The 2013 World Congress on
Advances in Structural Engineering and Mechanics (ASEM13)
Jeju, Korea, September 8-12, 2013

Convolution Quadrature Time-Domain Boundary Element Method for 3-D Wave Propagation in Fluid-Saturated Porous Media

*Fumika Chikazawa[1)], Takahiro Saitoh[2)] andSohichi Hirose[3)]

[1), 3)] *Department of Mechanical and Environmental Informatics,*
Tokyo Institute of Technology, Tokyo 152-8552, Japan
[2)] *Division of Environmental Engineering Science,Gunma University,*
Gunma 376-8515, Japan
[1)] *chikazawa.a.ff@m.titech.ac.jp*

ABSTRACT

This paper presents a convolution quadrature time-domain boundary element method (CQ-BEM) in fluid-saturated porous mediagoverned by Biot’s theory.The classical time-domain BEM (TD-BEM) has been applied to various wave analyses.However, it cannot be used for wave propagation in fluid-saturated porous media, because of the following reasons: 1) no time-domain fundamental solutions are known for the problem, 2) the method sometimes suffers from numerical instability. To overcome these difficulties, a convolution quadrature method (CQM) developed by Lubich is applied to the TD-BEM. The scattering problems of an incident plane wave by a cavity in poroelastic media are solved to validate proposed method.

1. INTRODUCTION

The dynamic analysis for the porous solid has been studied in soil mechanics and rock engineering fields. In particular, the dynamic analysis of ground liquefaction arose from earthquakes requires the consideration of the fluid-solid interaction.

Biot(1956) proposed a dynamic poroelasticity formulation for the fluid saturated porous media with the fluid-solid interaction.The formulation is based onTerzaghi's consolidation theory, the total and effective stress principles, and effective porosity. According to Biot’s theory, two differentlongitudinal waves and one transvers wave exist in poroelastic solids, and all the waves have the dispersion property.

Wave propagation in fluid-saturated porous media has been analyzed by the finite element method (FEM).However,the conventional time-domain boundary element method (TD-BEM), which is suitable for wave analysis, cannot be applied because no time-domain fundamental solutions are known for the problem. In addition,the method sometimes suffers from numerical instability when small time increments are used.

Recently, a convolution quadrature boundary element method (CQ-BEM) has been proposedby several researchers(Schanzet al.1997 and Saitohet al. 2009). In the

---

[1)] Graduate Student
[2)]Associate Professor
[3)]Professor

formulation of the CQ-BEM, the convolution integrals of the time-domain boundary integral equations are numerically approximated using a convolution quadrature method (CQM). The CQ-BEM requires Laplace-domain fundamental solutions. Therefore, the use of the CQ-BEM is particularly helpful for poroelastic wave scattering problems where no time-domain fundamental solutions exist.

In this paper, a CQ-BEM formulation for 3-D wave propagation in fluid-saturated porous media is presented. Numerical examplesare shown to validate the proposed method.

2. BIOT'S THEORY

The small and large indices used throughout this paper, such as $(\ )_{,i}$ and $(\ )_{,I}$, range from 1 to 3 and from 1 to 4, respectively, unless otherwise stated.Additionally, summation over repeated subscripts is implied throughout thispaper.

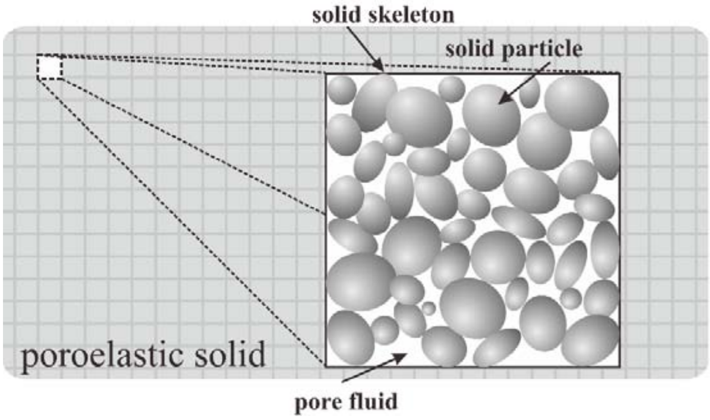


Fig. 1 A schematic of the model of poroelastic solid.

2.1 Compatibility and constitutive equations

Let us consider 3-D fluid-saturated porous media. The formulation ofporoelasticity is based on Biot's theory. (Biot 1956) In this study, we assume thatporoelastic media consist of solid skeletons, solid particles, and pore fluids, asshown inFig. 1. Assuming that the displacements for a solid skeleton and forpore fluid can be represented as $\boldsymbol{u} = \{u_1, u_2, u_3\}^T$ and $\boldsymbol{U} = \{U_1, U_2, U_3\}^T$, respectively.The compatibilityand constitutive equations are defined as follows:

$$e_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right), \qquad e = e_{kk} = u_{k,k} \tag{1}$$

$$\sigma_{ij} = 2\mu\epsilon_{ij} + (\lambda e - \alpha p)\delta_{ij}, \qquad p = -\alpha M e + M\zeta \tag{2}$$

whereσ_{ij} and $e_{ij}$are the total stress and strain for a solid skeleton, respectively.Thesuperscript $(\quad)^T$ denotes the transpose of a vector. In addition, $p$is the fluid pressure, ζ is the variation of the fluid volume per unit referencevolume, M is Biot's material con-stant, α is Biot's effective stress coefficient,and λ and μ are Lamé constants. The vari-able δ_{ij} is the Kronecker deltaand e is the dilatation strain. The variable $(\quad)_{,i}$ denotes the partial derivativewith respect to x_i.

2.2 Compatibility and constitutive equations

Now, we consider the problem of 3-D poroelastic wave scattering by theobject $\bar{D}$ in an exterior poroelastic medium D, as shown in Fig. 2. Whenthe incident wave $\boldsymbol{q}^{\mathbf{in}}$ hits the boundary surface S of the object $\bar{D}$, scatteredwaves are generated by the interac-tion between the object $\bar{D}$ and the incidentwave$\boldsymbol{q}^{\text{in}}$, as shown in Fig.2. Assuming the zero initial conditions, the equationsof motion for the solid and fluid can be written as follows:

$$\sigma_{ij,j} + \rho b_i = \rho \ddot{u}_i + \rho_f \ddot{w}_i \tag{3}$$

$$p_{,i} + \rho_f c_i = -\rho_f \ddot{u}_i - \frac{\rho_f}{\beta} - b\dot{w}_i \tag{4}$$

whereb_i and c_i are the body forces for the solid and fluid, respectively. The density of a poroelastic medium ρ is calculated by$(1-\beta)\rho_s + \beta\rho_f$, where β is the porosity, and ρ_s and ρ_f are the densities of a solid skeleton and pore fluid, respectively. In addition, $(\dot{\ })$ denotes the partial derivative with respect to the time t.The vector$\boldsymbol{w} = \{w_1, w_2, w_3\}^T$ denotes the relative displacement between the solid skeleton and the pore fluid,which has the relation with $\dot{w}_i = -(k/\eta)p_{,i}$where k is the permeability and η is the viscosity coefficient. The dissipation parameter b is defined by $b = \eta/k$.

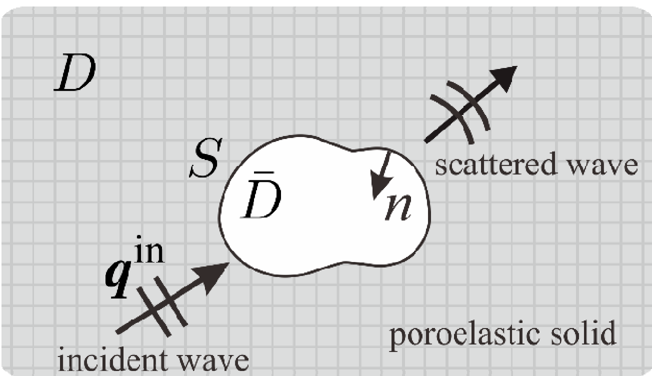


Fig. 2 Wave scattering in a poroelastic medium.

2.3 Wave propagation in poroelastic media

According to Biot's theory, two longitudinalwaves (L_1 and L_2 waves) and one transvers wave (T wave) exist in 3-D poroelastic solids. Thisdiscrepancy is the most significant feature of wave propagation in 3-D fluid-saturatedporous media, and these characteristics can be easily confirmed analytically in the frequency domain.

Considering a planewave with wave number k_ξ that is propagating to the x_i direction as follows:

$$\boldsymbol{q} = \{\boldsymbol{u}^T, p\}^T = \left\{\boldsymbol{A}^T e^{\mathrm{i}k_\xi x_1}, B e^{\mathrm{i}k_\xi x_1}\right\}^T \tag{5}$$

where $\boldsymbol{A} = \{A_1, A_2, A_3\}^T$ and B are the amplitudes of the solid displacementand fluid pressure, respectively. The vector $\boldsymbol{q}$ appearing in Eq. (5) is definedas a generalized displacement consisting of a solid displacement u and fluidpressure p. Eqs.(3)and(4) with no body force $(\mathrm{i.e.}, b_i = c_i = 0)$ yields the following governing equations for generalized displacements,

$$L_{IJ}\boldsymbol{q} = 0 \tag{6}$$

where$L_{IJ}$are the differential operators given by

$$L_{IJ} = \begin{bmatrix} [L_{ij}] & \{L_{i4}\} \\ \{L_{4j}\}^T & L_{44} \end{bmatrix} = \begin{bmatrix} \left[\dfrac{\partial}{\partial x_k} C_{kilj} \dfrac{\partial}{\partial x_l} + \omega^2 \tilde{\rho} \delta_{ij}\right] & \left\{-\tilde{\alpha} \dfrac{\partial}{\partial x_i}\right\} \\ \left\{\tilde{\alpha} \dfrac{\partial}{\partial x_j}\right\}^T & \dfrac{1}{\omega^2 \widetilde{m}} \Delta + \dfrac{1}{M} \end{bmatrix} \tag{7}$$

where C_{kilj} is the elastic constant defined by $C_{kilj} = \lambda\delta_{ki}\delta_{lj} + \mu(\delta_{kl}\delta_{ij} + \delta_{kj}\delta_{il})$. The variables Δand ωdenote the Laplacian and angular frequency,respectively. In addition, the parameters $\widetilde{m}$, $\tilde{\alpha}$, and $\tilde{\rho}$ are given by

$$\widetilde{m} = m + \mathrm{i}\frac{b}{\omega}, \quad \tilde{\alpha} = \alpha - \frac{\rho_f}{\widetilde{m}}, \quad \tilde{\rho} = \rho - \frac{\rho_f^2}{\widetilde{m}}, \quad m = \frac{\rho_f}{\beta}. \tag{8}$$

Eqs.(5) and (6) yields different three wave numbersk_T, k_{L_1},and k_{L_2}. The transverse wave number$k_T$is

$$k_T^2 = \frac{\tilde{\rho}}{\mu}\omega^2 \tag{9}$$

andtwo longitudinal wavesk_{L_1} and k_{L_2} are obtained as follows:

$$\begin{Bmatrix} k_{L_1}^2 \\ k_{L_2}^2 \end{Bmatrix} = \frac{1}{2}\left[\left(k_{L_{10}}^2 + k_{L_{20}}^2 + k_Q^2\right) \mp \left\{\left(k_{L_{10}}^2 + k_{L_{20}}^2 + k_Q^2\right)^2 - 4k_{L_{10}}^2 k_{L_{20}}^2\right\}^{\frac{1}{2}}\right] \tag{10}$$

where$k_{L_{10}}$, $k_{L_{20}}$ and k_Q are given by Fukui et al. (1996) and Yamamoto et al.(2002) as

$$k_{L_{10}}^2 = \frac{\tilde{\rho}}{\lambda + 2\mu}\omega^2, \ \ k_{L_{20}}^2 = \frac{\widetilde{m}}{M}\omega^2 m, \ \ k_Q^2 = \frac{\widetilde{m}\tilde{\alpha}^2}{\lambda + 2\mu}\omega^2. \tag{11}$$

Eqs. (9)and(10)imply that three varieties of waves exist inporoelastic solids, and k_{L_1} and k_{L_2}correspond to the fast longitudinal wave, and the latter one.Here, we define wave velocitiesfor these three kinds of wave numbers as $c_\kappa = \omega/k_\kappa(\kappa = L_1$, L_2,or$T)$.Therefore, in this paper, we referto the wave propagating with the velocity c_{L_1} as the "L_1 wave" ,c_{L_2} asthe "L_2 wave"and $c_T$as " T wave".In addition, in order to later use, the wave velocities corresponding to the wave number$k_{L_{10}}$, $k_{L_{20}}$ and k_Q is also defined as $c_\kappa = \omega/k_\kappa(\kappa = L_{10},\ L_{20}$,or$Q)$.

3. CQ-BEM FOR 3-D POROELASTODYNAMICS

3.1 Time-domain boundary integral equations

Consideringan elastic wave scattering problem in an infinite poroelastic medium as shown in Fig. 2, the time-domain boundary integral equations are derived as follows:

$$\begin{aligned} C_{IJ}(\boldsymbol{x})q_J(\boldsymbol{x}) &= q_I^{\text{in}}(\boldsymbol{x},t) \\ &+ \int_S U_{IJ}(\boldsymbol{x},\boldsymbol{y},t) * v_J(\boldsymbol{y},t)dS_y - \int_S W_{IJ}(\boldsymbol{x},\boldsymbol{y},t) * q_J(\boldsymbol{y},t)dS_y \end{aligned} \tag{12}$$

where$q_I^{\text{in}}(\boldsymbol{x},t)$ shows incident wave, $v_J(\boldsymbol{y},t)$ is generalized traction, which isgiven by $\boldsymbol{v} = \{\boldsymbol{t}^T, p_n\}^{\text{T}}$,$U_{IJ}(\boldsymbol{x},\boldsymbol{y},t)$ and $W_{IJ}(\boldsymbol{x},\boldsymbol{y},t)$ are the time-domain fundamental solutionsand its double-layer kernels for 3-D poroelastodynamics, respectively. In addition,C_{IJ} is the free term (Brebbia 1984) which depends on the shape of boundary at observation point$\boldsymbol{x}$.

Normally, the time-domain boundary integral equations (12) are discretizedby using the appropriate interpolation functions for the unknown values andsolved by a time-stepping algorithm. However, the boundary integral equationscannot be solved using such a scheme because there areno explicit time-domain fundamental solutions for 3-D poroelastic wave propagation. To overcome this difficulty, the CQM is applied to Eq.(12).

3.2 Convolution quadrature method

The convolution quadrature method (CQM), firstproposed by Lubich (1988), approximates the convolution $f * g(t)$ by a discrete convolution using theLaplace transform of the time-dependent function $f(\tau - t)$. In general, the convolution integralis approximated by CQM as follows:

$$f * g(n\Delta t) \simeq \sum_{j=0}^{n} \omega_{n-j}(\Delta t) g(j\Delta t),\ \ n = 0,1,\dots,N \tag{13}$$

where t is divided into N equal steps Δt and $\omega_j(\Delta t)$ are the quadrature weights. The quadrature weights are determined with the Laplace transform of the original time-dependent function f and given as follows:

$$\omega_n(\Delta t) \simeq \frac{\mathcal{R}^{-n}}{L} \sum_{l=0}^{L-1} \hat{f}\left(\frac{\gamma(\zeta_l)}{\Delta t}\right) e^{-\frac{2\pi i n l}{L}} \tag{14}$$

where $\hat{f}$ is the Laplace transform of f and $\gamma(\zeta)$ is the quotient of the generating polynomials of linear multistep method given by $\gamma(\zeta) = \sum_{i=1}^{k}(1-\zeta)^i/i$ using backward differential formulas (BDF) and ζ_l is given by $\zeta_l = \rho e^{2\pi i l/L}$. Inaddition, $\mathcal{R}$ is the radius of a circle in the analyticity domain of $\hat{f}$. ϵ is the error of the numerical calculation of Eq.(14),given by $\mathcal{R}^L = \sqrt{\epsilon}$ and the parameter L is set as $L = N$ to accelerate the calculation of Eq.(14)by using FFT.

3.3 Discretization of BIEs using the CQM

If we discretize the boundary surface S into M boundary elements using apiecewise constant approximation of the unknown generalized displacement q_I and traction v_I and using the CQM for the convolutions of Eq.(12), theboundary integral equationscan be discretized as follows:

$$\begin{aligned} &C_{IJ}(\boldsymbol{x}) q_J(\boldsymbol{x}, n\Delta t) = q_I^{\mathrm{in}}(\boldsymbol{x}, n\Delta t) \\ &+ \sum_{\alpha=1}^{M} \sum_{k=1}^{n} \left[A_{IJ;\alpha}^{n-k}(\boldsymbol{x}) v_{J;\alpha}(k\Delta t) - B_{IJ;\alpha}^{n-k}(\boldsymbol{x}) q_{J;\alpha}(k\Delta t)\right]. \end{aligned} \tag{15}$$

Here, $A_{IJ;\alpha}^{m}(\boldsymbol{x})$ and $B_{IJ;\alpha}^{m}(\boldsymbol{x})$ are influencefunctions defined by

$$A_{IJ;\alpha}^{m}(\boldsymbol{x}) = \frac{\mathcal{R}^{-m}}{L} \sum_{l=0}^{L-1} \left[\int_{S_\alpha} \widehat{U}_{IJ}(\boldsymbol{x}, \boldsymbol{y}, s_l) dS_y\right] e^{-\frac{2\pi i m l}{L}} \tag{16}$$

$$B_{IJ;\alpha}^{m}(\boldsymbol{x}) = \frac{\mathcal{R}^{-m}}{L} \sum_{l=0}^{L-1} \left[\int_{S_\alpha} \widehat{W}_{IJ}(\boldsymbol{x}, \boldsymbol{y}, s_l) dS_y\right] e^{-\frac{2\pi i m l}{L}} \tag{17}$$

where $\widehat{U}_{IJ}(\boldsymbol{x},\boldsymbol{y},s)$ and $\widehat{W}_{IJ}(\boldsymbol{x},\boldsymbol{y},s)$ are the Laplace-domain fundamentals solutions and its double-layer kernels, respectively. In addition, s_l denotes the Laplace parameter given by $s_l = \gamma(\zeta_l)/\Delta t$.

3.4 *Laplace-domain fundamental solutions and double-layer kernels*

As mentioned in the previous section, the time-domain BEM based on the CQM requires the fundamental solutions and double-layer kernels in the Laplace-domain. The Laplace-domain fundamental solutions $\widehat{U}_{IJ}(\boldsymbol{x},\boldsymbol{y},s)$ with Laplace parameter s for 3-D poroelastic wave propagation satisfy the following equation:

$$\hat{L}_{IJ}\widehat{U}_{IJ}(\boldsymbol{x},\boldsymbol{y},s) = -\delta_{IK}\delta(\boldsymbol{x}-\boldsymbol{y}) \tag{18}$$

where $\delta(\boldsymbol{x})$ shows the Dirac delta function. The differential operator $\hat{L}_{IJ}$ in Eq.(18) is Laplace-domain of L_{IJ} in Eq.(7) and given by

$$\hat{L}_{IJ} = \begin{bmatrix} \left[\dfrac{\partial}{\partial x_k} C_{kilj} \dfrac{\partial}{\partial x_l} - s^2\tilde{\rho}\delta_{ij}\right] & \left\{-\tilde{\alpha}\dfrac{\partial}{\partial x_i}\right\} \\ \left\{\tilde{\alpha}\dfrac{\partial}{\partial x_j}\right\}^T & \dfrac{1}{s^2\widetilde{m}}\Delta + \dfrac{1}{M} \end{bmatrix}. \tag{19}$$

Hormander's theoremis useful to derive the Laplace-domain fundamental solutionsfrom Eq.(18). In fact, the Laplace-domain fundamentalsolutions can be derived from Eqs.(18) and (19) as follows:

$$\widehat{U}_{ij}(\boldsymbol{x},\boldsymbol{y},s) = \frac{1}{4\pi\mu}\left(\frac{e^{-S_T r}}{r}\delta_{ij} - \frac{1}{S_{L_1}^2 - S_{L_2}^2}\left[\frac{S_{L_{10}}^2 - S_{L_2}^2}{S_T^2}\left(\frac{e^{-S_T r}}{r} - \frac{e^{-S_{L_1} r}}{r}\right) - \frac{S_{L_{10}}^2 - S_{L_1}^2}{S_T^2}\left(\frac{e^{-S_T r}}{r} - \frac{e^{-S_{L_2} r}}{r}\right)\right]_{,ij}\right) \tag{20}$$

$$\widehat{U}_{i4}(\boldsymbol{x},\boldsymbol{y},s) = -\frac{1}{4\pi\tilde{\alpha}}\frac{S_Q^2}{S_{L_1}^2 - S_{L_2}^2}\left[\frac{e^{-S_{L_1} r}}{r} - \frac{e^{-S_{L_2} r}}{r}\right]_{,i} \tag{21}$$

$$\widehat{U}_{4j}(\boldsymbol{x},\boldsymbol{y},s) = \frac{1}{4\pi\tilde{\alpha}}\frac{S_Q^2}{S_{L_1}^2 - S_{L_2}^2}\left[\frac{e^{-S_{L_1} r}}{r} - \frac{e^{-S_{L_2} r}}{r}\right]_{,j} \tag{22}$$

$$\widehat{U}_{44}(\boldsymbol{x},\boldsymbol{y},s) = -\frac{M}{4\pi}\frac{S_{L_{20}}^2}{S_{L_1}^2 - S_{L_2}^2}\left[\left(S_{L_{10}}^2 - S_{L_2}^2\right)\frac{e^{-S_{L_2} r}}{r} - \left(S_{L_{10}}^2 - S_{L_1}^2\right)\frac{e^{-S_{L_1} r}}{r}\right]. \tag{23}$$

Here, r is given by $r = |\boldsymbol{x}-\boldsymbol{y}|$ and the parameters, $S_{L_{10}}, S_{L_{20}}, S_Q, S_{L_1}, S_{L_2}$ and S_T aredefined by $S_\kappa = s/c_\kappa(\kappa = L_{10},\ L_{20}, Q, L_1, L_2, \text{or} T)$, where $c_\kappa = \omega/k_\kappa$.The parameter S_Q in Eqs.(21) and (22) implies the effect of solid-fluid interaction for 3-D poroelastodynamics. Eqs.(21)and(22), which are thecoupling terms between solid and fluid regions, approach to zero if $S_Q \to 0$.

Next, we define the double-layer kernels$\widehat{W}_{IJ}(\boldsymbol{x},\boldsymbol{y},s)$, for 3-D poroelastodynamics using the Laplace-domain fundamental solutions $\widehat{U}_{IJ}(\boldsymbol{x},\boldsymbol{y},s)$. Defining the generalizedtraction $\boldsymbol{v}$as $v_I = B_{IJ}q_J$acting on a plane with normal vector $n_i$gives the double-layer kernels$\widehat{W}_{IJ}(\boldsymbol{x},\boldsymbol{y},s)$if the following equation is solved:

$$\widehat{W}_{KI}(\boldsymbol{x},\boldsymbol{y},s) = \widehat{B}_{IJ}^{y}\widehat{U}_{JK}(\boldsymbol{y},\boldsymbol{x},s) = \widehat{B}_{IJ}^{y}\widehat{U}_{KJ}(\boldsymbol{x},\boldsymbol{y},s). \tag{24}$$

Here, the differential operator $\widehat{B}_{IJ}^{y}$ derived from the Laplace transform of $\boldsymbol{B}$, is given by

$$\widehat{B}_{IJ} = \begin{bmatrix} \left[C_{kilj}\dfrac{\partial}{\partial x_l}n_k\right] & \{-\tilde{\alpha}n_i\} \\ \left\{-\dfrac{\rho_f}{\widetilde{m}}n_j\right\}^T & -\dfrac{1}{s^2\widetilde{m}}\dfrac{\partial}{\partial x_k}n_k \end{bmatrix}. \tag{25}$$

Therefore, each component of the generalized traction$\boldsymbol{v} = \{t_1, t_2, t_3, p_n\}^T$is

$$t_i = \sigma_{ij}n_j - \alpha p n_i,\ \ p_n = -w_k n_k \tag{26}$$

where t_i and p_n showsthe traction relation for total stress and fluid pressure flux respectively.

4. NUMERICAL EXAMPLES

In this section, we show numerical examples using theproposed CQ-BEM.In all numerical examples, the boundary surface of anobject with radius $a = 1$ was discretizedinto 836 boundary elements usinga piecewise constant approximation. The material parameters used in theanalyses are listed in Table 1.The accuracy parameter $\epsilon$is$\epsilon = 1.0 \times 10^{-12}$.

Table. 1 Material parameters

μ /(+	λ /(+	M /(+		ρ_f $/\rho_s$
$1/3$	$1/3$	$5/6$	0.2	1 $/3$

4.1 Incident waves in the time-domain

As mentioned in the section 0, two longitudinal waves and one transvers wave exist in 3-D porous medium and these waves possess the dispersion property.Therefore, we assume that the time-domain incident wave $\boldsymbol{q}^{\text{in}} = \left\{\left[u_i^{in}(\boldsymbol{x},t)\right]^T, p^{\text{in}}(\boldsymbol{x},t)\right\}^T$ is a plane fastlongitudinal wave (L_1 wave)with the central period $T_0(= 2\pi/\omega_0)$ and angularfrequency ω_0 propagating in the x_1 direction and is calculated via the inverseFourier transform$\mathcal{F}^{-1}$of the original wave$\psi(\omega)$ as

$$u_i^{\text{in}}(\boldsymbol{x},t) = \delta_{i1}\mathcal{F}^{-1}[\psi(\omega)] \tag{27}$$

$$p^{\text{in}}(\boldsymbol{x},t) = \mathcal{F}^{-1}\left[-\text{i}\,\frac{k_{L_{10}}^2 - k_{L_1}^2}{k_{L_1}}\frac{\lambda + 2\mu}{\tilde{\alpha}}\psi(\omega)\right] \tag{28}$$

$$\psi(\omega) = \frac{u_0\omega_o^2\left(1 - e^{\text{i}\omega T_0}\right)}{2\text{i}\omega(\omega^2 - \omega_0^2)}e^{\text{i}kx_1} \tag{29}$$

whereu_0 is the displacement amplitude and is set as $u_0 = 1$. In addition, the coefficient of $\psi(\omega)$ in Eq.(28) shows the amplitude ratio of fluid pressure to the solid displacement for L_1 wave.

4.2 Accuracy of the proposed CQ-BEM

The scattering problem of an incident plane wave hitting an object $\bar{D}$in aporoelastic solid D, as shown in Fig.3, was solved by the proposed CQ-BEM to verify the computational accuracy.In this problem, if the material parameters ofthe$\bar{D}$ are the same as thoseof the poroelastic solid D and the continuous boundary conditions on the boundary S are imposed,the solutions of the boundary value problems are equivalent to those for theincident wave $\boldsymbol{q}^{\text{in}}$.

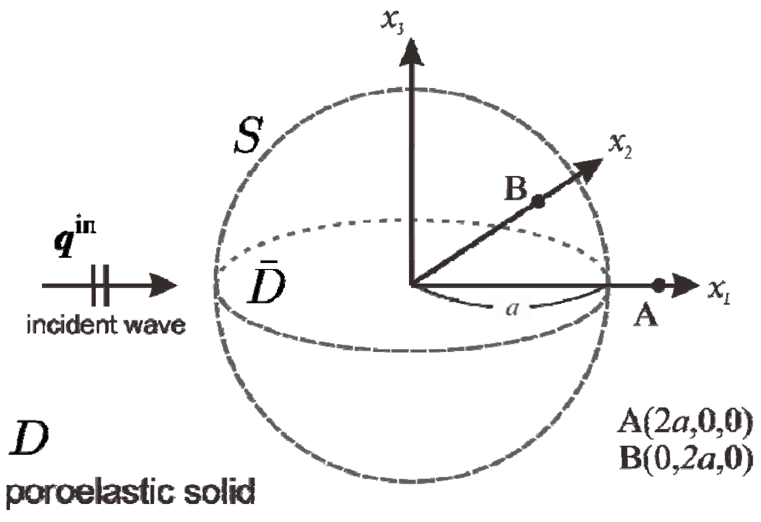
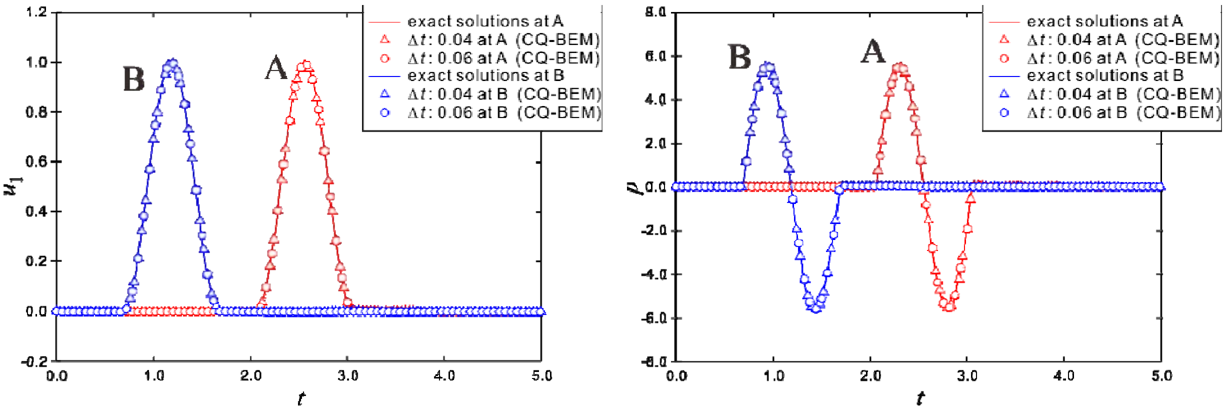


Fig. 3Analysis model for verifying the accuracy of the proposed method.

Fig. 4shows the time histories of the solid displacements u_1 and the fluid pressure $p$at the points A and B in Fig. 3 obtained by using second-order BDF.Solutions using two different time increments, $\Delta = 0.06$ and 0.04, are plotted, and the analytical solutions (incident wave) are also shown by the solid line for comparison. Here, theparameters N and L are given by $N = L = 256$. Moreover, $\alpha = 1, b = 10$,and $u_0 = 1$ are considered.It can be observed that the results obtained by proposed method are in good agreement with the exact solutions.



(a) (b)

Fig. 4The time histories of (a)u_1 and (b)p at the points A and Bin Fig. 3.

5. CONCLUSIONS

In this paper, the CQ-BEM was developed for 3-D wave propagation in fluid-saturated porous media. In the proposed method, the convolution integrals in the time-domain boundary integral equations are discretized using the CQM. The numerical resultsobtained by the proposed method are good agreement with the exact solutions. In the future,the fast multipolemethod (FMM) is applied to accelerate the proposed method for more efficient calculations.

REFERENCES

Biot, M.A. (1956), "Theory of Propagation of Elastic Waves in a Fluid-Saturated Porous Solid I. Low-frequency range", *J.Acoust. Soc. Am.,***28**, 168-178.

Brebbia, C.A., Telles, J.C.F. andWrobel C.L. (1984), *Boundary element techniques, Theory and applications in engineering*, Springer, Berlin, Germany.

Fukui, T., Funato, K. and Inoue, K. (1996), "Frequency domain boundary element method for wave propagation in Biot material", *J. Bound. Elem.*, **13**, 149-154 (in Japanese).

Lubich, C. (1988), "Convolution quadrature and discretized operational calculus I",*Numer. Math.,***52**, 129-145.

Saitoh, S., Hirose, S. and Fukui, T., (2009), “Convolution quadrature boundary element method and acceleration by fast multipole method in 2-D viscoelastic wave propagation”, *Theoretical and Applied Mechanics Japan*, **57**, 385-393.

Schanz, M. and Antes, H. (1997), “Application of Operational quadrature methods in time domain boundary element method”, *Mec.*,**32**, 179-186.

Yamamoto, K. and Kitahara, M. (2002), “Elastic wave scattering analysis of cavities in poroelastic media using three-dimensional boundary element formulation”, in J.-L. Auriault, et al. (eds.), *PoromechanicsII*,Balkema, 857-863.