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On the numerical simulation of structural pounding in three dimensions

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ABSTRACT

This paper presents a simple and efficient methodology for the numerical simulation of earthquake-induced pounding of adjacent buildings in three dimensions (3D). In the frames of this methodology, a new force-based impact model, which is the main focus of the current paper, is proposed. The proposed impact model does not require an 'a priori' determination of the contact points, since the location of the acting point of the impact forces is determined dynamically, based on the arbitrary position of the colliding structures at the time of impact. Furthermore, the frictional forces between the colliding structures are considered, while the geometry at the vicinity of impact is also taken into account for the calculation of the impact forces. Finally, the advantages of the new methodology, in contrast to other common approaches in structural pounding, are identified and discussed.

1. INTRODUCTION

The problem of earthquake-induced pounding of structures has been extensively examined through several types of numerical studies during the past few decades due to several observations from real seismic events and the great attention that this issue has received. Actually some pounding occurrences have been identified in reconnaissance reports of recent earthquakes (EERI 2009; Cole et al 2012). In their great majority, the performed numerical studies simulate structures in two dimensions (2D), while the limited studies that used three-dimensional (3D) simulations did not conduct extensive parametric investigation of the pounding effects. It is evident that, although some basic effects of pounding on the dynamic response of structures can be identified using 2D simulations, other important factors that are directly associated to the spatial movement of the structures cannot be taken into account due to this simplification. For example, the use of both orthogonal seismic components of an excitation, which is enabled only in the case of 3D simulations, has a significant effect

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on the overall response of the simulated building. Furthermore, any eccentricities, irregularities or asymmetries in plan, which may excite the torsional vibration of a building and increase the possibility of impacts during earthquakes, are essential parameters that can be taken into account only through a 3D analysis.

However, in contrast to the case of 2D simulations, the numerical modeling of impacts in 3D poses some significant difficulties due to the inherent complexities of the problem. For example, while in 2D simulations the impacts are considered to be central, i.e. without frictional forces developed in a tangential direction and one-dimensional (1D) impact models are used, in 3D simulations the tangential impact forces and frictional forces have to be also taken into account. Moreover, considering the spatial movement of the colliding structures, the exact location of the impact in plan and the contact geometry play a significant role on the computed response during pounding.

As mentioned above, very limited research studies have been conducted considering 3D earthquake-induced structural pounding, apparently due to the involved complexities and the consequent computational cost. In general, the numerical studies that refer to the 3D simulation of pounding of either buildings or bridge girders can be categorized in two major groups regarding the methodology of simulating impacts. The first one includes the studies where impact is simulated using the ‘stereomechanical’, also known as impulse-based, approach (Papadrakakis et al 1996; Leibovich et al 1996; Liolios 2000; Mouzakis and Papadrakakis 2004), while the second group involves research studies that use force-based, also known as ‘penalty’, methods (Goyal et al 1994; Fujino et al 2000; Zhu et al 2002; Gong and Hao 2005; Wei et al 2009; Guo et al 2011).

According to the former approach, it is assumed that the duration of an impact is zero and instantaneous changes of the velocities are computed based on the preservation of momentum, taking also into account the coefficient of restitution, which is defined as the ratio of the relative velocity between the colliding bodies after and before impact. However, this approach cannot handle multiple impact incidences at any time instance and does not provide the impact forces acting on the colliding bodies at the time of impact.

On the other hand, the ‘penalty’ methods allow a minor interpenetration between the colliding bodies, which is used together with an impact spring stiffness to assess the impact force at each time-step. In contrast with the impulse-based approach, these methods allow the efficient simulation of dynamic systems with the possibility of multiple impacts occurring at the same time, due to the fact that the computed impact forces are superimposed in the formulation of the corresponding equations of motion. This considerable advantage of the force-based impact models renders them more suitable for simulating pounding of buildings in series and, thus, the methodology that is presented herein follows this approach.

In the frames of the current research, a simple but efficient methodology was needed, which would enable us to effectively perform large number of dynamic analyses of buildings in 3D, considering pounding, in order to parametrically investigate the effects of this phenomenon on the overall structural response.

2. STRUCTURAL MODELING

In the proposed methodology the buildings are considered as three-dimensional multi-degree-of-freedom (MDOF) systems with shear-type behavior for their stories in the horizontal direction. The slab at each floor level is represented by a rigid diaphragm that is mathematically simulated as a convex polygon for specific reasons that are associated with the proposed impact model, which is described in the next section of this paper. The masses are considered to be lumped at the floor levels, having three dynamic degrees of freedom (DOFs), i.e. two translational, parallel to the horizontal global axes, and one rotational along the vertical axis (Fig. 1). Therefore, considering ground excitations only in the horizontal directions, which are the most important to be taken into account, no displacement occurs in the vertical direction, since the translational dynamic DOF of the structure refer only to horizontal planes. Accordingly, it is assumed that the impact forces occur only in the horizontal planes. Both linear elastic and non-linear inelastic behavior can be considered for the columns of the simulated buildings.

The differential equations of each system are directly integrated using the Central Difference Method (CDM), computing the displacements at time $t+\Delta t$. At each time-step of the analysis, the algorithm performs a check for detecting potential impacts, based on the deformed position of each floor diaphragm in space. When an impact is detected, the resulting impact forces are computed according to the impact model and the methodology that is presented in the next section.

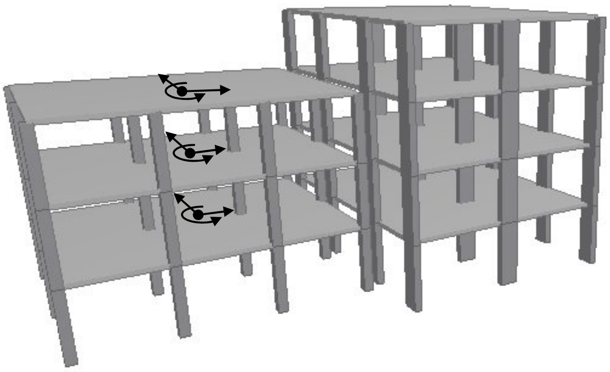

Fig. 1 Three-dimensional modeling of adjacent buildings.

Considering the specific needs and demands of this numerical problem, as well as the limited flexibility and efficiency of the available general-purpose commercial software applications, a specialized software application has been developed in order to implement the proposed methodology. In particular, the developed software application enables the effective and efficient performance of 3D numerical simulations and parametric analyses of buildings with contact detection capabilities, according to the above structural modeling assumptions and the following impact model. Modern object-oriented design and programming approaches are utilized, while the Java programming language is employed in the development of the software application, taking into account the significant advantages that these technologies offer.

2. PROPOSED METHODOLOGY FOR MODELING IMPACTS IN 3D

The force-based impact models that are available in the literature calculate the impact force as a function of the interpenetration depth between the colliding bodies. However, this approach has a significant drawback in the case of 3D impact modeling. Specifically, this approach assumes that the calculated impact force depends only on the indentation and not the geometry at the contact region. This would have been true only if the later was taken into account for the calculation of impact stiffness at each time-step based on the deformed position of the colliding structures, but at least for the aforementioned studies that is not the case. For example, considering the two cases of impact between the two rigid plates presented in Fig. 2, assuming a constant impact stiffness parameter and taking into account only the interpenetration depth, the impact force would be the same for the two cases. However, in reality, it is evident that the impact force in the first case (Case A) would be greater than in the second case (Case B), since the overlapping area is greater. Based on this observation, it is considered to be more appropriate to take into account the area of the overlapping region, instead of the interpenetration depth, in the estimation of the impact force, since it is widely accepted that the impact stiffness depends on the geometry of the contact region (Goldsmith 1960).

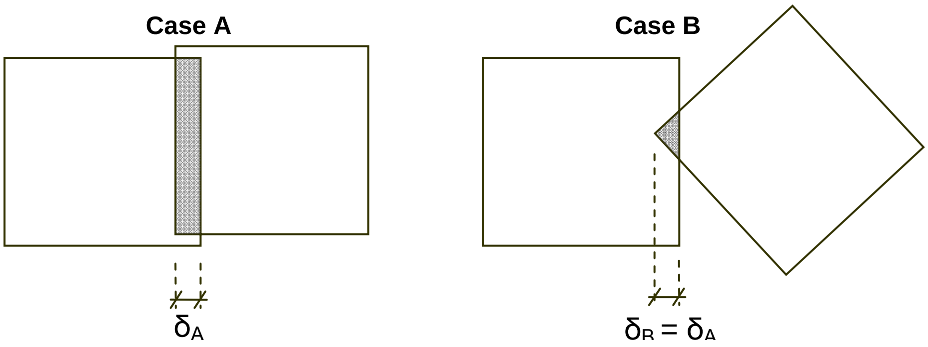

Fig. 2 Two different cases of impact geometry between two rigid plates, where the maximum indentation is the same.

Fig. 3 represents schematically the theory of the proposed impact model. In particular, when two slabs that are modeled as polygons come in contact they form an overlapping region which in the most of the cases is either a triangle (Fig. 3(a)) or a quadrilateral (Fig. 3(b)). The algorithm uses the geometry of the overlapping region at each time-step in order to determine: (i) the location of the action point of the impact forces, (ii) the direction of the impact forces and (iii) the magnitude of the impact forces.

2.1 Location of the impact forces

The location of the action point of the impact forces is a very important issue in the case of simulating poundings of buildings in 3D. While in the case of 1D impact models the location of the resultant force vector clearly is at the point of contact, in the case where contact conditions exist over a finite surface area on both bodies, the exact point where the contact force should be applied is not so obvious. For the specific problem of modeling impact between rigid diaphragms, the contact forces in the normal and tangential directions are assumed to act on the centroid of the overlapping region, and applied at the corresponding position of the bodies in contact.

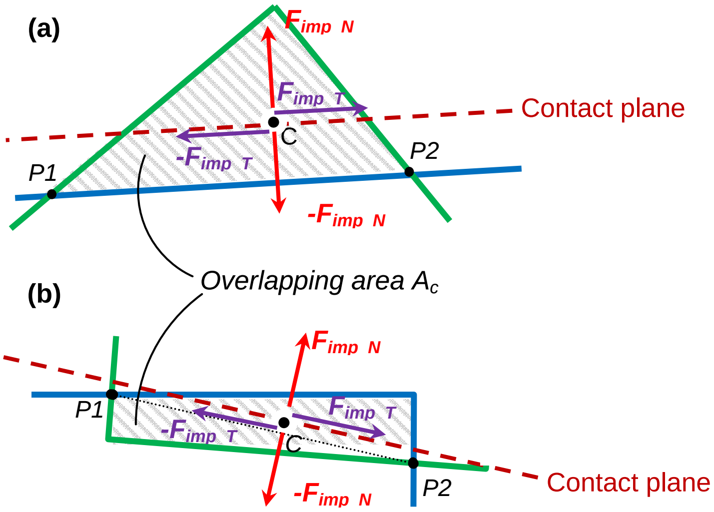

Fig. 3 Schematic representation of the proposed impact model. (a) The overlapping region forms a triangle; (b) the overlapping region is a quadrilateral.

2.2 Direction of the impact forces (contact plane)

For the proposed impact model, it is also necessary to determine the normal and tangential contact directions in order to be able to apply the corresponding normal and tangential impact forces as well as the Coulomb's Law of Friction. Therefore, the contact plane is assumed to be parallel to the line that is determined by the two nodes *P1* and *P2* at the intersections between the boundaries of the two colliding bodies (see Fig. 3). Since the impact forces will be applied at the centroid *C* of the overlapping region, the contact plane is passing through that point. The methodology that is used defines a normal and a tangential direction in such a way to ensure that no directional jump occurs, between two sequential time-steps of the analysis. Specifically, the contact plane smoothly changes direction, while the overlapping contact area changes from triangular to quadrilateral and vice-versa.

2.3 Estimation of the magnitude of the impact forces

According to the basic concepts of the widely known ‘penalty’ method, contact springs are automatically formed when two rigid bodies are in contact in order to calculate the resulting impact force that pushes them apart. In the current case, the stiffness of the impact spring is used along with the area (A_c) of the overlapping region to calculate the elastic impact force. Since the impact response differs between the normal and tangential directions, two different equations are needed to calculate the normal and tangential impact forces, respectively, at each iteration time-step:

$$^{(t+\Delta t)}F_{imp,N} = {}^{(t)}A_c \cdot k_{imp,N} \tag{1}$$

$$^{(t+\Delta t)}F_{imp,T} = {}^{(t)}F_{imp,T} + {}^{(t)}u_{rel,T} \cdot k_{imp,T} \tag{2}$$

The indices N and T in the above equations indicate the normal and the tangential directions, respectively, as indicated in Fig. 3. The coefficients $k_{imp,N}$ (in kN/m^2) and $k_{imp,T}$ (in kN/m) are the impact stiffness coefficients in the normal and tangential directions, respectively. A_c is the area of the contact region and $u_{rel,T}$ is the relative displacement along the tangential direction. The time instance ($t+\Delta t$) represents the current time-step, since the CDM is used for the numerical integration, while (t) represents the previous time-step.

The Coulomb friction law is used to limit the tangential impact force below a certain magnitude, taking into account the magnitude of the normal impact force and the static and kinetic friction coefficients of the contact surface:

$$\begin{array}{l} \text{If } \left|{}^{(t+\Delta t)}F_{imp,T}\right| \le \left|{}^{(t+\Delta t)}F_{imp,N} \cdot \mu_s\right| \quad \rightarrow \quad \text{use Equation (2)} \\ \text{If } \left|{}^{(t+\Delta t)}F_{imp,T}\right| > \left|{}^{(t+\Delta t)}F_{imp,N} \cdot \mu_s\right| \quad \rightarrow \quad {}^{(t+\Delta t)}F_{imp,T} = {}^{(t+\Delta t)}F_{imp,N} \cdot \mu_k \end{array} \tag{3}$$

where μ_s and μ_k are the static and kinetic, respectively, coefficients of friction, which are applied in the ‘stick’ and ‘slide’ mode of contact, respectively.

2.4 Impact damping

As in the case of 1D impact models, a viscous dashpot can be used, in parallel with the impact spring, to represent the dissipation of energy during impact (e.g. thermal and acoustic energy) and, based on the relative velocity of the bodies in contact, provide the corresponding damping impact force. Therefore, the corresponding total impact forces in the normal and tangential directions, respectively, taking into account the impact damping, are given by the following expressions:

$$F_{imp,N} = F_{imp,N}^{elastic} + F_{imp,N}^{damp} \tag{4}$$

$$F_{imp,T} = F_{imp,T}^{elastic} + F_{imp,T}^{damp} \tag{5}$$

Since damping is assumed to be velocity-proportional, the magnitude of the damping force in each impact direction (normal and tangential) is proportional to the corresponding relative velocity of the bodies that are in contact:

$$^{(t+\Delta t)}F_{imp,N}^{damp} = {}^{(t)}\dot{u}_{rel,N} \cdot c_{imp,N} \tag{6}$$

$$^{(t+\Delta t)}F_{imp,T}^{damp} = {}^{(t)}\dot{u}_{rel,T} \cdot c_{imp,T} \tag{7}$$

where $\dot{u}_{rel,N}$, $\dot{u}_{rel,T}$, $c_{imp,N}$ and $c_{imp,T}$ are the relative velocities and the damping coefficients in the normal and tangential directions, respectively. The values of the impact damping coefficients can be approximated in the same manner as in the case of 1D impact models (Anagnostopoulos 2004; Muthukumar and DesRoches 2006; Jankowski 2006).

It is evident that the proposed methodology cannot be implemented using a common commercial analysis software application due to its special modeling assumptions. Therefore, as aforementioned, a specialized software application has been developed in order to implement the proposed impact model, following a complete methodology for simulating earthquake-induced pounding of structures in 3D.

3. ADVANTAGES AND CAPABILITIES OF THE PROPOSED IMPACT MODEL

3.1 Location of the acting point of the impact forces

Usually, the various commercial finite element analysis (FEM) software applications use typical 1D contact elements (or gap-elements) to model impact situations, such as pounding, between structures. In particular, those are two-joint non-linear gap elements with user-specified gap width and impact spring stiffness. However, these contact elements have some significant drawbacks compared to the aforesaid proposed methodology. Primarily, the position of the two joints of a contact element determines the acting points of the arising impact forces that are calculated after the closure of the specified gap, regardless of the deformed position of the colliding structures. For better understanding, let’s consider the case of two symmetrical, non-eccentric buildings (see Fig. 4), excited by a bidirectional seismic action, resulting to pounding. In case of using 1D contact (gap) elements, those would be applied between the joints A1-A2 and B1-B2 (Fig. 4(a)), determining ‘a priori’ the acting points of the computed impact forces. Therefore, in the case of a pounding incidence, such as the one shown in Fig. 4(b), no torsional vibration would be exhibited by the colliding buildings, since two equal impact forces would be applied at the two corners of each structure. On the other hand, in the case of using the proposed impact model, the computed impact will be applied at the centroid of the overlapping region *C* (Fig. 4(b)), resulting to the torsional vibration of the two buildings due to eccentric pounding, which is the most realistic case.

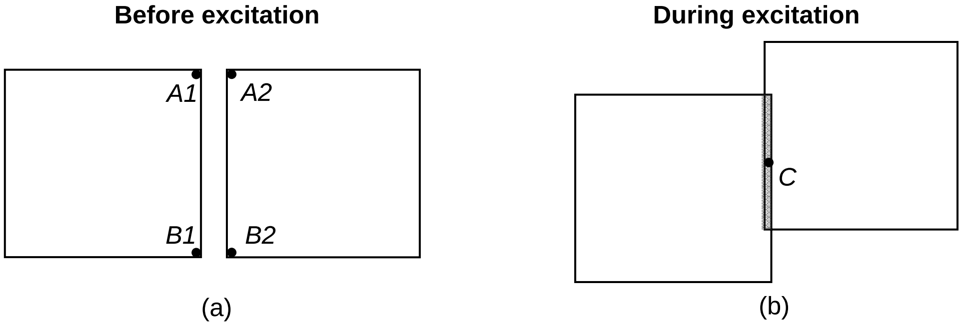

Fig. 4 Schematic example of the position, in plan, of two adjacent buildings before and during an excitation.

3.2 Friction and tangential impact forces

As mentioned in previous, 1D impact models do not provide the ability to take into account the tangential impact force and, therefore, the friction between the colliding structures. Consequently, for the same reasons, the common gap-elements, used by several commercial FEM programs do not simulate friction, which in many cases is an important factor in 3D simulations involving pounding and should be taken into account. For example the friction between the structures may excite or increase the torsional vibration of the adjacent buildings during pounding.

3.3 Geometry at the vicinity of impact

The deformed position of the colliding structures and consequently the geometry at the vicinity of impact plays also a significant role in the estimation of the impact forces. One aspect of this issue has been explained in a previous paragraph, using Fig. 2 as an example, demonstrating the importance of using the area of the overlapping region in the calculation of impact forces, instead of the interpenetration depth, which is used in the case of 1D impact models.

Another issue that has to do with the geometry of impact is the relation between the magnitude of the impact force and the interpenetration depth. For example, if we consider the geometry of Case A in Fig. 1 the impact force should be linearly depended by the interpenetration depth, while in Case B the relation should be non-linear. If those two cases were part of the same analysis, then a common 1D impact model would't be appropriate to represent in sufficient accuracy both situations, since the relation of the magnitude of the impact force and the indentation is pre-determined by the type (linear or non-linear) of the impact model. However, the proposed impact model can capture both cases in a more realistic manner by relating the magnitude of the impact force with the area of the overlapping region.

4. CONCLUSIONS

A new methodology for simulating earthquake induced pounding of buildings that are modeled as 3D-MDOF systems has been presented. Some significant disadvantages of the available impact models in the literature have led us to propose a new approach to the numerical problem of impact modeling. Specifically, following the ‘penalty’ method the impact forces are calculated based on the area of the overlapping region, instead of the overlapping depth that is usually used in previous similar studies. This assumption takes into account the geometry at the vicinity of impact, a factor that is omitted when using only the indentation depth, and determines the relation (linear or non-linear) between impact force and indentation. Another advantage of the proposed methodology is that the location of impacts is not known ‘a priori’, since the impact detection is based on the spatially arbitrary location of each of the rigid diaphragms that are at the same level. Therefore, there is no need for contact elements to be applied at certain locations of each diaphragm, which actually omit the location and the direction of the impact forces since in their majority such contact elements have only one dimension.

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