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Reduction of Bending Vibrations of Slender Beam Structures with Actuated Piezoelectric Patches and Shunt Circuits

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ABSTRACT

Open-loop control is an easy and well-known method to control the motion of mechanical systems. A special open-loop control technique is known under the notion shape control, when an elastic system follows a certain trajectory. In this paper we answer the question how to control the motion of flexible systems which are equipped with piezoelectric transducers as actuating control agencies and electrical resistances between their electrodes. As a special case, a slender beam is under consideration. Based on an extended version of the governing equations of motion of a beam within the framework of Bernoulli-Euler, we determine how to choose the configuration of the electric circuit and the values of its resistors and the proper value for the reference voltage, in order to annihilate deformations at several locations along the beam axis. A solution is derived by finding the influence functions of the external mechanical loads and of the piezoelectric patches. Mathematically, this corresponds to the solution of an inverse problem. The validity of the proposed method strictly holds in the static regime only, but it can be shown that structural vibrations can be approximately nullified as long as a certain non-dimensional parameters, containing the number and the capacitance of the piezoelectric patches, the sum of the resistors of the circuit and the excitation frequency, is small. Furthermore, an outlook for the practical realization of the proposed method is given.

1. INTRODUCTION

Piezoelectric transducers are nowadays commonly accepted elements for sensors and actuators. The use of piezoelectric transducers can be divided into three different fields. First the indirect piezoelectric effect is exploited if the motion of an elastic system is to be controlled (actuation). Second, states of the system are monitored and observed by taking advantage of the direct piezoelectric effect (sensor). Third, energy might be transferred from the mechanical to the electrical domain or vice versa (energy harvesting). For a review the reader is referred to (Mason 1981), (Chopra 2002) and (Crawley 1994).

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In the following of this contribution, we are interesting in beam- or plate type structures. Bonding piezoelectric transducers onto elastic frames or plates, the motion of flexible systems might be controlled by using advanced feedforward or feedback control algorithm (Irschik et al 1998), or their motion might be monitored by intelligently distributed the piezo-transducers (Krommer and Irschik 2013). Mechanical models for the physical interaction of piezoelectric patches bonded onto elastic beams have been developed by (Crawley 1987) and (Chandra 1993).

In this contribution we are concerned in controlling the displacement of beam-type structures, which are subjected to a certain load distribution. The motion of the elastic beam, which is modeled within the framework of Bernoulli-Euler, is also influenced by piezoelectric patches that are glued onto the surface. The electrodes are connected to resistances, whose other endings are either linked to ground, to the voltage source or to the electrodes of another transducer. By a proper design of the electric circuit (resistances) and the applied voltage source, one is able to annihilate the structural deformation at several locations (=sensor locations). The underlying theory for modeling beam-type structures within the geometrically linear domain, within the framework of the Bernoulli-Euler kinematics and the negligence of rotary inertia, is based on some preliminary works of our group (Krommer 2001), (Schoeftner and Irschik 2011) and (Schoeftner and Buchberger 2013). Based on these theories the influence functions of the external load and of the voltage acting across the electrodes of a piezoelectric patch are calculated. In order to cancel the lateral deformation at n locations, one has to control the voltage of n piezoelectric actuators by solving an inverse mechanical problem. The voltage of only one actuator may be prescribed in practical applications, therefore the voltages of the other actuators are obtained by connecting the other actuators to the shunts, which cause a well-defined voltage drop if current flows through them. Then the resulting voltage values are between the reference signal and the electric ground. As a simple example, a clamped-free beam is considered. On its surface eight piezoelectric patches are glued, and the developed shape control theory with resistively interconnected shunts is validated. Furthermore, this theory is also verified by an experimental setup: it is shown that bending vibrations at the first resonance are significantly reduced.

2. MODELING

In the following the governing equation of a laminated slender beam equipped with piezoelectric patch actuators is given. The width and the height of the patches are constant and read b_{p} and $h_{\mathrm{p}} = z_{2\mathrm{p}} - z_{1\mathrm{p}}$ (see the finite element model of the three-layer beam in Fig. 1). Within the framework of Bernoulli-Euler, the equation of motion reads

$$M_w \ddot{w}_0 - M_{,xx} = q_z, \qquad (1)$$

where w_0, M, q_z are the lateral deformation, the bending moment and the external force load (see Schoeftner and Buchberger 2013). The mass per unit length is given by M_w, which is the cross- section area multiplied by the density of each layer.

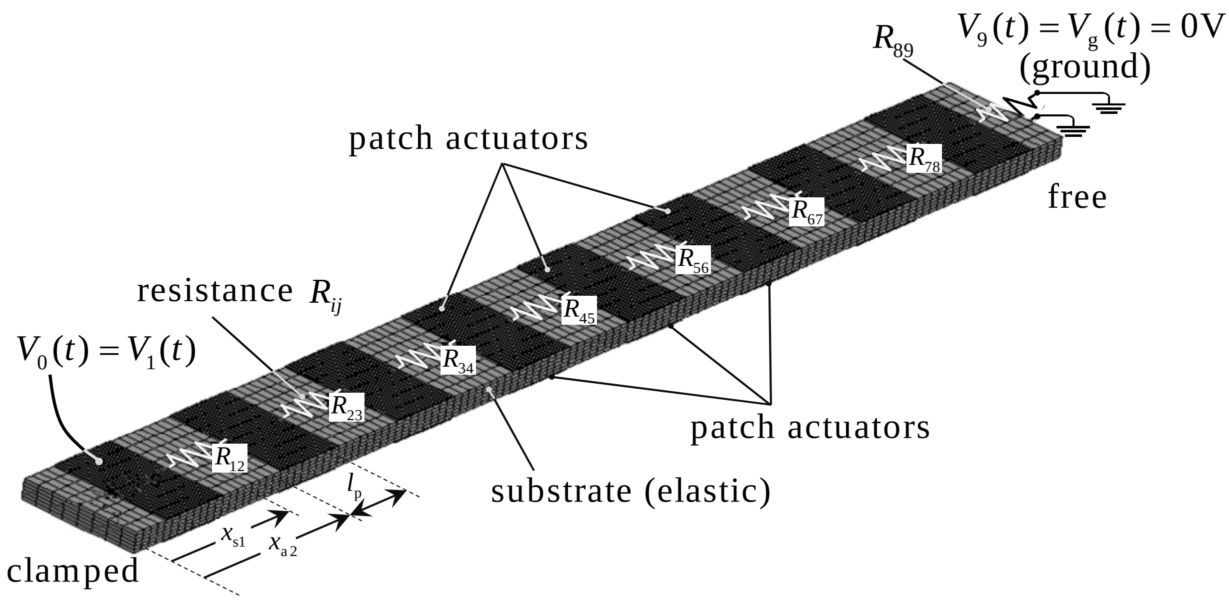


Fig. 1 Cantilever beam equipped with piezoelectric patch actuator (inner electrodes are grounded, external electrodes connected to a shunted circuit)

For the bending moment the relation $M(x) = -K_M(x)w_{0,xx}(x) + 2\tilde{e}_{31}z_{mp}b_pV_n$ holds, where K_M is the bending stiffness. The second term is the electrical part of the bending moment that is only present, when patch $\#n$ is located at position x, i.e. $x_{an} < x < x_{an} + l_p = \hat{x}_{an}$ holds. At $x = x_{an}$ and at $x = x_{an} + l_p$, the continuity relations holds

$$
\begin{aligned}
&w_0(x_{an}^-) = w_0(x_{an}^+) \\
&w_{0,x}(x_{an}^-) = w_{0,x}(x_{an}^+) \\
&-K_M(x_{an}^-)w_{0,xx}(x_{an}^-) = -K_M(x_{an}^+)w_{0,xx}(x_{an}^+) + 2\tilde{e}_{31}z_{mp}b_pV_n \\
&-K_M(x_{an}^-)w_{0,xxx}(x_{an}^-) = -K_M(x_{an}^+)w_{0,xxx}(x_{an}^+)
\end{aligned} \quad (2)
$$

and

$$
\begin{aligned}
&w_0(\hat{x}_{an}^-) = w_0(\hat{x}_{an}^+) \\
&w_{0,x}(\hat{x}_{an}^-) = w_{0,x}(\hat{x}_{an}^+) \\
&-K_M(\hat{x}_{an}^-)w_{0,xx}(\hat{x}_{an}^-) + 2\tilde{e}_{31}z_{mp}b_pV_n = -K_M(\hat{x}_{an}^+)w_{0,xx}(\hat{x}_{an}^+) \\
&-K_M(\hat{x}_{an}^-)w_{0,xxx}(\hat{x}_{an}^-) = -K_M(\hat{x}_{an}^+)w_{0,xxx}(\hat{x}_{an}^+).
\end{aligned} \quad (3)
$$

In words, the displacement and its spatial derivative, the bending moment and the shear force remain continuous. Since the voltage V_n cannot be prescribed independently in our case for each patch, one may derive additional differential equations from Kirchhoff's voltage and current rule, which couple the mechanical (the displacement $w_0(x)$) to the electrical domain (the patch voltage V_n)

$$V_n - R_{n\,n+1}\, i_{n\,n+1} = V_{n+1} \qquad n = \{0,1,\ldots,N-1,N\} \tag{4}$$

and

$$\begin{aligned} i_{n\,n+1} - i_{n-1\,n} &= \dot{Q}_n = \dot{Q}_{n,\mathrm{elast}} - C\dot{V}_n \qquad n = \{1,2,\ldots,N\} \\ Q_{n,\mathrm{elast}} &= -\tilde{e}_{31} z_{\mathrm{mp}} b_{\mathrm{p}} \left(w_{0,x}(\hat{x}_{\mathrm{a}n}) - w_{0,x}(x_{\mathrm{a}n}) \right). \end{aligned} \tag{5}$$

Eq.(4) states that there is a voltage drop $V_{n+1} - V_n$ in the direction of the electric current $i_{n\,n+1}$, which is proportional to the resistance $R_{n\,n+1}$. Eq.(5) states that the difference of out- and ingoing current is equal to the current produced by the transducers.

3. SHAPE CONTROL – ANNIHILATION OF BEAM VIBRATIONS

The solution for problems governed by Eqs.(1)-(5) is given by

$$w_0(x) = G_F(x) F_0 + \sum_{n=1}^{N} G_{Vn}(x) V_n \,, \tag{6}$$

where N is the number of piezoelectric patches used for the actuation. The expression $G_F(x)$ and $G_{Vn}(x)$ describe the deformation of the beam at location x due to a unit force load $F_0 = 1\mathrm{N}$ and $V_n = 1\mathrm{V}$. They are called influence functions. The form in Eq.(6) holds, if the external load and the voltage is a sinusoidal excitation. Nevertheless, the influence functions can be easily computed in the static case for beam-type structures, when the time-derivatives in Eqs.(1)-(5) vanish. For the sake of simplicity, the reader is referred to (Schoeftner et al 2013b) for the solution of the static influence functions for various mechanical boundary conditions.

Assuming that one intends to annihilate the displacement at N different locations, i.e. $w_0(x_{\mathrm{s}1}) = 0$, one may set up N equations which read in matrix notation

$$\underline{W}_0 = \underline{G}_F F_0 + \underline{\underline{G}}_V \underline{V} \quad \text{with} \quad \underline{W}_0^T = \left[w_0(x_{\mathrm{s}1}),\ldots,w_0(x_{\mathrm{s}N}) \right] \qquad \underline{V}^T = \left[V_1, V_2, \ldots, V_N \right]. \tag{7}$$

The matrices of the influence functions $\underline{G}_F, \underline{\underline{G}}_V$ contain the impact of the load and of all piezoelectric patches on the deformation of the systems. Demanding that $\underline{W}_0 = \underline{0}$ should hold, one solves (7) for the voltage vectors

$$\underline{V} = -\underline{\underline{G}}_V^{-1} \underline{G}_F F_0 \,. \tag{8}$$

Since we are mainly interested in the practical realization of this idea, we skip the mathematical question, if and under which circumstances the inverse $\underline{\underline{G}}_V$ of exists. Further investigation on this important topic will be postponed and treated in the future.

Since the necessary voltage vector is given by (8), one may easily calculate the resistances of the electric circuit and also its architecture (note due to the negligence of the time-derivative, the current through each resistor is equal $i = i_{01} = \ldots = i_{\mathrm{N\,N+1}}$). The prescribed voltage signal provided by the voltage supply is the maximum of the necessary patch voltages $V_0 = \max\{V_1, V_2, \ldots, V_N\}$. Assuming a non-monotonically increasing or decreasing sequence for the patch voltages, the architecture of the resistive circuits and its relative resistance values, which connect the electrodes of the patches, is determined by the descending order of the patch voltages

$$\frac{V_i - V_j}{V_j - V_k} = \frac{R_{ij}}{R_{jk}} \quad \text{with} \quad V_i > V_j > V_k. \tag{9}$$

4. EXAMPLE

In this section, we show the correctness of our derived theory by a cantilever, which is equipped with $N = 8$ piezoelectric patches. Consequently, the displacement at eight sensor locations will be annihilated. The length and the width of the beam is $l = 0.5\mathrm{m}$ and $b_{\mathrm{p}} = 0.05\mathrm{m}$. The material of the substrate is aluminum and that of the piezoelectric transducers is PZT-5A. The length of each patch is $l_{\mathrm{p}} = 0.03\mathrm{m}$, the height of the substrate and of the patch $h_s = 0.008\mathrm{m}$ and $h_{\mathrm{p}} = 0.0004\mathrm{m}$, respectively. The tip-load $F_0 = 0.027\mathrm{N}$ is realized by a solenoid attached at the free end of the beam (see Fig. 5). The additional mass of the solenoid has to be taken into account, reducing the first eigenfrequency from $26.5\mathrm{Hz}$ (without consideration of the solenoid mass) to $18.8\mathrm{Hz}$. For the given external load, one calculates the necessary voltage vector from Eq.(8)

$$\underline{V}^T = [V_1, V_2, \ldots, V_N] = [6.76, 5.56, 4.99, 3.80, 3.24, 2.04, 1.48, 0.28]\mathrm{V}. \tag{10}$$

Obviously the voltage provided by the voltage supply is $V_0 = 6.76\mathrm{V}$. Substituting Eq.(10) into Eq.(9) one finds the proper ratio of the resistances for the electric circuit. Demanding the total resistance to be $R_{\mathrm{tot}} = 20{,}000\Omega$, one calculates the values given in Table 1.

Table 1 Values for the resistances of the circuit ($R_{\mathrm{tot}} = 20{,}000\Omega$)

resistance (unit)	value	resistance (unit)	value	resistance (unit)	value
$R_{01}(\Omega)$	0	$R_{12}(\Omega)$	3552	$R_{23}(\Omega)$	1674
$R_{34}(\Omega)$	3536	$R_{45}(\Omega)$	1656	$R_{56}(\Omega)$	3548
$R_{67}(\Omega)$	1660	$R_{78}(\Omega)$	3544	$R_{89}(\Omega)$	830

Results for the static displacement are shown Fig.2. One can see that the deflection of the tip-loaded beam and of the voltage-loaded beam (with $V_0 = 6.76\mathrm{V}$ and the resistances from Table 1) are approximately equal, but changed in sign. Superposing both results, one finds the shape control results. It becomes clear from the figure below, that the displacement is exactly annihilated, as desired, at the 8 sensor locations (x-mark symbol).

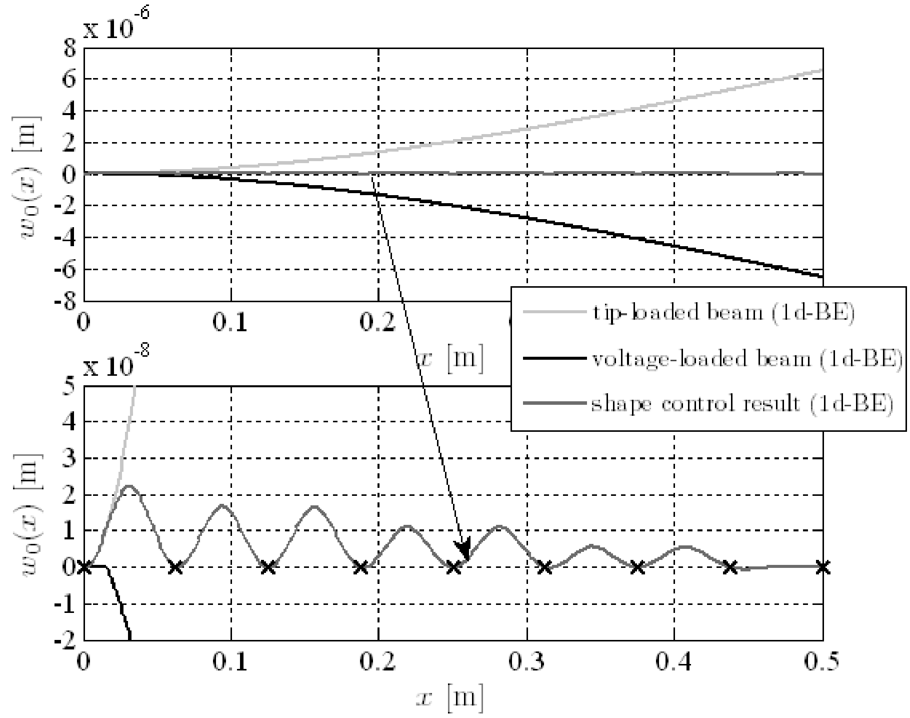


Fig. 2 Static deflection $w_0(x)$ of a tip-loaded beam (light gray-only tip-force, black-only voltage load, dark gray-shape controlled beam)

Next, we investigate if our shape control method can be also applied to time-harmonic loads. The displacement at $x = 0.5l$ and at the free end $x = l$ are shown in Fig.3. For low frequencies $f \to 0\text{Hz}$, our feedforward control method works perfectly. For higher frequencies ($f > 75\text{Hz}$), our method fails. At the first resonance $f_1 = 18.8\text{Hz}$ the tip-displacement is reduced from 0.24mm to only 0.011mm (-95%). For higher frequencies or for the second eigenfrequency, vibrations may be even amplified. It can be shown that the shape control method is a well-working feedforward control technique in the dynamic regime, as long as the non-dimensional parameter

$$\pi = 8CR_{\text{total}}\omega \tag{11}$$

is not much higher than one, see (Schoeftner and Buchberger 2013b). This means that the dynamic electrical equations (4) and (5) can be replaced by the static ones (disregard the time-derivatives, the expression $\tau = 8CR_{\text{total}}$ is the time-constant of the electrical part). In our example one calculates $\pi \approx 1.52$ with $f = 18.8\text{Hz}$. It is noted that in the simulation model, the clamped end is not assumed to be perfectly rigid. The boundary stiffness has been adjusted (see the configuration Fig.4), so that the first eigenfrequencies of the simulation model and of the experiment match.

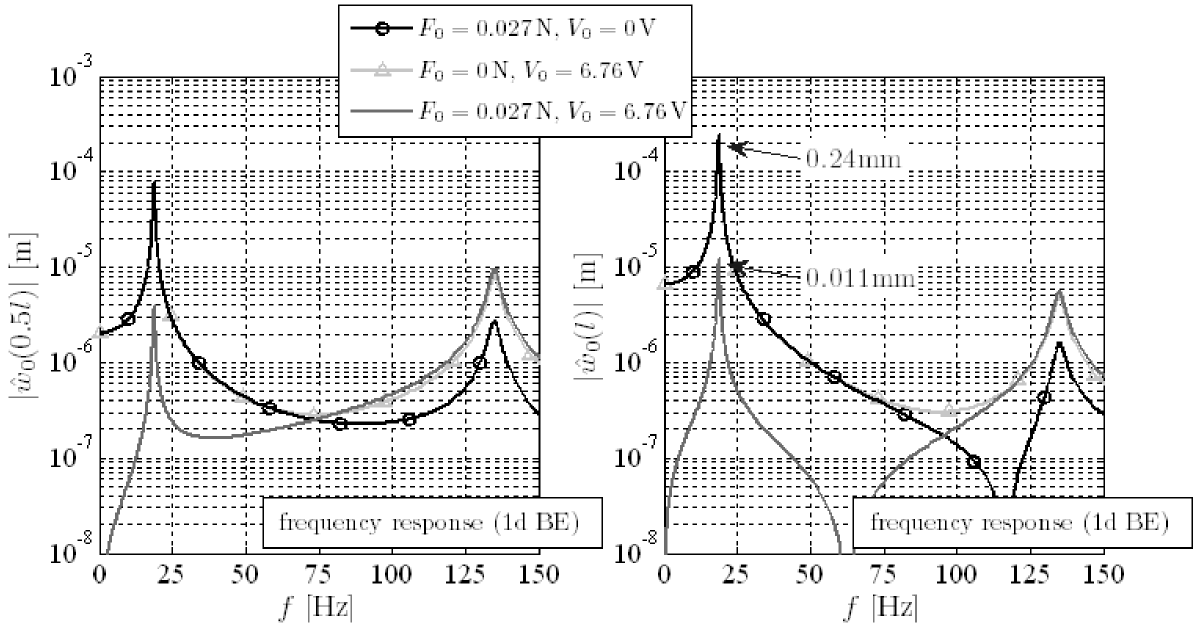
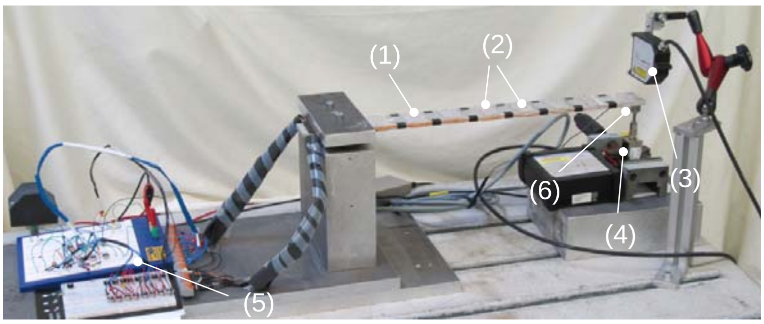


Fig. 3 Frequency response $\hat{w}_0(0.5l)$ and $\hat{w}_0(l)$ for a resistive circuit with $R_{\mathrm{tot}} = 20{,}000\Omega$ when the cantilever beam is excited by the tip-force excitation (black), by the voltage actuation (light gray) and by both the voltage and the tip-force excitation (shape control-gray)

Our last task is to verify the shape control method. Fig.4 shows the experimental setup. One recognizes the beam (1) with its 16 piezoelectric patches (2) attached on the upper and lower sides. All the internal electrodes are linked to ground. Both external electrodes of the first, the second, the third,… patch on the lower and upper sides are kept at the same potential, but both are linked via resistances (5) as indicated in Fig.1. A laser displacement sensor (3) measures the tip-displacement.



(1) cantilever beam with patches
(2) piezoelectric patches
(3) laser displacement sensor
(4) solenoid (shaker for tip-load)
(5) resistive circuit
(6) force sensor

Fig. 4 Experimental setup for verifying the shape control theory with resistive circuits

Fig. 5 compares measurement results with simulation results, when the beam is excited at the first resonance $f_1 = 18.8\mathrm{Hz}$ (solenoid force is measured, see (6) in Fig.4). After 10s the controller is activated. The simulation shows that the harmonic displacement is strongly suppressed from $0.24\mathrm{mm}$ to $0.011\mathrm{mm}$. The experiment shows a similar behavior: vibrations with amplitude of 0.27mm are also attenuated 0.016mm (-94%), if the shape control method is turned on. A visualization of the results with the laser scanning vibrometer is shown in Fig.6.

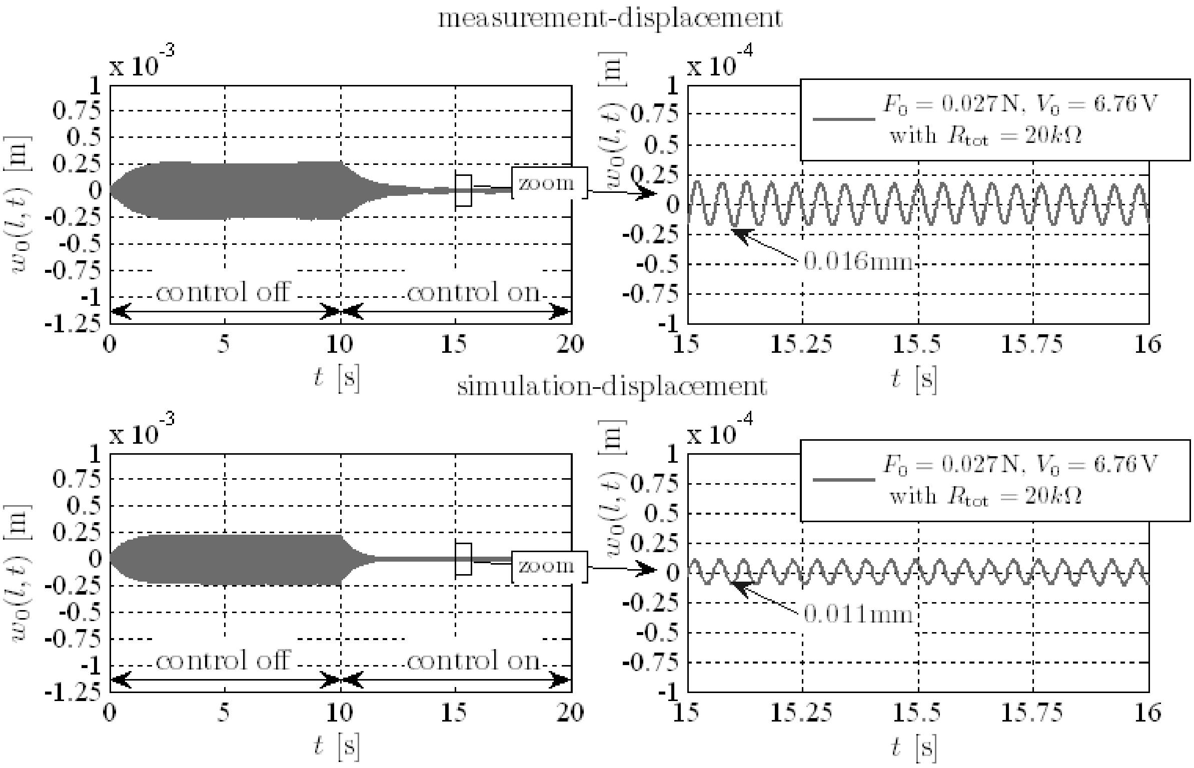


Fig. 5 Comparison of the transient response when the shape control method is activated at $t = 10\text{s}$ (above: measurement results, below: simulation results)

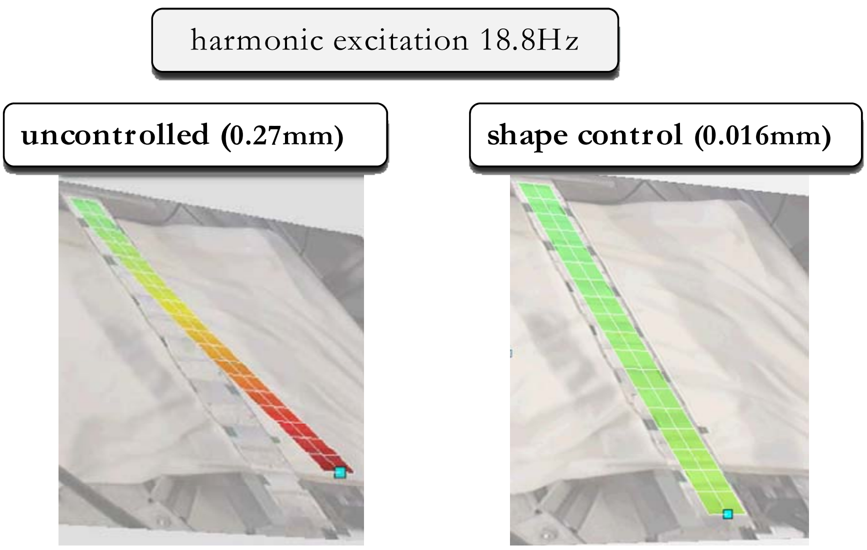


Fig. 6 Laser scanning vibrometer-visualization of the measured transient response with and without shape control

CONCLUSION

In this contribution a new method for shape control of beams is introduced. With the help of resistive shunts it is shown that it is possible to annihilate the deformation at certain locations along the beam axis, if only the voltage of one piezoelectric patch is be prescribed. The resulting voltage distribution for the remaining patches depends on the design of the resistive circuit. First, the basic equations for modeling of piezoelastic structures on beam-level are given. Then it is shown how to determine the proper voltage level for the actuation of the piezo-patches and to calculate the optimal values for the resistors of the circuit. This method is called shape control with resistive circuits. Then the theory is verified by a simple example, a cantilever beam is excited by the electromagnetic force of a solenoid at the free end. It is shown that the shape control method works perfectly in the static regime, but also approximately annihilates time-harmonic vibrations. Finally, the theory is experimentally validated. Results for the experiment and for the simulation model are compared, if the system is excited at the first resonance. It is shown that when the system is controlled (i.e. the shape control method is activated) vibrations can be significantly reduced (simulation: -95%, experiment: -94%).

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