemical potential is, the much more difficultly the pattern transformation appears. It can be explainedthat the modulus of the gel increases as the initial chemical potential increases, which is illustrated in Fig.4.The last and the most important one is that, the pattern transformation has a greater influence on mechanical properties of composite material than the water fraction of gel.

3.2. Poisson’s ratio

Numerous studies show that the porous material with arrays of holes has negative Poisson’s ratio when taking pattern switching. Compared with the negativePoisson’s ratio of structure with no gel inside, the pattern switching state exhibits a positive value of the Poisson’s ratio in gel composite material with square gel inclusions. Although the Poisson’s ratio is positive for the gel composite material, thetendency and shape of gel composite material are same with that of the porous material which was studied byBertoldi et al (2010) .

The numerical results of Poisson’s ratio are plotted as a function of nominal strainin Fig.6 for various values of chemical potential.A strong dependency on this parameter is evident.We find out thatthe Poisson’s ratio of the composite material decreases at which pattern transformationinduced by the instability occurs. It is also striking that simplyby the decreasing the solvent chemical potential of the gel, composite material can be more easily reconstructed and the Poisson’s ratio has a more sharp decline after switching.

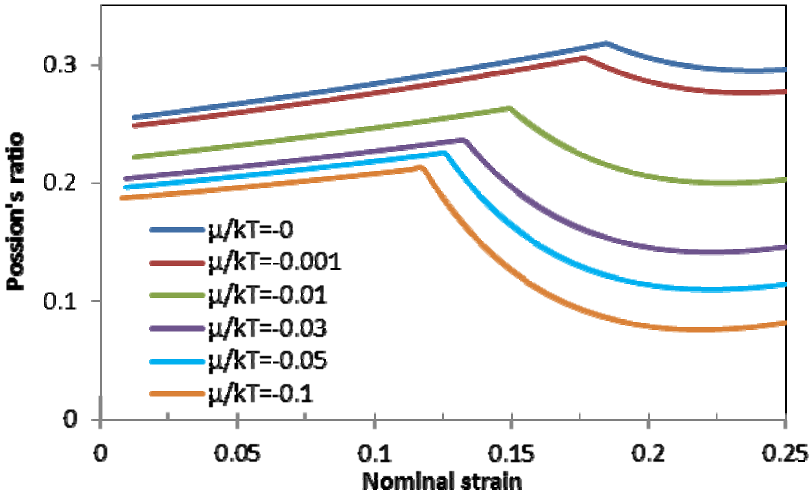


Fig.6 Poisson’s ratio as a function of nominal strain using the RVE model with array of square gel inclusions in an elastomeric matrixfor various initial chemical potentials.

4. CONCLUSION

We develop the novel composite material by filling the gel inclusions into the periodicelastomeric cellar structures, in which we combine the different shapes of gel material with the PMMA matrix. Numerical simulations of the designed composite material are carried out to investigate the mechanical properties. The behavior of the composite material is characterizedby initial almost linear elastic behavior with a sudden change to adifferent elastic relationship. It can be found that the internal structure of composite material greatly affects mechanical characteristics, so do the different shapes of gel inclusions in composite material.We hope this study can provide future perspectives for this novel composite material, as well as how their properties can be optimized and predicted.

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