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Behavior of PSC Structure Using Nonlinear Tendon Model

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ABSTRACT

Nonlinear tendon constitutive model for nonlinear analysis of prestressed concrete structures are presented. Since the post-cracking behavior of concrete structures, in which bonded reinforcements such as tendons and/or reinforcing steels are embedded, depends on many influencing factors(the tensile strength of concrete, anchorage length of reinforcements, concrete cover, and steel spacing) that are deeply related to the bond characteristics between concrete and reinforcements, consideration of the tension stiffening effect on the basis of the bond-slip mechanism is necessary to evaluate ultimate resisting capacity of structures. In this paper, an improved tendon model, which considering the slip effect between concrete and tendon, and effect of tension stiffening, is suggested. The validity of the proposed models is established by comparing between the analytical results and experimental results in prestressed concrete beams.

1. INTRODUCTION

Prestressed Concrete (PSC) Structures have been widely used in SOC structures such as long-span bridge, nuclear power plant containment building etc. PSC structures have many advantages such as effective deflection control, crack behavior control, activation of an entire concrete section, and lighter weight in comparison to reinforcement concrete (RC) structures. Since the post-cracking behavior of PSC structures, in which bonded reinforcements including reinforcement steels and/or tendon are embedded in concrete matrix, depends on many influencing factors that are related to the bond characteristic between concrete and steels, consideration of the tension stiffening effect on the basis of the bond-slip mechanism is necessary. To trace the nonlinear structural response with an increase of external load and to evaluate the structural strength, enough understanding for the structural characteristic of PSC structures from the behavior of composed material must be based. Many numerical models that can simulate the structural response of PSC structures have been

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proposed. One of the approaches is the use of a link element or a bond-zone element between the concrete and tendon in finite element analysis (FEA) of PSC structures. These approaches, however, cause the problem: (a) increasing the number of element, (b) complexity of creating input data, and (c) increasing the computation time.

2. MATERIAL MODEL

The envelope curve (Fig. 1.) to define the uniaxial stress-strain relation of concrete is used and also tension stiffening effect is considered. To simulate the stress state of the concrete under biaxial loading, the orthotropic model is adopted in this paper for its simplicity and computational efficiency. With reference to the principal axes of orthotropic, the incremental constitutive relationship can be expressed by

$$\begin{Bmatrix} d\sigma_1 \\ d\sigma_2 \\ d\sigma_3 \end{Bmatrix} = \frac{1}{1-\upsilon^2} \begin{bmatrix} E_1 & \upsilon\sqrt{E_1E_2} & 0 \\ \upsilon\sqrt{E_1E_2} & E_2 & 0 \\ 0 & 0 & (1\text{-}\upsilon^2)\mathrm{G} \end{bmatrix} \begin{Bmatrix} d\varepsilon_1 \\ d\varepsilon_2 \\ d\gamma_{12} \end{Bmatrix} \tag{1}$$

where $(1-u^2)G = 0.25[E_1 + E_2 - 2u(E_1E_2)^{\frac{1}{2}}]$, E_1 and E_2 are the secant moduli of the elasticity in the direction of the axes of orthotropic, which are oriented perpendicular and parallel to the crack direction. Additionally, G is the shear modulus of the elasticity and u is the Poisson's ratio.

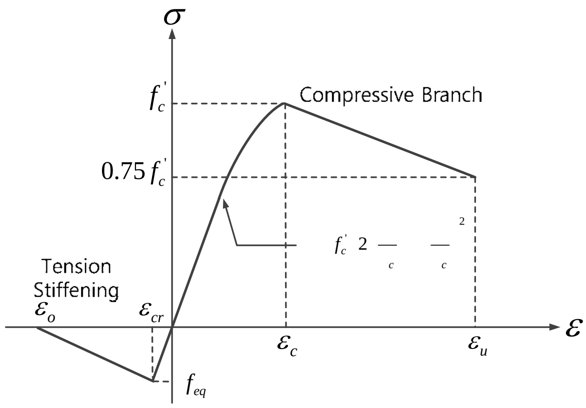


Fig. 1 Stress-strain relationship of concrete

On the other hand, the stress–strain curve for the mild steel is generally assumed identical in tension and compression. For simplicity in calculation, it is necessary to idealize the one-dimensional stress–strain curve for the steel element. In this study, mild steel for the reinforcing steel and liner plate is assumed a linear elastic and linear strain-hardening material. Because the mild steel embedded in the concrete matrix has been used as the secondary reinforcement and generally does not affect the ultimate resisting capacity in PSC structures. In advance, Von-Mises yield criterion is also used to express the material state of liner plate subjected to biaxial stress condition.

3. STRESS-STRAIN RELATIONSHIP FOR PRESTRESSING TENDON

3.1 Consideration of tension stiffening effect

In a cracked cross-section, all tensile force is balanced by the steel encased in concrete matrix only. However, between adjacent cracks, tensile forces are transmitted from the steel to the surrounding concrete by bond forces. This effect is called the tension stiffening effect. When a PSC member is subjected to uniaxial tension, the applied load N is carried partly by the prestressing tendon (N_s) and partly by the concrete matrix (N_c) at the uncracked region and is carried wholly by the prestressing tendon at the crack face. The equilibrium condition of $N = N_c + N_s$ has been maintained up to the yielding of prestressing tendon and gives the relation of $N = A_s E_s e_y = A_s E_s e_y^* + A_c f_c(e_y^*)$. From this relation, the yield stress of bare tendon (f_y) can be expressed in terms of the yield stress of embedded tendon (f_n) and the compressive stress of concrete (f_c):

$$f_y = f_n + \frac{f_c(e_y^*)}{r} \tag{2}$$

where $r = A_s / A_c$ is the steel ratio. Then, an equation that defines the tensile stress of concrete in the strain softening region beyond the cracking strain(e_{cr}) is substituted into Eq. (1). Since e_y can be replaced by $e_y^* = f_y^* \big/ E_s$ on the basis of the assumption for the linear elastic behavior of prestressing tendon up to yielding, the use of tension stiffening model to minimize the difference in numerical results according to the finite element mesh size gives the following relation of :

$$\frac{f_n}{f_y} = 1 - \frac{nk\, f_{cr} \big/ f_y - 1}{nr(k-1) - 1} \tag{3}$$

where $b = e_o / e_{cr}$, $e_o = 2G_f \ln(3/b) \big/ f_t(3-b)$, $n = E_s / E_c$, $f_{cr} = E_c e_{cr}$ in which f_{cr} is the tensile stress of concrete.

3.2 Consideration of bond-slip effect

From the strain distribution, the local slip can be defined as the total difference in elongations between the prestressing steel and the concrete matrix measured over the length between a distance x from a crack face and the center of the segment($x = s/2$).

$$d(x) = \int_x^{l_t} (e_s(x) - e_c(x))\,dx \tag{4}$$

where s is the length between two adjacent cracks, which is equivalent to the crack spacing, $e_s(x)$ and $e_c(x)$ are the strain distributions of steel and concrete, respectively. On the basis of the force equilibrium and the relation of Eq. (4) with the assumption for the linear bond stress-slip, the very well-known following governing differential equation for the bond-slip can be obtain

$$\frac{d^2 s(x)}{dx^2} - k^2 s = 0 \tag{5}$$

where $k^2 = S_0 E_b (1+nr) / A_s E_s$, the steel ratio $r = A_s / A_c$, S_0 is the perimeter of the tendon, and E_s and A_s are Young's modulus and sectional area of tendon.

Upon the evaluation of $e_{s,eq}$ corresponding to the yielding of prestressing tendon, the second modification for the linear stress-strain relation of prestressing tendon up to reach the yielding has been performed on the basis of the consistent force carrying capacity of prestressing tendon, $F_{sy} = f_{s,eq} A_s = E_{s,eq} A_s e_{s,eq} = E_s A_s e_{sy}$ regardless of the bond-slip behavior. That is, the modified elastic modulus of tendon can be determined by $E_{s,eq} = E_s e_{sy} / e_{s,eq}$ and its graphical expression can be found in Fig. 2.

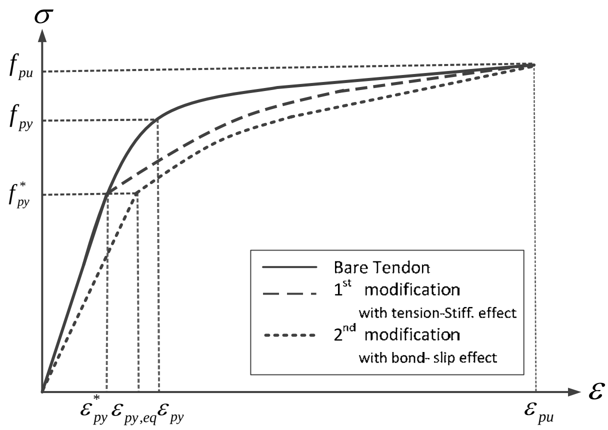
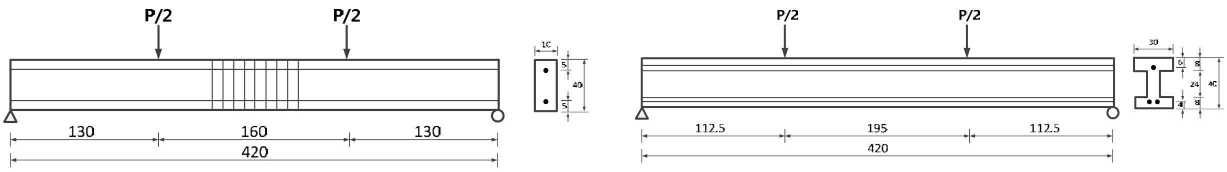


Fig. 2 Modification for the stress-strain relationship of tendon

4. CONCLUSIONS

4.1 PSC beams

The proposed analytical model for prestressing tendon is applied to two simply supported PSC beams which analyzed by Rabczuk and Eibl (2004). The geometry, the cross-section dimensions, the boundary conditions and the loading arrangement of the adopted beams are presented in Fig. 3. Beam I was reinforced with stirrups at mid-span region and was prestressed with $F_p = 26.25kN$ for the upper tendons and $F_p = 11.25kN$ for the lower tendons. On the other hand, Beam II was prestressed with $F_p = 80kN$ for lower tendons but the upper tendon was placed for the test setup only without introducing the prestressing force.



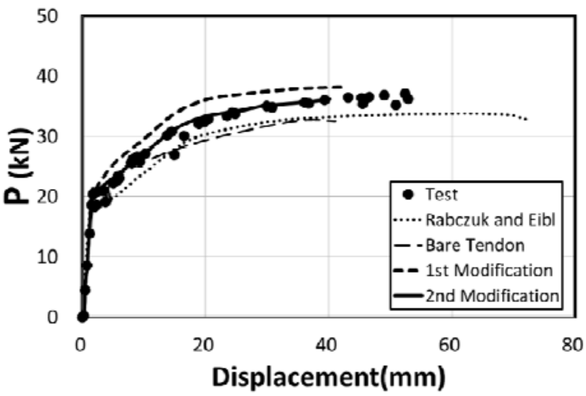
(a) Beam I (b) Beam II

Fig. 3 Test setup of PSC beams (unit : cm)

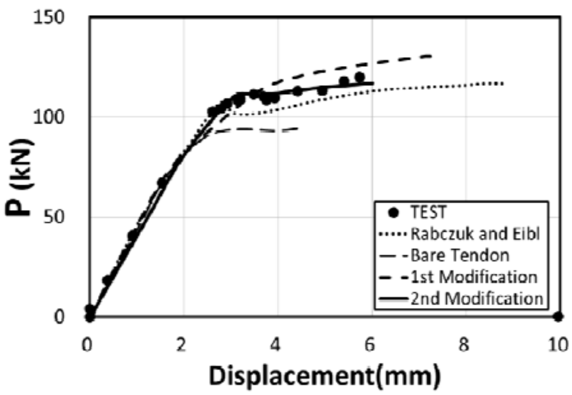
4.2 Analysis Results

Comparisons for the measured load to the mid-span deflection are shown in Fig. 4. for both beams. As shown in Fig. 4., the numerical results obtained by using the modified stress-strain relation of tendon according to the introduced numerical approach give very good agreements with experimental results throughout the entire loading history. On the other hand, the inclusion of tension stiffening effect without reduction of yielding stress in tendon produces an overestimation of the ultimate resisting capacity due to an incorrect prediction of average stress in tendon between adjacent two cracks. Moreover, the exclusion of bond-slip caused even at the bonded internal tendon will also produces an overestimation of the ultimate resisting capacity.

It is clear from the comparison of these numerical results with the experimental data that an exact consideration of the tension stiffening effect together with the bond-slip effect through the modification of stress-strain relation of tendon yields a very good agreement for the structural behavior and an exact estimation of the ultimate resisting capacity.



(a) Beam I



(b) Beam II

Fig. 4 Comparison of test results and analytic results

5. CONCLUSIONS

A modified stress-strain relationship of tendon is proposed in this paper for the non-linear finite element analyses of PSC containments with tendon. The proposed models do not require a double node to simulate the tension stiffening or the bond-slip effect developed at the interface of two adjacent materials of concrete and tendon, and as such they can effectively be used in modeling a large three-dimensional PSC structures. The introduced tendon model has been verified through a comparison of experimental data and numerical results. From the numerical analyses, the following conclusions were also obtained: (1) the tension stiffening and slip effects are more dominant in a PSC structures regardless of tendon type; (2) ignoring the tension effect clearly leads to underestimation of the stiffness and ultimate resisting capacity of PSC structures.

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