

Behavior of RC elements subjected to tri-directional loads

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ABSTRACT

The two-dimensional design and behavior of typical reinforced concrete (RC) structures has been extensively studied in the past several decades. Such design requires knowledge of the constitutive behavior of reinforced concrete elements subjected to a biaxial state of stress. These constitutive models were accurately derived from experimental test data on representative reinforced concrete panel elements. The true behavior of many large complex structures however, requires knowledge of the constitutive laws of RC elements subjected to a triaxial state of tension/compression stress. The goal of the proposed work is to develop new constitutive relations for RC elements subjected to a triaxial state of stress. To accomplish this task, large-scale tests on representative concrete panels need to be conducted. The University of Houston is equipped with a unique universal panel testing machine that was used for this purpose. The panel tester enhanced the understanding of the in-plane shear behavior of reinforced concrete elements. Recently, twenty additional hydraulic cylinders were mounted in the out of plane direction of the universal panel tester to facilitate testing concrete elements subjected to tri-directional shear stresses. An experimental program was conducted in order to evaluate the behavior of reinforced concrete elements subjected to tri-directional loads. The results of the tests revealed that the application of out-of-plane shear loads clearly reduced the in-plane shear strength. Based on these results, an interaction diagram between the three tri-directional shear stresses acting on an RC element was developed. An element-based approach was then adopted in which these constitutive relations were integrated using the finite element method to predict the overall behavior of the structure. The newly developed three-dimensional finite element model is based on fiber beam-column formulations, in which the new constitutive laws are accounted for at the fiber level. The presentation concludes with correlation studies of RC columns subjected to three-dimensional loads. These studies proved the newly developed model can provide reasonable estimates when compared to experimental results.

1. INTRODUCTION

The evaluation of the constitutive behavior of reinforced concrete under 3-dimensional loading conditions has not been properly established due to lack of proper experimental data. As such, analysis of complex 3-dimensional RC structures is currently being conducted using simplified assumptions that do not capture the true behavior. For instance, the presence of out-of-plane loads results in a different crack pattern that alters the main characteristics of the concrete material, an issue currently not accurately represented in concrete models. Specifically, the smeared stress-strain relation of concrete, the softening behavior of concrete under triaxial loading, and the post-cracking Poisson ratio are fundamentally different than their corresponding values under 2-dimensional loadings only. As a result, a rational design cannot be accurately developed. Rahal and Collins (1995) stated that 3D stresses on an element in a real structure cannot be determined easily with the current knowledge of cracked concrete behavior, while only 3D analysis can fully represent all aspects of the response of concrete structures. The lack of adequate test data on triaxial behavior of plain concrete still poses a major difficulty in developing proper constitutive relations under complex loading conditions (Barzegar and Maddipudi, 1997).

Several numerical models of concrete subjected to multi-directional loads exist in the literature. Vecchio and Selby (1991) developed a finite element model based on the Modified Compression Field Theory for 3D analysis of concrete structures with eight node regular hexahedral elements. Rahal and Collins (1995) developed a simplified 3D truss model also based on the Modified Compression Field Theory to analyze members subjected to 3D loads. Cocchi and Volpi (1996) developed a nonlinear model of RC members subjected to combined axial, shear, bending and torsional loads, based on an extension of the Diagonal Compression Field theory. Maekawa et al. (1997) used the concept of stress decomposition/re-composition, in which the 3D stress field is decomposed into 3 in-plane sub-spaces. Gregori et al. (2007) analyzed the section of a concrete column subjected to triaxial loads by subdividing it into several regions that are subjected to either uniaxial, biaxial, or triaxial state of stress.

The objective of this paper is to develop an efficient model for analysis of concrete members subjected to multi-directional loads. The model is based on fiber beam-column element formulations. Fiber-based beam-column models have the advantage of reduced computational cost and are therefore practical for large structural applications. A fiber model is developed by dividing each element into several sections along the member, and each section is then further divided into several fibers which represent either concrete and steel (Fig. 1). The strain in each fiber is calculated from the centroidal section strain and curvature with the help of the assumption that plane sections remain plane. Stresses and stiffnesses in the corresponding fibers are calculated from the fiber strain values. The constitutive relation of the section is derived by integration of the response of the fibers and the response of the element is derived by integration of the response of the sections along the length of the element. In this paper, two fiber beam-column elements are developed, a simplified model and a more sophisticated one. The first model is described next and is based on accounting for the effect of transverse out-of-plane loads on the in-plane behavior.

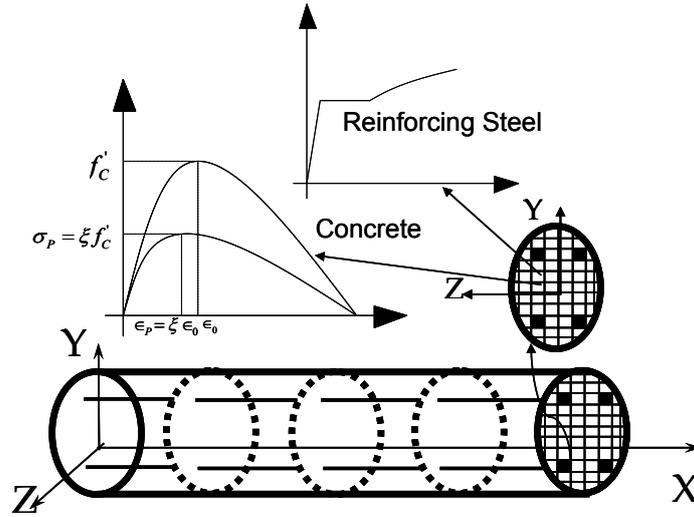


Fig. 1 Finite Element Model using Fiber Formulation

2. RC IN-PLANE MODEL ACCOUNTING FOR TRANSVERSE SHEAR

Labib et al. (2013a) tested experimentally a series of RC panels in order to investigate the behavior of elements subjected to combination of out-of-plane and in-plane shear stresses. The response of specimens OP0, OP1, OP2, OP3, OP4, OP5, OP6, OPR with different levels of out-of-plane loads is shown in Fig. 2. Labib et al. (2013a) found that out-of-plane transverse shear caused a reduction in the in-plane shear capacity of the panels. Based on his results, an interaction diagram between in-plane and out-of-plane shear loads was developed (Fig. 3).

The reduction in the in-plane shear strength resulted in an additional softening parameter for the concrete compression strut of the membrane element. Based on the test data, Labib et al. (2013) modified the softening coefficient ζ proposed by Hsu and Zhu (2002) with an additional parameter $f(\Gamma)$ to account for the effect of out-of-plane shear:

$$\zeta = \left(\frac{5.8}{\sqrt{f'_c} (MPa)} \leq 0.9 \right) \left(\frac{1}{\sqrt{1 + 400 \bar{\varepsilon}_1}} \right) \left(1 - \frac{|\beta|}{24} \right) f(\Gamma) \quad (1a)$$

$$\beta = 0.5 \tan^{-1} \left(\frac{\gamma_{12}}{\varepsilon_1 - \varepsilon_2} \right), \quad f(\Gamma) = 1 - \left(0.0178\Gamma - \frac{\Gamma^2}{10000} \right), \quad \Gamma = \frac{(\tau_o)_{total}}{\tau_{ou}} \times 100 \quad (1b)$$

where ζ is the softening coefficient, β is the deviation angle in degrees between the applied and principal stress angles, $\bar{\varepsilon}_1$ is the lateral tensile strain, f'_c is the concrete compressive strength, $\zeta f'_c$ is the softened concrete compressive strength, $\varepsilon_1, \varepsilon_2, \gamma_{12}$ represent the strain state in the principal directions, τ_{ou} is the out-of-plane shear

strength, and $(\tau_o)_{total}$ is the total applied shear stress in the out-of-plane direction.

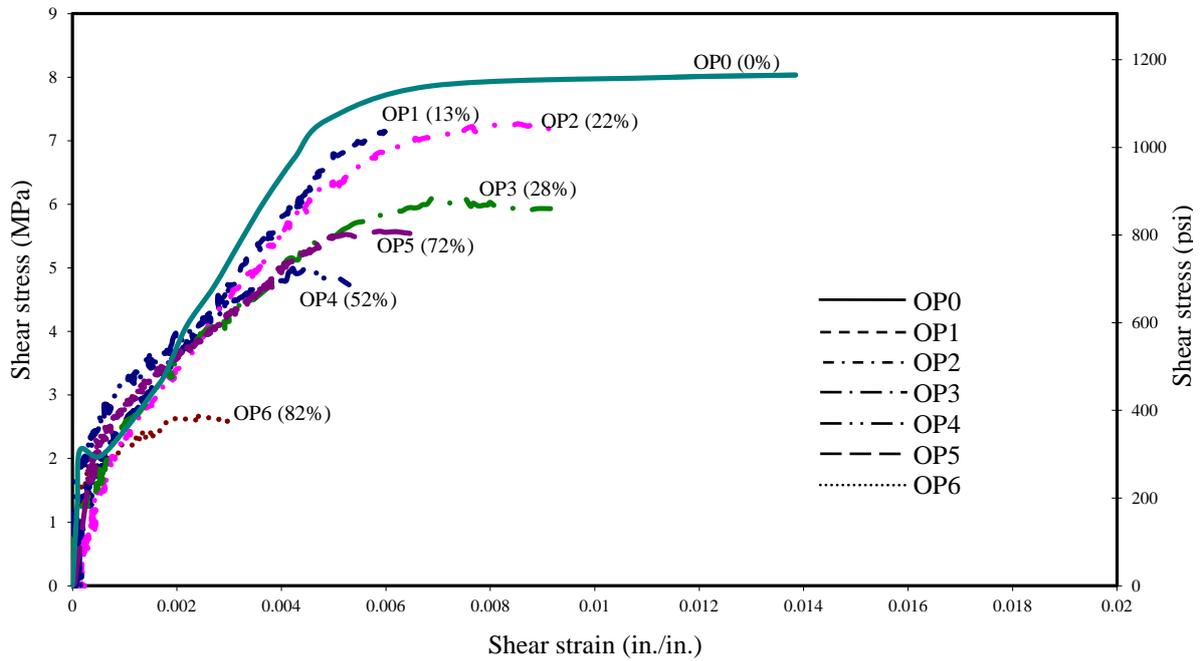


Fig. 2 Response of specimens with different levels of out-of-plane load

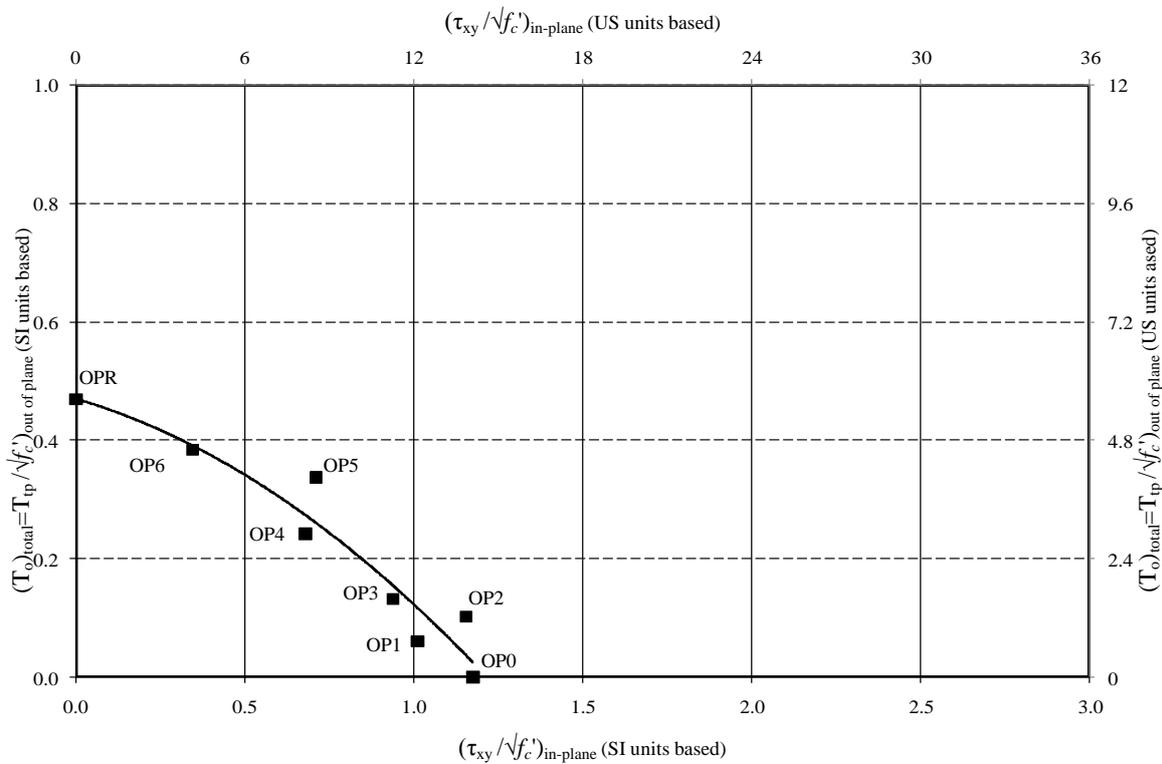


Fig. 3 Interaction diagram between in-plane and out-of-plane shear loads
To account for the effect of out-of-plane shear, the concrete softening coefficient

used in 2D shear-critical fiber elements (Mullapudi and Ayoub 2010) is modified to account for the effect of out-of-plane shear loads, through the use of Eq. (1) (Labib et al. 2013b). To validate this model, correlation studies with an experimentally tested RC column by Kobayashi et al. (1986) is conducted.

Kobayashi et al. (1986) performed an experimental program to obtain the shear interaction surface of columns subjected to bi-directional horizontal forces. The specimen cross section is 150 mm x 150 mm as shown in Fig. 4. The shear span to depth ratio (a/d) was 2.0. To increase the bending strength, the specimen was longitudinally reinforced with 8 deformed bars of 10.2 mm diameter. The lateral stirrups were 4.1 mm diameter and with a spacing of 50 mm. The yield stress for the longitudinal steel was 841.4 MPa and the concrete compressive strength was 23.4 MPa.

In the experiment, the horizontal force in one direction was kept constant, and the force in the orthogonal direction was increased until failure. The number associating the specimen name in this case shows the rate of shear to the shear strength in the first direction (BS00, BS02, BS04, BS06, BS08). All specimens exhibited a shear failure mode. At the top and bottom of the specimen, crushing of the concrete in the compression zone occurred at the maximum strength. Figs. 5-8 show the experimental and analytical load-deformation behavior of the specimens. The figures revealed the model was able to accurately capture the reduction in strength due to the presence of the out of plane load.

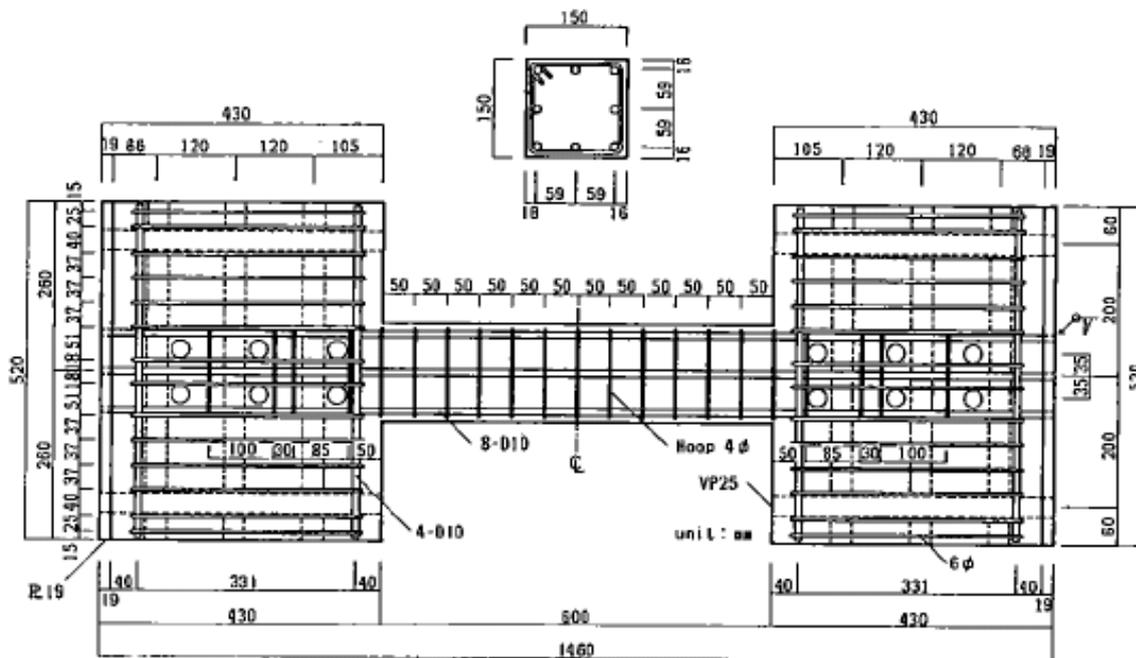


Fig. 4 Kobayashi et al. Column (1986)

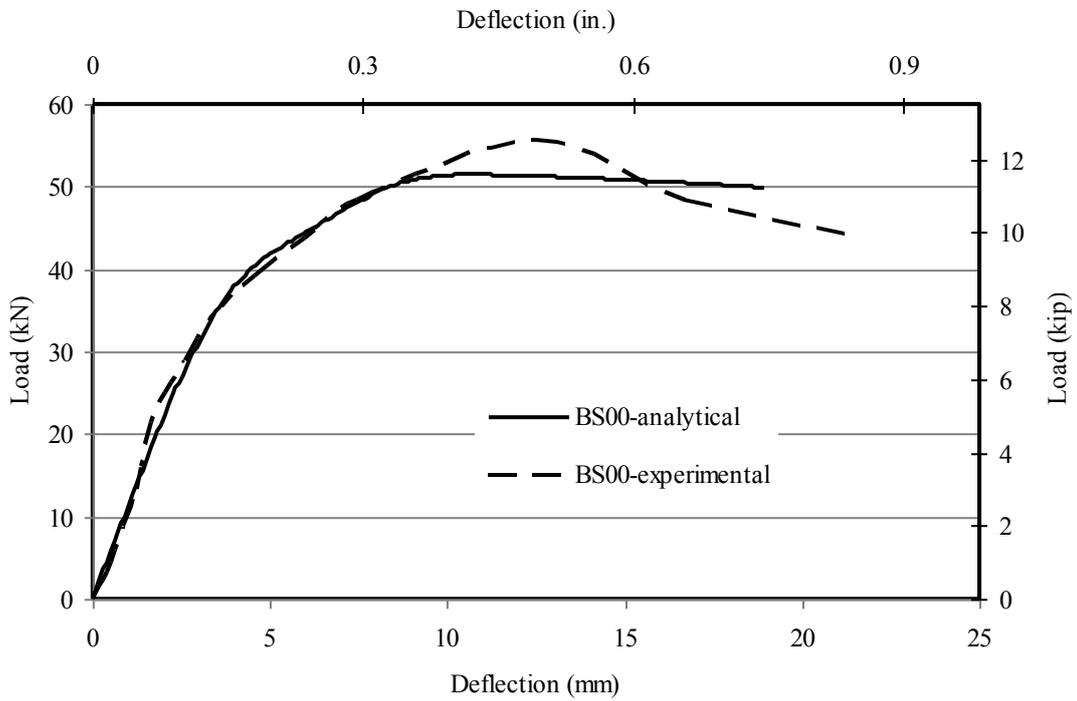


Fig. 5 Load-Deflection Behavior of Kobayashi et al. Specimen BS00

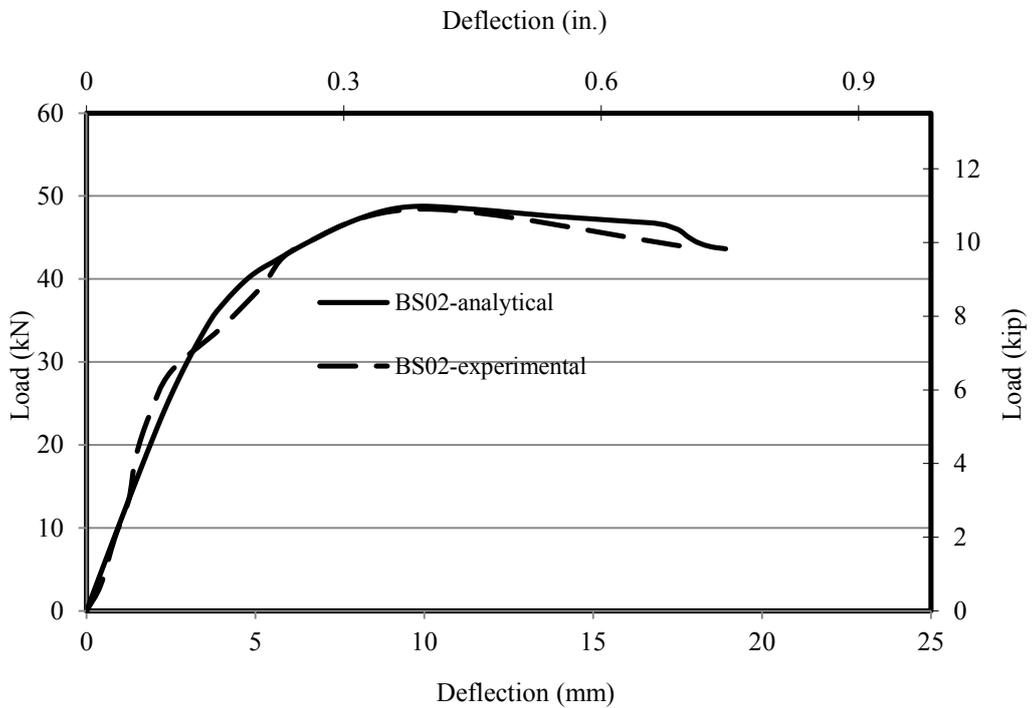


Fig. 6 Load-Deflection Behavior of Kobayashi et al. Specimen BS02

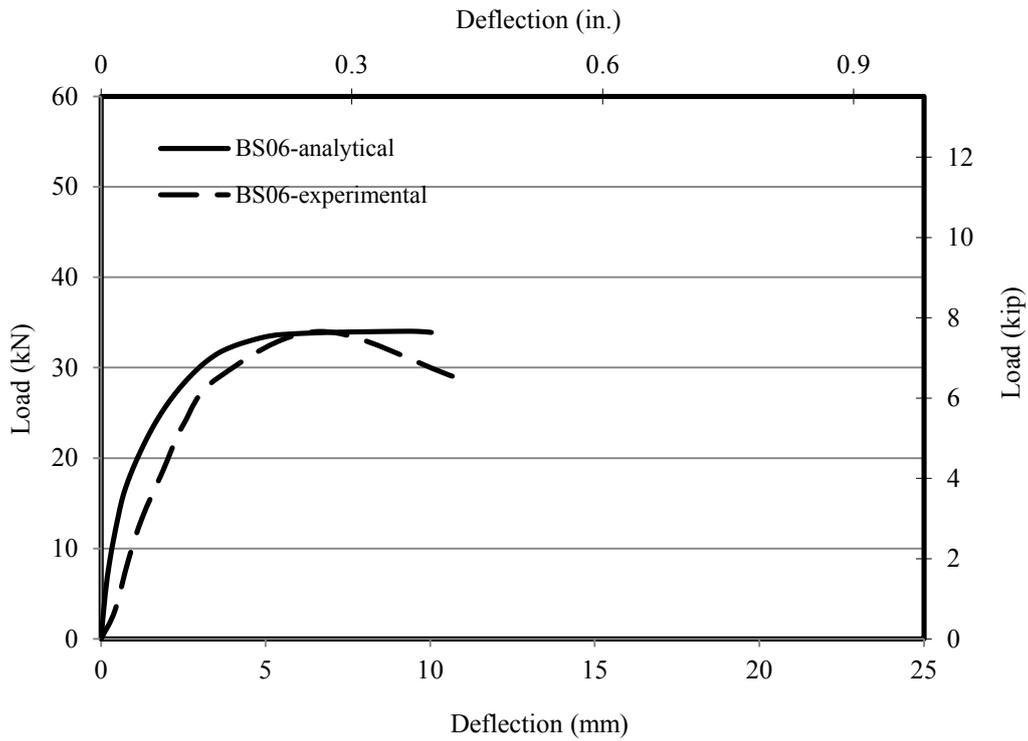


Fig. 7 Load-Deflection Behavior of Kobayashi et al. Specimen BS06

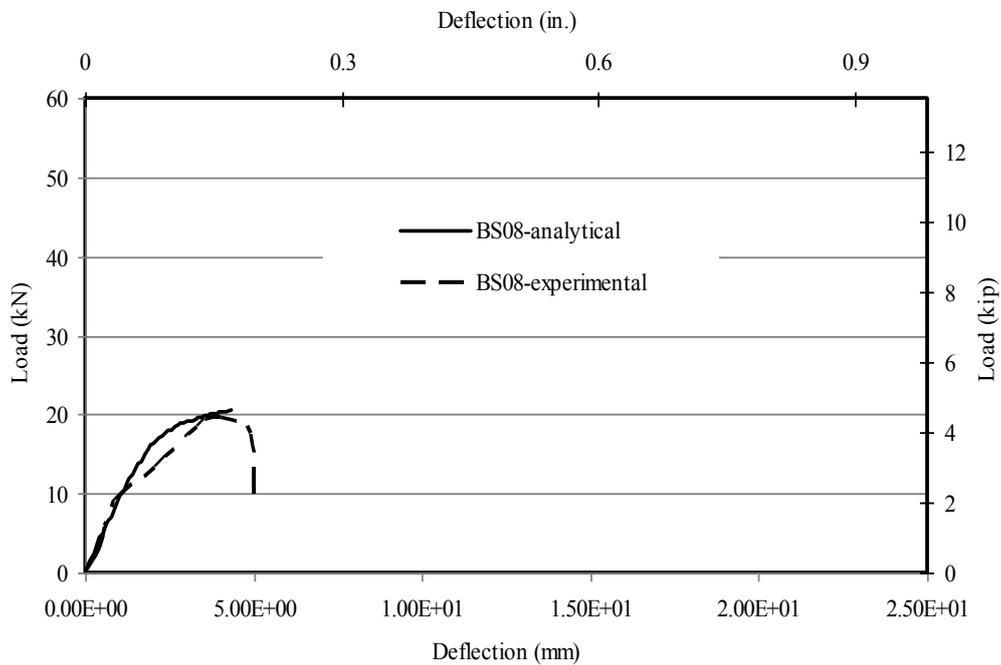


Fig. 8 Load-Deflection Behavior of Kobayashi et al. Specimen BS08

3. FULL 3-DIMENSIONAL RC FINITE ELEMENT MODEL

The previous model was based on 2D analysis in the presence of a fixed out-of-plane load. In order to develop a full 3D model, the response needs to be described by defining six degrees of freedom at each section (Fig. 9) consisting of three translations u_0, v_0, w_0 and three rotations $\theta_{x0}, \theta_{y0}, \theta_{z0}$ with the corresponding forces N, V, W and moments T, M_y, M_z respectively.

The kinematic assumption is used to relate the six displacements with the three translations at any point of the section according to the following equations:

$$\begin{aligned} u(x, y, z) &= u_0(x) + z\theta_{y0}(x) - y\theta_{z0}(x) \\ v(x, y, z) &= v_0(x) - z\theta_{x0}(x) \\ w(x, y, z) &= w_0(x) + y\theta_{x0}(x) \end{aligned} \quad (2)$$

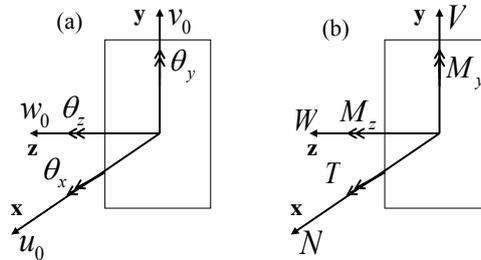


Fig. 9 (a) Section displacements (b) Section forces

Section deformations and forces are related to element deformations and forces through displacement shape functions (Mullapudi and Ayoub, 2009).

Eq. (1) is used to derive the three strain components $\varepsilon_x, \gamma_{xy}$ and γ_{xz} . The remaining strain vectors $\varepsilon_y, \varepsilon_z$ and γ_{yz} are determined based on equilibrium between the concrete and steel by satisfying the equations $\sigma_y = 0, \sigma_z = 0$, and $\sigma_{yz} = 0$ respectively. The process of deriving these equations is explained in details by Mullapudi and Ayoub (2013).

To establish the concrete constitutive laws in 3D, the values of the concrete uniaxial strains in principal directions 1, 2 and 3 have six conditions. The strength in one direction is affected by the strain state in the other directions. The concrete strain conditions are as follow:

1-TENSION, 2-COMPRESSION:

In this case, the equivalent uniaxial strain of concrete $\bar{\varepsilon}_1$ in principal direction 1 is in tension, and the equivalent uniaxial strain $\bar{\varepsilon}_2$ in principal direction 2 is in compression. Due to this condition, the uniaxial concrete stress σ_1^c in direction 1 is calculated from $\bar{\varepsilon}_1$, and is not a function of the perpendicular concrete strain $\bar{\varepsilon}_2$. The compressive strength

in principal direction 2 however, σ_2^c softens due to the tension in the orthogonal direction following Eq. (1).

1-TENSION, 2-TENSION:

The equivalent uniaxial strain of concrete $\bar{\varepsilon}_1$ in direction 1 is in tension, and the equivalent uniaxial strain $\bar{\varepsilon}_2$ of concrete in direction 2 is also in tension. In this case, the uniaxial concrete stress σ_1^c in direction 1 is evaluated from $\bar{\varepsilon}_1$, and σ_2^c in direction 2 is evaluated from $\bar{\varepsilon}_2$. Both σ_1^c and σ_2^c are functions of the orthogonal concrete strains $\bar{\varepsilon}_1$ and $\bar{\varepsilon}_2$ respectively.

1- COMPRESSION, 2- COMPRESSION:

The uniaxial strains of concrete in principal directions 1 and 2 are both in compression. The current research uses the Vecchio's (1992) simplified version of Kupfer et al. (1969) biaxial compression strength equation. The concrete compressive strength increase in one direction depends on the confining stress in the orthogonal direction. Concrete in compression exhibits lateral expansion and increase in the value of Poisson's ratio. An upper limit of 0.5 has been considered for Poisson's ratio.

The stress σ_3^c is calculated based on the same relationships as above through generation of another three conditions between principal directions 1 and 3 respectively.

To validate the finite element model, it was used to simulate the behavior of a reinforced concrete bridge columns (H/D(6)-T/M(α) and H/D(6)-T/M(0.2)) tested by Shanmugam et al. (2010) at the Missouri University of Science and Technology.

The test specimen had a 24 in. diameter and was 144 in. long. The reinforcement consisted of twelve No. 8 longitudinal bars, and No. 3 spiral transverse reinforcement spaced at 2.75 in.

A concrete compressive strength of $f'_c=3.9$ ksi and tensile strength of $f_t=0.5$ ksi are used to analyze the RC column. A yield stress $f_y=67.0$ ksi was used in the analysis for both the longitudinal and transverse reinforcement. The Young's modulus of concrete and steel are $E_c = 4000$ ksi and $E_s = 29000$ ksi respectively.

Column H/D(6)-T/M(α) is the control specimen and is tested under pure torsion. The experimental force-displacement result is shown in Fig. 10. The numerically-derived Torque-Twist plot matched well with the experimental results. The Torque-Twist curve is linear up to the formation of diagonal cracks, which is followed by a drop in the torsional stiffness. After cracking, the stirrups got engaged in resisting the torque and a significant amount of yielding was observed.

Column H/D(6)-T/M(0.2) has an applied torsion to moment (T/M) ratio of 0.2. Fig. 11 shows the comparison of the column behavior along with that of a column tested under pure bending conditions. Because of the induced torsion, the bending strength and stiffness were reduced considerably. The results confirmed the model was able to simulate that reduction in strength. Fig. 12 shows the longitudinal steel strain

distribution at 17 in. above the foundation. The figure shows that the experimental strain gauge reading matched with the analytical results and that the model captured the yielding rather well.

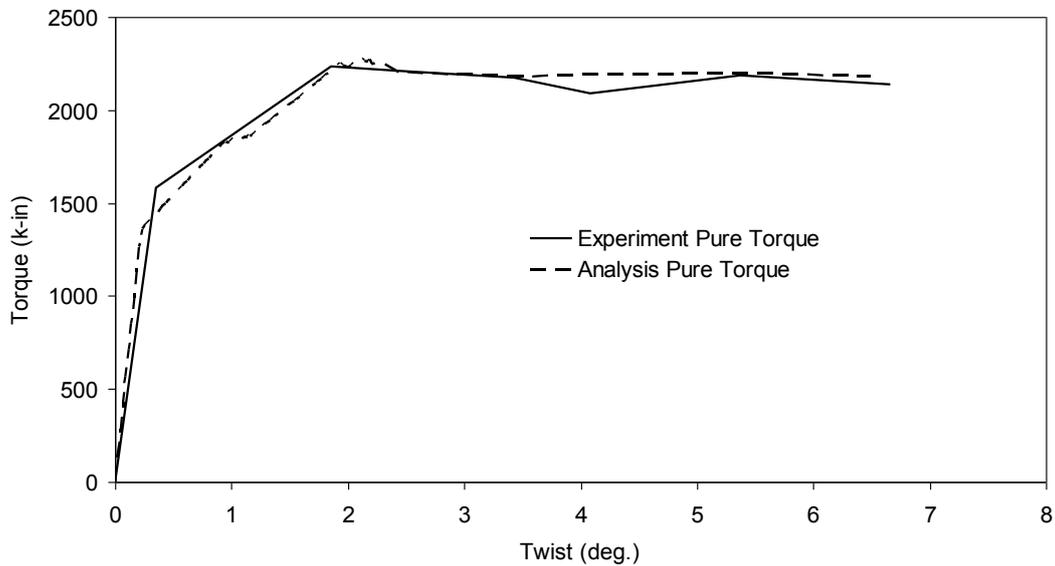


Fig. 10 Monotonic torque-twist curve of Shanmugam et al. specimen (2010)

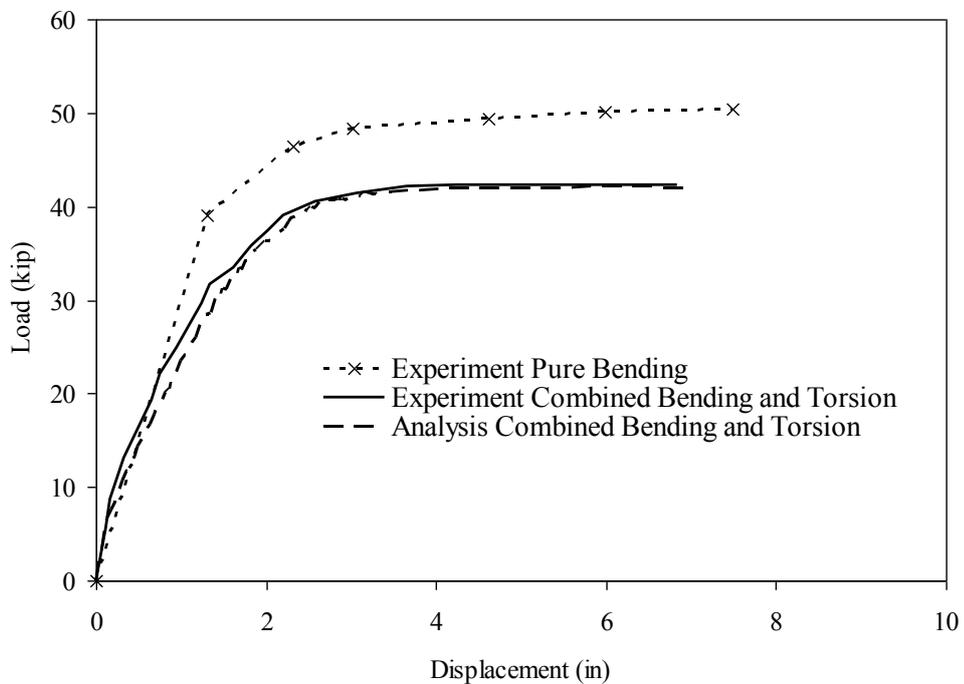


Fig. 11 Monotonic load-displacement curve of Shanmugam et al. specimen (2010)

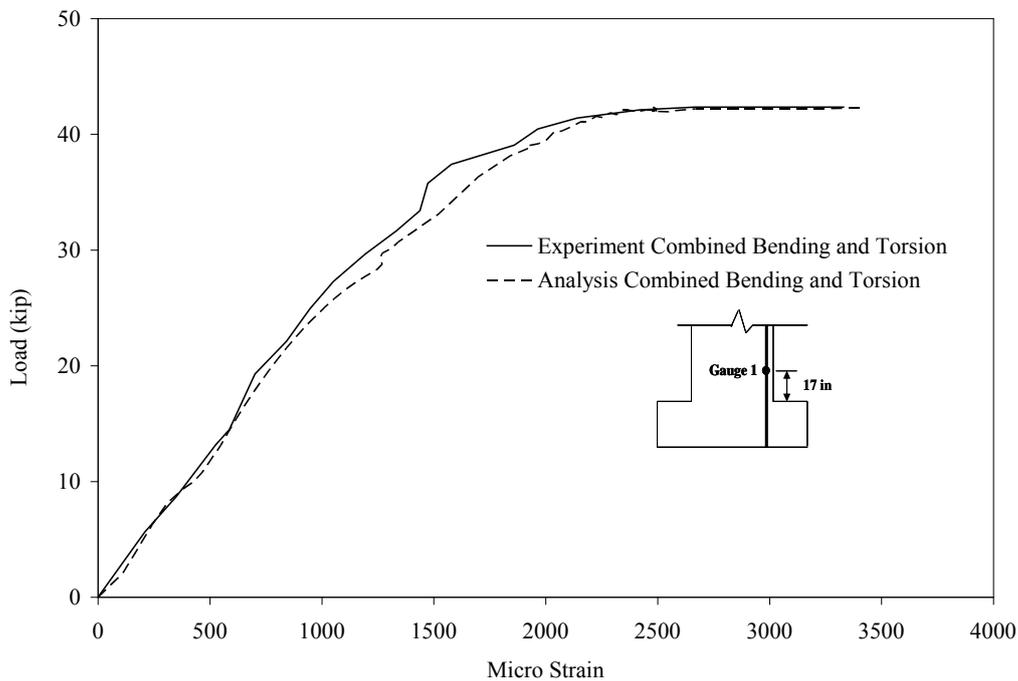


Fig. 12 Longitudinal strain distribution at Gauge 1 location of Shanmugam et al. specimen (2010)

4. CONCLUSIONS

This paper presents two new fiber-based beam-column models for numerical analysis of RC 3-dimensional structures. Both models are based on an extension of the Softened Membrane Model. The first model is essentially a 2D element that captures the effect of out-of-plane loads on the in-plane behavior, while the second model is a complete 3D element. Correlation studies with experimentally tested RC specimens confirmed both models can accurately simulate the local and global response of the member. The combined loading effect significantly affects both the flexural and torsional strength of RC sections, which was properly captured by the models. The study confirmed that the use of realistic constitutive laws in RC finite element models, which incorporates stress and strain softening of concrete, is essential to reaching a good agreement between experimental and analytical responses.

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