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Application of response surface methodology: Optimum mix design of concrete with slag as coarse aggregate

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Abstract

The optimum mix design of slag in concrete is one of the best ways in identifying which mixture will yield high compressive strength without compromising good behavior and significance of each variable in every compressive strength test when a certain percentage of slag is being mixed in concrete. To determine the mix design that will yield the optimum compressive concrete strength, response surface methodology (RSM) is explorer in this study.

RSM is an optimization tool explored in the study because it interprets experimental results even in a non-linear response surface manner and it provides sufficient experimental interpretation as part of the conclusive result [1]. It has modern optimization features that can be useful in most complicated experimental design. Its most important applications are in the field where variables have potential significance in predicted system behavior called response. The combination of factorial application and modern experimental design has outstanding contribution in optimizing experimental procedures in a reduced number of studies and the response is easy to interpret.

RSM was used on the data obtained from laboratory experiments conducted by the researchers. The experiments conducted include the influencing factors: slag percentage (50%, 75%, and 100%), curing period (14 days, 21 days, and 28 days), and types of cement (1P, I, and IP), and the interaction effects of these factors in compressive strength test are analyzed in this paper through response surface methodology. The responses of each specimen have showed significant increase in attained strength with respect to the control specimens.

Keywords: concrete, slag, optimization, Response Surface Methodology, aggregate, Design of Experiment

1. Introduction

The construction industries are growing faster. The use of concrete as a construction material is in great demand, thus requiring the industry to make a wide choice in the selection of its building components. In order to meet the increasing demand on the performance of these components, it is necessary to adapt waste material recycling to compensate the lack of natural resources and obtain alternative ways conserving the environment.

Concrete is the most widely used construction material all over the world. The raw materials needed are available in most parts of the world and it does not require complex or expensive equipment to make concrete. But due to its popular and in demand supply as construction material, some of its component should have an alternative source aside from the conventional one.

Many engineering researchers and studies have been developed in using locally available materials for construction due to these economic problems [2,3]. In an attempt to undergo development in construction materials technology that provide economical building materials with good quality and standard, studies about by-product waste, such as slag, is done as an alternative material for construction, both horizontal and vertical purposes.

Slag is a by-product waste material from steel manufacturers. It is often being recycled, treated, and disposed. Since there are lots of studies about slag's applications as substitute to various construction materials, manufacturers these days rarely dispose this waste; instead sell it in a low cost.

Improper disposal of slag is the main problem in the industrial world. Its large amount is produced by steel makers yearly and has been dumped unsuitably without proper implementation and remediation measures. It was then found out that slag is one of the hazardous elements in the environment if not disposed appropriately. Due to its increasing demand, disposal of slag as solid waste material is a serious problem.

In addition, another environmental problem involved in the field is the production of coarse aggregates. In the absence of timber in construction, demand for concrete increases. As expected, demand for aggregates increases also. Although gravel is the conventional coarse aggregate being mixed to produce concrete, its highly increasing cost in the construction market and geologic and geomorphic implications on gravel supply are some of the problems nowadays. There was a forecast made by Dunne et. al [4] that the demand of gravel as one of the construction materials could lead to scarcity of supply in every country and importation would eventually take place. The authors also discussed the constraints of the supply not only to gravel but also to sand in the river channels, which is the very well known source for these materials.

2. Methodology

Response Surface Methodology and Design of the Experiment, specifically, Box - Behnken Design were used as framework of the study.

2.1 Factors and Levels

The low level values of the numerical factors are the lowest possible and acceptable level in each of the factors. 50% slag content and 14-day curing period could already attain concrete strength. The maximum levels where tested and were proven to achieve the desired quality for each concrete combination. Therefore exceeding these values will result to undesirable compressive strength.

Table 1 Values of each factor per level

	Factors	Low Level	Middle Level	High Level
Numerical Factors	Slag Content (%)	50	75	100
	Curing Period (days)	14	21	28
Categorical Factor	Cement (type)	1P	I	IP

Table 2 Result of the experiment

Specimen	**Specimen Code**	**Curing Period**	**Compressive Strength (tons)**		
			1	**2**	**3**
1P 50%	1P50 14	14	27.1	29.0	31.4
1P 50%	1P50 21	21	59.5	55.3	55.0
1P 50%	1P50 28	28	40.7	47.2	41.6
1P 75%	1P75 14	14	18.0	23.6	20.1
1P 75%	1P75 21	21	52.1	58.3	54.0
1P 75%	1P75 28	28	54.8	54.2	55.4
1P 100%	1P100 14	14	25.7	26.4	19.8
1P 100%	1P100 21	21	52.5	44.9	47.6
1P 100%	1P100 28	28	46.7	52.8	53.4
I 50%	I50 14	14	26.0	20.9	25.4
I 50%	I50 21	21	55.2	51.8	60.2
I 50%	I50 28	28	60.3	61.8	60.9
I 75%	I75 14	14	22.4	21.0	18.6
I 75%	I75 21	21	58.0	56.9	52.2
I 75%	I75 28	28	63.1	58.4	48.1
I 100%	I100 14	14	46.0	42.0	46.8
I 100%	I100 21	21	49.9	50.3	50.1
I 100%	I100 28	28	50.3	53.2	55.0
IP 50%	IP50 14	14	34.7	36.4	30.3
IP 50%	IP50 21	21	41.3	43.7	38.4
IP 50%	IP50 28	28	42.7	43.8	43.1
IP 75%	IP75 14	14	35.3	44.0	39.2
IP 75%	IP75 21	21	47.5	48.7	44.5
IP 75%	IP75 28	28	52.8	51.9	53.6
IP 100%	IP100 14	14	32.3	46.2	30.1
IP 100%	IP100 21	21	46.1	46.8	49.3
IP 100%	IP100 28	28	50.8	43.2	45.9

2.2 Sampling Procedures and Runs

The performance of the different factors was evaluated independently using the runs randomly ordered by Design Expert software for Response Surface Design.

2.3 Experimental Procedures

To minimize the bleeding of the concrete in the experiment, 2 inches slump height was used for all combinations as the optimum slump. All batches were produced under good weather and clean environment to avoid impurities. The specimen preparation and testing standards are all in accordance with ASTM and AASHTO.

Curing and inspection for produced concrete were done right after the mixing process. Universal Testing Machine was used to measure the final compressive strength of each concrete mixes. Compression test was done right after the respective curing periods of each concrete mixes.

3. Analysis

The experiment produced 81 concrete mixes at various levels of the three factors (Table 2).

3.1 Response Surface Formula

$$Formula = f \sim ct + SO\ (days,\ slagcont),\ data = cx \qquad (1)$$

Equation (1) was used in analyzing response – surface model components. The second – order response surface (SO) was used to capture the curvature immediately. Each type of cement has different analysis to relate the interaction between the slag content and curing period (Tables 3 to 5).

Table 3 Analysis of Type 1P cement using Equation (1)

	Estimate	Std. Error	t-value	Pr(>|t|)	
Intercept	53.55278	12.40093	4.3184	0.0001675	***
cement type	0.62222	6.65238	0.0935	0.9261227	
days	9.02	0.99786	9.0394	6.197^{-10}	***
slag content	1.76667	3.79244	0.4658	0.6228104	
days : slag content	3.45167	0.81475	4.2365	0.0002098	***
days^2	-17.72917	1.57775	-11.237	4.393^{-12}	***
slag content^2	-1.35556	1.82183	-0.7441	0.4628249	

Table 4 Analysis of Type IP cement using Equation (1)

	Estimate	Std. Error	t-value	Pr(>|t|)	
Intercept	61.54722	7.46878	8.2406	4.375^{-09}	***
cement type	-16.85556	8.48397	-1.9868	0.0564705	.
days	5.405	1.2726	4.2472	0.0002037	***
slag content	12.00556	4.83661	2.4822	0.0190923	*
days : slag content	0.14833	1.03907	0.1428	0.8874702	
days^2	-7.97083	2.01215	-3.9614	0.0004443	***
slag content^2	-4.99444	2.32343	-2.1496	0.0400625	*

Table 5 Analysis of Type I cement using Equation (1)

	Estimate	Std. Error	t-value	Pr(>|t|)	
Intercept	37.95556	10.5543	3.5962	0.001183	**
cement type	17.98889	11.9888	1.5005	0.144304	
days	11.66333	1.79833	6.4856	4.242^{-07}	***
slag content	-6.46111	6.83471	-0.9453	0.352297	
days : slag content	-0.47667	1.46833	-0.3246	0.74779	
days^2	-13.5	2.84341	-4.7478	5.111^{-05}	***
slag content^2	3.81667	3.28329	1.1625	0.254529	

Significant codes: 0 ‘***’ | 0.001 ‘**’ | 0.01 ‘*’ | 0.05 ‘.’ | 0.1 ‘’ | 1

3.2 Analysis of Variance

The Analysis of Variance (ANOVA) indicates how the three factors affect the strength of concrete. The analysis includes the first – order response surface (FO), two – way interaction (TWI), and pure quadratic (PQ) of each concrete mixes. Tables 6 to 8 are the respective analysis of 3 types of cement.

Table 6 ANOVA Table of Type 1P cement

	Dof	Sum Square	Mean Square	F - value	Pr (>F)
Cement type	1	71.38	71.38	3.5843	0.068347
FO (days, slag content)	2	2787.41	1393.70	69.9849	7.97^{-12}
TWI (days, slag content)	1	357.42	357.42	17.9479	0.0002098
PQ (days, slag content)	2	2525.61	1262.81	63.4119	2.58^{-11}
Residuals	29	577.52	19.91		
Lack of fit	5	344.50	68.90	7.0966	0.0003358
Pure error	24	233.01	9.71		

Table 7 ANOVA Table of Type IP cement

	Dof	Sum Square	Mean Square	F - value	Pr (>F)
Cement type	1	98.80	98.80	3.0505	0.0912996
FO (days, slag content)	2	793.72	396.86	12.2525	0.000139
TWI (days, slag content)	1	0.66	0.66	0.0204	0.8874702
PQ (days, slag content)	2	657.94	328.97	10.1565	0.0004538
Residuals	29	939.31	32.39		
Lack of fit	5	602.34	120.47	8.5799	9.019^{-05}
Pure error	24	336.97	14.04		

Table 8 ANOVA Table of Type I cement

	Dof	Sum Square	Mean Square	F - value	Pr (>F)
Cement type	1	391.02	391.02	6.0455	0.021505
FO (days, slag content)	2	3157.47	1578.73	24.4084	6.084^{-07}
TWI (days, slag content)	1	6.82	6.82	0.1054	0.7477903
PQ (days, slag content)	2	1545.40	772.70	11.9465	0.0001642
Residuals	29	1875.72	64.68		
Lack of fit	5	1588.79	317.76	26.5789	4.738^{-09}
Pure error	24	286.93	11.96		

The quadratic model of each cement type has F-value of 3.5843 for 1P, 3.0505 for IP, and 6.0455 for I. This implies that the cement type is not significant aside from cement I (values of Pr > F less than 0.0500 indicate model terms are significant) and the lack of fit tests are all significant. This only means that the approach of analyzing the result in terms of cement type is correct.

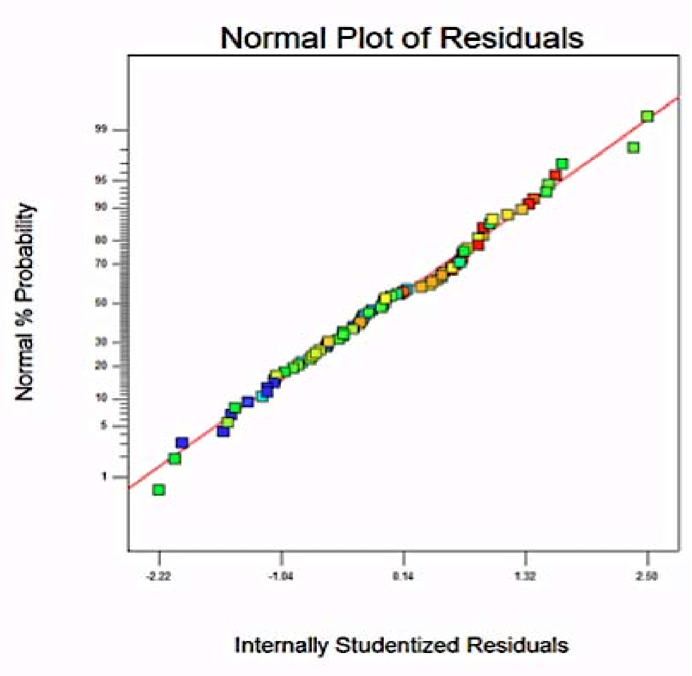

Figure 1 Normal probability plot of the concrete experiment.

3.3 Diagnostic Plots

Diagnostic plots are useful to see whether assumptions are met. Figure 1 shows the normal probability plot of the residuals. There is no significant defection from the normal probability line and it can fairly conclude that the assumption of normality is satisfied.

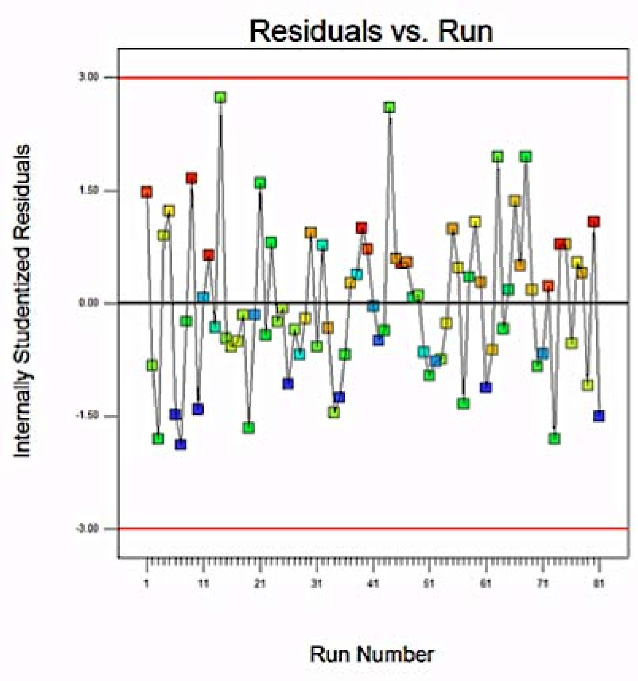

Figure 2 Residual vs Run (Order) plot

Figure 2 shows the Residuals vs. Run plot and no significant pattern or structure is observed. As the run order is increased, the residual values did not exhibit significant patterns of increase or decrease.

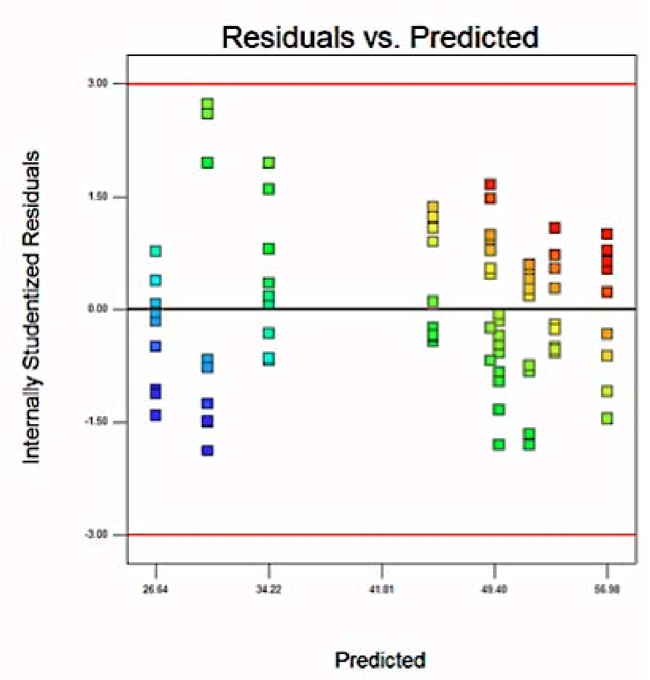

Figure 3 Residual vs Predicted (Fits) plot

Figure 3 shows the Residuals vs. Predicted plot and the illustration shows no significant pattern of a “megaphone”. Only means that when the predicted values increase, residual values show no sign of significant pattern of increase or decrease.

3.4 Response Surface Model

Since ANOVA tables showed that interactions were deemed not significant but having two factors significant, the optimum mix may be in the region between the lowest and middle values of curing period, and in the middle and highest region for slag content. To illustrate this in numbers, Numerical Optimization tool of design expert software was used to find the optimal point on the response surface that will maximize the compressive strength of concrete. The selected values were indeed followed the region were the maximum compressive strengths can be seen.

The contour plots, in Figures 1 to 3, give an idea to the variation of strength when slag content and curing period vary.

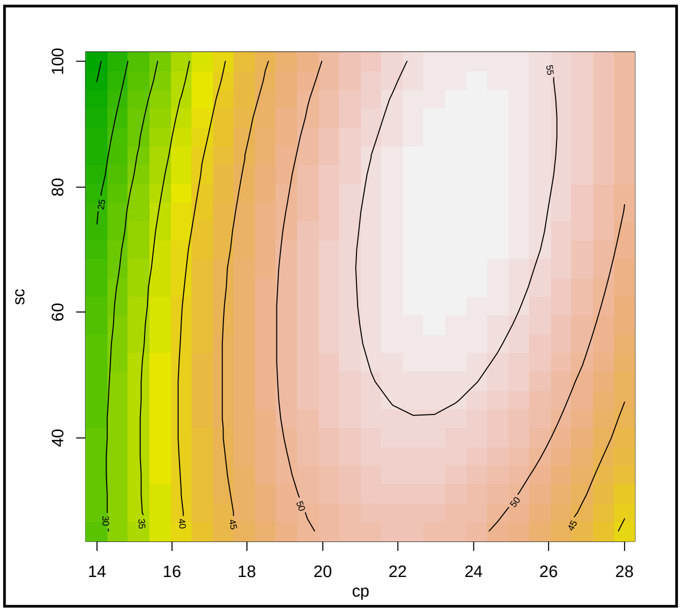

Figure 4 Response Surface of Type 1P cement in region of optimum combination.

3.4.1 Interpretation for Type 1P

The predicted strength is given by the equation

$$Strength = 53.6 + 9.02\ days + 3.45\ (days*slagcont) - 17.7 days^2 \qquad (2)$$

The stationary point in response surface is 0.362777 for days and 1.113511 for slag content. The stationary point in original units is 23.53944 for curing period and 77.83777 for slag content. Table 9 shows the Eigenvalues of Type 1P. Since the

Eigenvalues are both negative (-1.175624 and -17.909098), the stationary point in original units is now the optimal combination for Type 1P cement.

Table 9 Eigen Analysis for Type 1P

Values	**[1]**	**-1.175624**	**-17.909098**
Vectors		[,1]	[,2]
	days	-0.1036956	-0.9946091
	slag content	-0.9946091	0.1036956

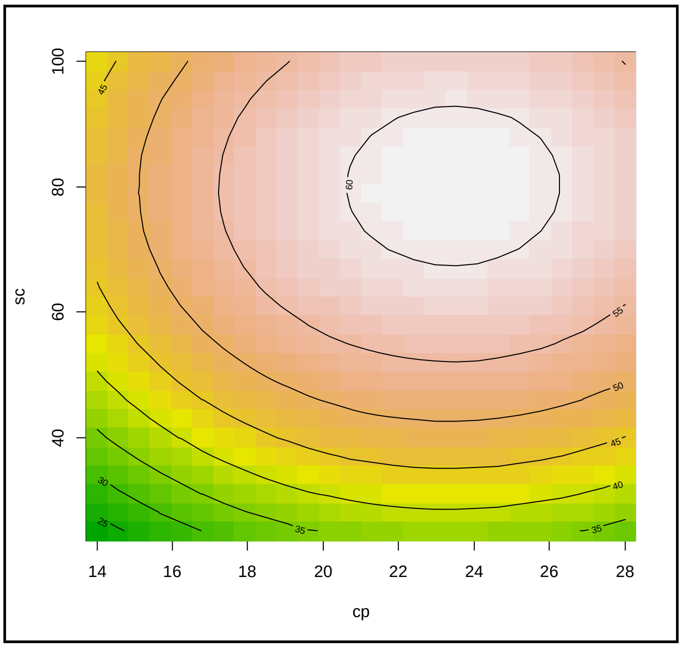

Figure 5 Response Surface of Type IP cement in region of optimum combination.

3.4.2 Interpretation for Type IP

The predicted strength is given by the equation

$$Strength = 61.5 + 5.4\ days + 0.15\ (days*slagcont) - 7.97\ days^2 \qquad (3)$$

The stationary point in response surface is 0.3502803 for days and 1.2070926 for slag content. The stationary point in original units is 23.45196 for curing period and 80.17731 for slag content. Table 10 shows the Eigenvalues of Type IP. Since the Eigenvalues are both negative, the stationary point in original units is now the optimal combination for Type IP cement.

Table 10 Eigen Analysis for Type 1P

Values	[1]	-4.992597	-7.97268
Vectors		[,1]	[,2]
	days	-0.02489517	-0.99969007
	slag content	-0.99969007	0.02489517

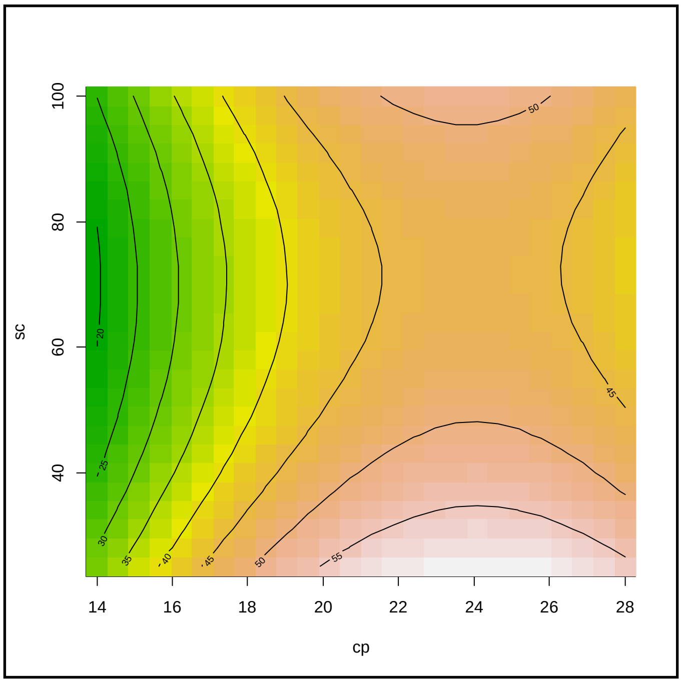

Figure 6 Response Surface of Type I cement in region with saddle response.

Table 11 Eigen Analysis for Type I

Values	[1]	3.819946	-13.50328
Vectors		[,1]	[,2]
	days	0.01375933	-0.99905340
	slag content	-0.99905340	-0.01375933

3.4.3 Interpretation for Type I

The predicted strength is given by the equation

$$Strength = 37.96 + 11.66\ days - 0.48\ (days*slagcont) - 13.5\ days^2 \qquad (4)$$

The stationary point in response surface is 0.4165729 for days and 0.8724468 for slag content. The stationary point in original units is 23.916101 for curing period and 71.81117 for slag content. Table 11 shows the Eigenvalues of Type I. Since the Eigenvalues are not all negative, then there is no optimal combination of slag content and curing period. However, the canonical path analysis gives an idea as to the

Table 12

	Curing period	Slag content
1	23.436	196.800
2	23.485	184.300
3	23.534	171.800
4	23.576	159.300
5	23.625	146.800
6	23.674	134.300
7	23.723	121.800
8	23.772	109.300
9	**23.821**	**96.800**
10	**23.870**	**84.300**
11	**23.919**	**71.800**
12	**23.961**	**59.300**
13	**24.010**	**46.825**
14	**24.059**	**34.325**
15	**24.108**	**21.825**
16	**24.157**	**9.325**
17	24.206	-3.175
18	24.255	-15.675
19	24.304	-28.175
20	24.346	-40.675
21	24.395	-53.175

possible combinations for the next phase of the experiment (Table 12).

Looking at the table above, the only relevant combinations are those from 9 to 16 because 1-8 combinations suggest slag content exceeding 100% and combinations 17-21 give negative percentages for slag content.

4. Conclusions

This study proved that slag content and curing period significantly affect the compressive strength of concrete, and the types of cement serve as the binding component of all materials thus creates no significance.

The relationships of these factors against the response (compressive strength) are not all linear. Slag content and curing period have a non-linear relationship and therefore should not be treated directly proportional against responses relative to the varying levels of the factors.

However, one of the cement types exhibits saddle response in the analysis. It is highly recommended to perform the same experimental procedure with these combinations provided by canonical path analysis and find out the optimum combination with highest possible compressive strength.

The optimum combination for maximizing strength is in the region between the lowest and middle values of curing period, and in the middle and highest region for slag content specifically, 14 days to 24 days and 75% to 100% respectively.

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