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Application of support vector machine in health monitoring of plate structures

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ABSTRACT

This paper demonstrates the use of Support Vector Machine (SVM) for detection of damage location and its intensity in an aluminum plate. Twelve damage locations and nine damage intensities have been simulated by reducing thickness of the plate at various locations using the finite element analysis package Abaqus. The first mode shape data is extracted at various points on the plate and it has been used as input data for SVM to predict the damage locations and their intensities. This approach does not require data of the plate in damaged state. In order to make the mode shape data more realistic in nature, Gaussian noise from 30dB to 80dB has been added. The results demonstrate that SVM can be used as a tool for structural health monitoring without using data of healthy (undamaged) state.

1. INTRODUCTION

Structural Health Monitoring (SHM) is of great importance in civil, mechanical and aerospace structures for safety purpose and to avoid economical loss. The process of implementing a damage identification strategy for above mentioned structures is referred to as SHM (Farrar et al., 2007). The presence of damage in the structure leads to change in the modal parameters (natural frequency, damping and stiffness), and interpreting the changes in these parameters one can ensure whether the structure is damaged or intact. The change in the natural frequency was not sufficient to locate the damage, hence, there was need to develop methods based on mode shape data and Frequency Response Function (FRF) data of the structure (Banerjee et al., 2005, 2009).

The use of SVM for prediction of fault in power systems has been demonstrated by

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Kumar et al. (2011). They used support vector classification to predict the damage location. The inputs used for SVM model are Power and Voltage Values. Bulut et al. (2007) demonstrates the damage detection in civil structure using SVM classifier and wavelets. They found that the SVM was a robust classifier in presence of noise whereas wavelet-based compression gracefully degrades its classification accuracy. The present article uses vibration data (mode shape data) for regression analysis using SVM in order to locate damage and its intensity in the rectangular plate.

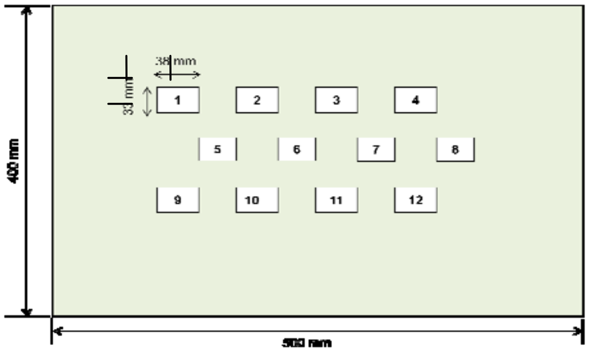

Figure 1 Damage locations

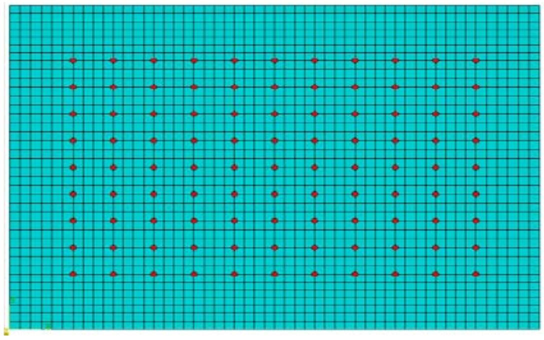
Figure 2 FE mesh & data acquisition points

Table 1 modified location labels in reference to Figure 1

Locations from center of plate		***Rearranged location***	
Radial distance (mm)	***Location label (as per Fig. 1)***	***Radial distance (mm)***	***Location label (as per Fig. 1)***
156	1	25	6
100	2	53	11
84	3	59	7
140	4	76	10
96	5	84	3
25	6	96	I5
59	7	100	2
134	8	124	12
142	9	134	8
76	10	140	4
53	11	142	9
124	12	156	1

2. FINITE ELEMENT MODELLING AND ANALYSIS

A simply supported plate of dimensions 500mm x 400mm x 3mm, with following properties: Young's modulus = 70GPa, Density = 2700 Kg/m3, Poisson's ratio = 0.3 is considered. FE modeling and analysis of the plate is carried out in ABAQUS® using 4 node rectangular shell element of size 10mm X 10mm. In Figure (1) damage locations are shown which are simulated by reducing the thickness from 10% to 50 % of the original plate thickness in steps of 5%.

However for better understanding of results, the plate centre is taken as reference (0, 0) and locations are defined as per their radial distance from center. The purpose of this arrangement is to highlight trend of error in damage prediction with respect to location from the centre of the plate. The arrangement can be explained from Table 1.

3. OVERVIEW OF SUPPORT VECTOR MACHINE FOR REGRESSION

A brief formulation on SVM for regression analysis given by Vojislav (2001) is presented in this section.SVM is initially developed for solving classification problems, and successfully applied in regression problems. The general formulation of regression learning is carried out as follows. Given *l* training data set for learning the machine (algorithm), it attempts to learn the input-output relationship ***f(x)***. A training data set **D =** $\{[\boldsymbol{x}(i), y(i)] \in \Re^n, i = 1, \cdots, l\}$ consists of *l* pairs $(\boldsymbol{x}_1, y_1), (\boldsymbol{x}_2, y_2), \cdots, (\boldsymbol{x}_l, y_l)$, where the inputs $\boldsymbol{x}$ are *n*- dimensional vectors $\boldsymbol{x} \in \Re^n$, and the system responses $y \in \Re$ are continuous values. Here first linear regression problem formulation is considered and extended to non-linear problem.

$$f(x,w) = w^T x + b \tag{1}$$

where, ***x*** is input vector, ***w*** is weight vector and ***b*** is bias term.

Typically regression analysis is associated with approximating input-output relationship considering *error of approximation.* The *linear loss (error) function* with *ε-insensitivity* zone introduced by Vapnik is given as

$$|y - f(x,w)|_\varepsilon = \begin{cases} 0 & |y - f(x,w)| \le \varepsilon \\ |y - f(x,w)| - \varepsilon & \text{otherwise} \end{cases} \tag{2}$$

The *linear loss (error) function* with *ε-insensitivity* zone is shown graphically in the Figure (4).

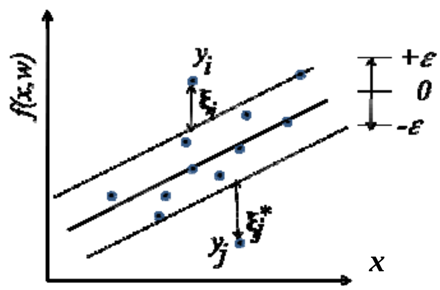

Figure 3 parameters used in (1D) SV regression

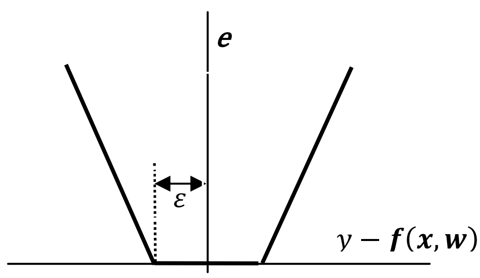

Figure 4 Loss (error) function

The value given by the Eq. (1) is predicted one and y is the actual value of the system response for given input $\boldsymbol{x}$. The loss or error is equal to zero if the difference between predicted and actual value is less than ε tube. Vapnik's ε-*insensitivity* loss function allows us to set limit or some measure of error which can be tolerated and given by a small value ε. If the predicted point lies outside the ε tube, then the loss is equal to magnitude of the difference between the predicted value and the radius of the ε tube which termed as slack variable and is given by

$$\left|y - f(x,w)\right| - \varepsilon = \zeta \text{ for data "above" an } \varepsilon \text{ tube} \tag{3}$$

$$\left|y - f(x,w)\right| + \varepsilon = \zeta^* \text{ for data "below" an } \varepsilon \text{ tube} \tag{4}$$

A new empirical risk is introduced in order to perform SVM regression and is given as

$$R_{emp}^{\varepsilon}(w,b) = \frac{1}{l}\sum_{i=1}^{l}\left|y - w^T x - b\right|_{\varepsilon} \tag{5}$$

The objective of SVM regression is to minimize the empirical risk R_{emp}^{ε} and norm of weigh vector $\|w\|^2$ simultaneously. Thus, main goal is to estimate a linear regression hyperplane $f(x,w) = w^T x + b$ by minimizing

$$R = \frac{1}{2}\|w\|^2 + C\left(\sum_{i=1}^{l}\left|y - w^T x - b\right|_{\varepsilon}\right) \tag{6}$$

Using expressions for slack variables the empirical risk becomes

$$R = \frac{1}{2}\|w\|^2 + C\left(\sum_{i=1}^{l}\zeta + \sum_{i=1}^{l}\zeta^*\right) \tag{7}$$

Under the constraints

$$\begin{aligned} y_i - w^T x_i - b &\le \varepsilon + \zeta,\ i = 1\cdots l \\ w^T x_i + b - y_i &\le \varepsilon + \zeta^*,\ i = 1\cdots l \end{aligned} \tag{8}$$

$$\begin{aligned} \zeta &\ge 0,\ i = 1\cdots l \\ \zeta^* &\ge 0,\ i = 1\cdots l \end{aligned} \tag{9}$$

There are mainly two parameters which have to be tuned to get good performance from SVM regression analysis. The constant C influences the trade-off between an approximation error and the weight vector norm$\|\boldsymbol{w}\|$. Another parameter ε which has to choose by the user, that defines the precision required in prediction.

This constrained problem is solved by forming primal Lagrangian (L_p) function, and is given by

$$L_p\left(w,b,\zeta,\zeta^*\alpha_i,\alpha_i^*,\beta_i,\beta_i^*\right)=\frac{1}{2}w^Tw+C\left(\sum_{i=1}^{l}\zeta+\sum_{i=1}^{l}\zeta^*\right)-\sum_{i=1}^{l}\alpha_i^*\left[y-w^Tx-b+\varepsilon+\zeta_i\right]-\sum_{i=1}^{l}\left(\beta_i^*\zeta_i^*+\beta_i\zeta_i\right) \quad (10)$$

This primal Lagrangian function has to be minimized with respect to primal variables ***w*****,** *b*, **ξ**, and ξ^* and maximized with respect to $\alpha_i,\alpha_i^*,\beta_i,\beta_i^*$. The problem is solved in its dual form and is given as follows,

Maximize

$$L_d\left(\alpha,\alpha^*\right)=-\varepsilon\sum_{i=1}^{l}\left(\alpha_i^*+\alpha_i\right)+\sum_{i=1}^{l}\left(\alpha_i^*-\alpha_i\right)y_i-\frac{1}{2}\sum_{i,j=1}^{l}\left(\alpha_i^*-\alpha_i\right)\left(\alpha_j^*-\alpha_j\right)x_i^Tx_j \quad (11)$$

Subject to

$$\sum_{i=1}^{l}\alpha_i^*=\sum_{i=1}^{l}\alpha_i \quad (12)$$

$$\begin{aligned}0\le\alpha_i^*\le C,\ i=1\cdots l\\ 0\le\alpha_i\le C,\ i=1\cdots l\end{aligned} \quad (13)$$

If we look at the dual form of problem it is expressed in terms of Lagrange multipliers α and α^* only. This standard optimization problem can be expressed in a matrix form and given as:

Minimize

$$L_d\left(\alpha\right)=0.5\alpha^TH\alpha-f^T \quad (14)$$

Subject to constraints Eqs. (12), (13).

Where for linear regression

$$H=x^Tx+1 \quad (15)$$

$$f=\left[\varepsilon-y_1\ \varepsilon-y_2\cdots\ \varepsilon-y_N\ \varepsilon+y_1\ \varepsilon+y_2\cdots\ \varepsilon+y_N\right] \quad (16)$$

The solution of above problem will give Lagrange multipliers pairs. The number of support vector is equal to the nonzero parameters $\alpha_i$orα_i^*. After calculating Lagrange multipliers the weight vector and bias term is found as follows

$$w=\sum_{i=1}^{l}\left(\alpha_i^*-\alpha_i\right)x_i \quad (17)$$

$$b=\frac{1}{l}\sum_{i=1}^{l}\left(y_i-x_i^Tw\right) \quad (18)$$

The best regression hyperplane in case of linear problem is given by

$$f(x,w)=w^Tx+b \quad (19)$$

While designing SV machines for non-linear regression analysis first map the input vectors $\boldsymbol{x} \in \mathfrak{R}^n$ in to vectors **z** of a higher-dimensional feature space $\boldsymbol{F}(\boldsymbol{z} = \boldsymbol{\phi}(\boldsymbol{x})$, where $\boldsymbol{\phi}$ represents a mapping), and solve a linear regression problem in this feature space. The most mapping (kernel) functions are polynomials and radial basis functions with Gaussian kernels. The given optimization problem is solved with change in only Hessian matrix ***H*** and is given as

$$H = \begin{pmatrix} G & -G \\ G & G \end{pmatrix} \tag{20}$$

Where ***G*** is the corresponding kernel matrix ***G*** $(\boldsymbol{x_k}, \boldsymbol{x_j})$ and weight vector and bias term is given by

$$w = \left(\alpha^* - \alpha\right) \tag{21}$$

$$b = \frac{1}{l}\sum_{i=1}^{l}\left(y_i - g_i\right) \tag{22}$$

$$g = Gw \tag{23}$$

and the best non-linear regression function is given by

$$z = f\left(x, w\right) = Gw + b \tag{24}$$

4. RESULTS AND DISCUSSION

Since SVM regression algorithm gives only single output, SVM regression analysis is carried out in two stages.

4.1 STAGE1: DAMAGE LOCATION PREDICTION

The damage location prediction was done in two steps. Step 1 involved predicting the X coordinate and step 2 involved predicting the Y coordinate of the damage location. The training input set used for step 1 is mode shape data for all damage intensities and training output set was corresponding X coordinate of damage locations. Test set was the mode shape data whose damage location and intensity was to be predicted. SVM now predicts X coordinate of damage location in step 1. Step 2 was similar to step one except that the Y coordinate of damage location was predicted. Stage 2 involved damage intensity prediction. In this stage, mode shape data for a particular location was used as input and damage intensity at that location was used as output. This was repeated for all the locations. Parameters for SVR are taken as: *C=2, e=0.0005, Radial Basis function (RBF) kernel, $\boldsymbol{\varepsilon}$ - insensitive loss function, kernel width=0.6 for damage location prediction, and C=10, kernel width=1 for damage intensity.* The percentage error is calculated as given below.

$$\% \text{ error} = \frac{\sqrt{(X_{predicted} - X_{actual})^2 + (Y_{predicted} - Y_{actual})^2}}{\text{maximum length of diagonal}} X\,100 \tag{21}$$

First mode shape data obtained at all the points of the simulated rectangular plate highlighted in the Figure 2 is considered as training input for the SVM, and corresponding damage location and/or intensity as training output. The damage location is represented by the midpoint of the damaged area in order to get single valued output for the SVM.

Table 2 Error in damage location prediction averaged over damage location of plate

Intensity% \ noise level	no noise	80 dB	70dB	60 dB	50 dB	40 dB	30 dB
10	27.00	27.00	27.03	27.08	27.49	51.93	83.84
15	1.79	1.79	1.82	1.85	2.41	27.52	68.56
20	0.62	0.62	0.62	0.66	1.28	11.32	75.68
25	0.21	0.21	0.23	0.34	0.57	2.01	36.17
30	0.16	0.17	0.16	0.19	0.33	1.64	43.64
35	0.13	0.13	0.13	0.16	0.27	1.02	27.18
40	0.11	0.11	0.12	0.15	0.27	0.99	11.46
45	0.09	0.09	0.10	0.11	0.28	0.86	2.54
50	0.08	0.08	0.08	0.14	0.18	0.85	3.58

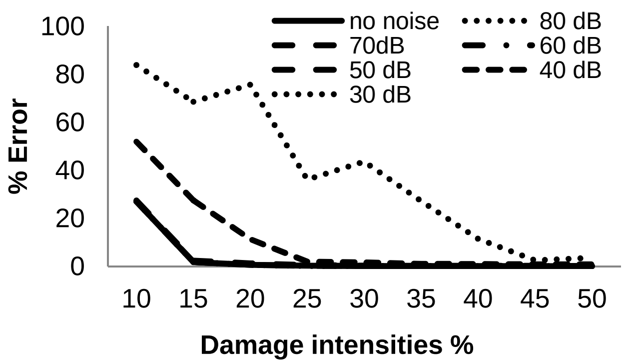

Figure 5 Error in damage location prediction averaged over damage intensities

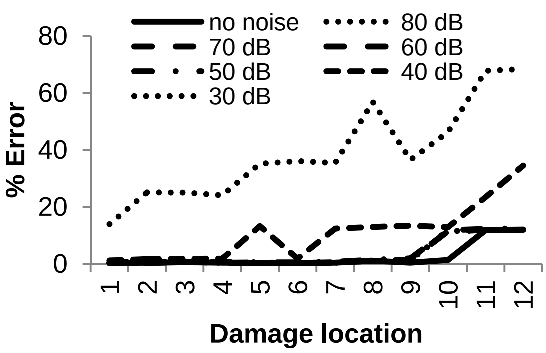

Figure 6 Error in damage location prediction averaged over damage location

The same % errors for considered noise level cases now are averaged over damage intensity and the variation of % error with the damage locations is tabulated in table 3 and is plotted in the Figure 5 averaged over damage intensities and averaged over damage locations in Figure 6 respectively. For no noise case, the % error remains below 2% up to damage location 134mm and it suddenly increases at the locations

142mm and 156mm which are far away from the center of the plate. As we add noise in the data for low noise levels the same error, which was up to 134mm in the case of no noise case, now it is at 125mm. The detailed results are given in the table 2.

Table 3 Error in damage location prediction averaged over damage intensity of plate

Distance (mm) / *noise*	25	53	59	75	84	96	100	107	125	134	142	156
no noise	0.19	0.35	0.60	0.38	0.36	0.30	0.41	1.06	0.38	1.36	11.83	11.97
80 dB	0.19	0.35	0.60	0.38	0.36	0.30	0.41	1.05	1.37	11.45	11.83	11.98
70 dB	0.20	0.36	0.62	0.39	0.36	0.31	0.42	1.08	1.36	11.47	11.85	11.97
60 dB	0.31	0.42	0.64	0.38	0.46	0.32	0.50	1.12	1.38	11.48	11.89	12.01
50 dB	0.44	0.72	0.64	0.74	0.54	0.51	0.70	1.48	1.91	11.75	12.25	12.42
40 dB	1.14	1.67	1.77	1.83	13.21	1.80	12.38	12.95	13.40	12.86	23.40	34.47
30 dB	13.93	25.06	24.98	24.06	35.10	36.06	35.40	56.82	36.48	46.25	67.73	68.36

4.2 STAGE2: DAMAGE INTENSITY PREDICTION

Table 3 summarizes the error in intensity prediction by SVM for those locations found in the stage 1. For noise level up to 50dB the % errors are almost same at low damage intensity.

Table 3 Error in damage intensity prediction averaged over damage location of plate

noise / **Intensity%**	**no noise**	**80 dB**	**70dB**	**60 dB**	**50 dB**	**40 dB**	**30 dB**
10	26.66	26.63	26.57	26.45	31.36	69.18	100.65
15	1.49	1.50	1.58	1.70	2.88	30.24	85.12
20	1.04	1.03	0.99	1.17	2.40	13.11	79.95
25	0.69	0.69	0.69	0.80	1.55	2.68	42.02
30	0.43	0.44	0.47	0.57	1.45	4.29	47.19
35	0.17	0.16	0.17	0.42	0.70	2.40	27.89
40	0.04	0.05	0.11	0.34	0.75	2.63	15.54
45	0.27	0.26	0.26	0.24	0.92	1.68	11.55
50	0.42	0.42	0.43	0.54	0.87	1.83	16.44

The percentage error in intensity prediction is calculated as given below

$$\%\ \text{error} = \frac{\left|\text{Damage intensity}_{\text{predicted}} - \text{Damage intensity}_{\text{actual}}\right|}{\text{Damage intensity}_{\text{actual}}} \text{X } 100 \qquad (22)$$

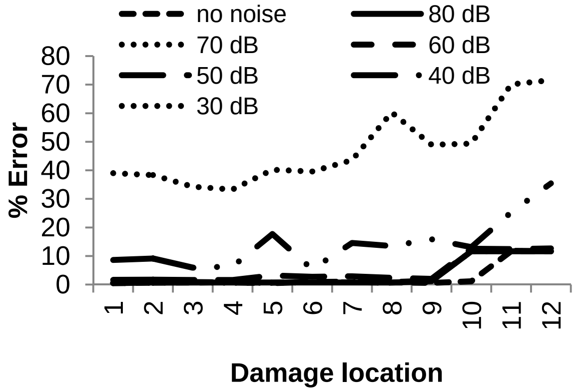

Figure 7 Error in damage intensity prediction averaged over damage location for low noise level

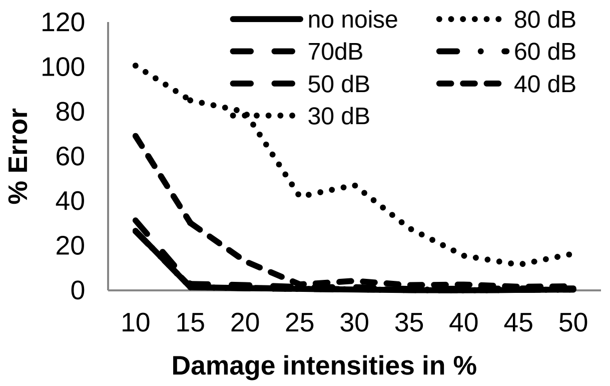

Figure 8 Error in damage intensity prediction averaged over damage location for high noise level

Table 4 represents the detailed values of % error for different noise levels including no noise case, and it is plotted in the Figure 7 and Figure 8. For no noise case the % error is high for only two locations (142mm and 156mm) but, when we add the noise, it is high for three locations for noise levels 80dB to 50dB. The error for the case of 40dB noise is acceptable only for the locations closer to the center of the plate i.e. up to 75mm from the center.

Table 4 Error in damage intensity prediction averaged over damage intensity of plate

Distance / *noise*	25	53	59	75	84	96	100	107	125	134	142	156
no noise	0.48	0.68	0.72	0.74	0.65	0.79	0.74	0.70	0.60	1.17	11.64	11.63
80 dB	0.47	0.67	0.72	0.74	0.67	0.78	0.72	0.70	1.19	11.64	11.63	11.64
70 dB	0.48	0.70	0.73	0.67	0.66	0.78	0.75	0.70	1.25	11.64	11.68	11.63
60 dB	0.62	0.93	0.87	0.71	0.49	0.90	0.96	0.85	1.32	11.77	11.79	11.71
50 dB	1.64	1.69	1.59	1.58	3.20	2.71	2.89	2.32	1.99	12.54	12.40	12.63
40 dB	8.58	9.10	5.93	6.32	17.72	5.36	14.57	13.51	15.83	13.09	25.28	35.44
30 dB	39.00	38.33	34.24	33.27	40.21	39.58	43.56	60.29	48.97	49.32	70.10	71.58

5. CONCLUSIONS

The SVM has been trained with vibration-induced displacements collected at 99 points for the first mode shape as input and damage intensity or location as output. After training, the SVM is able to predict any damage intensity or location of the training set data with almost negligible error. The % error in prediction of damage location and intensity is less at the center of the plate and goes on increasing away from the center. The prediction capability of SVM is degraded with addition of noise in the data. For low noise levels % error remains almost same as that of no noise case in the data that means SVM can tolerate such noise levels with less deviation in the errors.

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