

## **Kalman filter based data fusion for dynamic displacement estimation using LDV and LiDAR**

\*Kiyoung Kim<sup>1)</sup>, Junhee Kim<sup>2)</sup> and Hoon Sohn<sup>3)</sup>

<sup>1), 3)</sup> *Department of Civil Engineering, KAIST, Daejeon 305-600, Korea*

<sup>2)</sup> *Department of Architectural Engineering, Dankook University,*

*Kyonggi-do 305-600, Korea*

<sup>1)</sup> [kiyoungkim@kaist.ac.kr](mailto:kiyoungkim@kaist.ac.kr)

### **ABSTRACT**

A real-time dynamic displacement estimation technique is developed by data fusion of laser Doppler vibrometer (LDV) and light detection and ranging (LiDAR). The velocity measurement of LDV is low level of noise and sampled at high frequency, but has accumulated error during integration. Also, the LiDAR displacement measurement has high level of noise and low sampling frequency. The proposed technique combines the LDV velocity and LiDAR displacement measurements to estimate dynamic displacement with low noise level, high sampling frequency and no integration error. Kalman filter based smoothing algorithms are adopted to remove the accumulated error during the LDV velocity integration and the high noise of the LiDAR displacement in real time. To verify the estimation performance of the technique, a lab-scale test using a cantilever beam is performed.

### **1. INTRODUCTION**

Dynamic displacement is the most important response of a structure. It describes the movement of a structure directly and clearly, as well as can be converted to other dynamic responses such as deflection and strain. Moreover, Kim et al (2011) proposed that the physical parameters of a structure such as mass, stiffness and damping are directly estimated when dynamic displacement is used as input of the state-space model of the structure.

However, it is extremely difficult to directly measure the dynamic displacement of a structure due to its nature of relativeness. For example, very complex and cumbersome scaffold installation is indispensable for measuring displacement using linear variable differential transformer (LDVT). As an alternative, accelerometers are often used in indirect displacement calculation by the double integration of the measured acceleration, but the displacement calculated from the acceleration has large amount of accumulated integration error induced by the sensor bias error of accelerometers. Furthermore, the integration error is not linear in most cases since the noise component

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<sup>1)</sup> Graduate Student

<sup>2)</sup> Associate Professor

<sup>3)</sup> Professor

of the measured signal is not a zero-mean random process (Thong et al. 2004).

As a response of the necessity for easy and accurate dynamic displacement measurement, non-contact sensors have been introduced in recent years. For instance, researchers have tried to apply global positioning system (GPS), along with real-time kinematics (RTK), into displacement measurement (Hwang et al, 2012). Also, vision based sensor has been spotlighted due to its intuitiveness and noncontact nature (Lee and Shinozuka, 2006, Kim and Kim, 2011). In addition, many types of direct displacement measurement sensor have been introduced, such as radar based sensor, laser Doppler vibrometer (LDV) and light detection and ranging (LiDAR).

Although these sensors introduced some possible approaches to measure dynamic displacement in direct and convenient manners, most of them have entry barriers to be applied to displacement measurement in practice. GPS has too low resolution and too high degree of noise, for instance, even though RTK technology is utilized for enhancement of measurement quality (Tamura et al, 2002). Also, in the application of vision based sensor, cameras should be closer to the target plates attached on the structure for better resolution of obtained images; therefore it is not appropriate for measuring large scale structure such as bridges and dams. Although radar based sensor and LDV can measure displacement with high sampling rate, high resolution and low noise level, they cannot measure sudden change of displacement due to impact-like loading such as earthquake (Feigl and Thurber 2009), since the unwrapping algorithm for displacement measurement signal reconstruction utilizes arctangent method which does not give unique solution. In addition, light detection and ranging (LiDAR) can measure dynamic displacement by directly converting its repetitive scan data of target measurement points into displacement, but its precision is not sufficient for measuring displacement under 1cm.

In this paper, a novel dynamic displacement estimation technique using two non-contact sensors is presented. In the proposed method, the velocity measured by LDV and the displacement by LiDAR are combined using Kalman filter smoothing techniques, so that the drawbacks of LDV and LiDAR measurements can be minimized and the displacement with high sampling rate and low noise level can be estimated.

## **2. PROPOSED DYNAMIC DISPLACEMENT ESTIMATION METHOD**

As briefly mentioned before, the proposed method fuses the velocity measured by LDV and the displacement measured by LiDAR using Kalman filter smoothing algorithms. The whole procedure for the method is briefly described in Fig. 1. In this section, the working principles and measurement details of LDV and LiDAR are discussed, and the Kalman filter smoothing algorithms adopted is introduced.

### *2.1 Working principle of LDV*

LDV can measure the out-of-plane displacement and velocity with high sampling rate and low noise. The incident and reflective laser beam have different frequency due to Doppler effect, and the intensity of the interfered beam is captured by the photo detector. This intensity is divided into two orthogonal electric signals, which are used to calculate phase change through arctangent method. Since the phase change is

proportional to displacement, the dynamic displacement is calculated directly. However, when the phase experiences sudden changes greater than  $\pi$ , the phase is wrongfully reconstructed (or unwrapped) from the wrapped signal generated by the arctangent method.

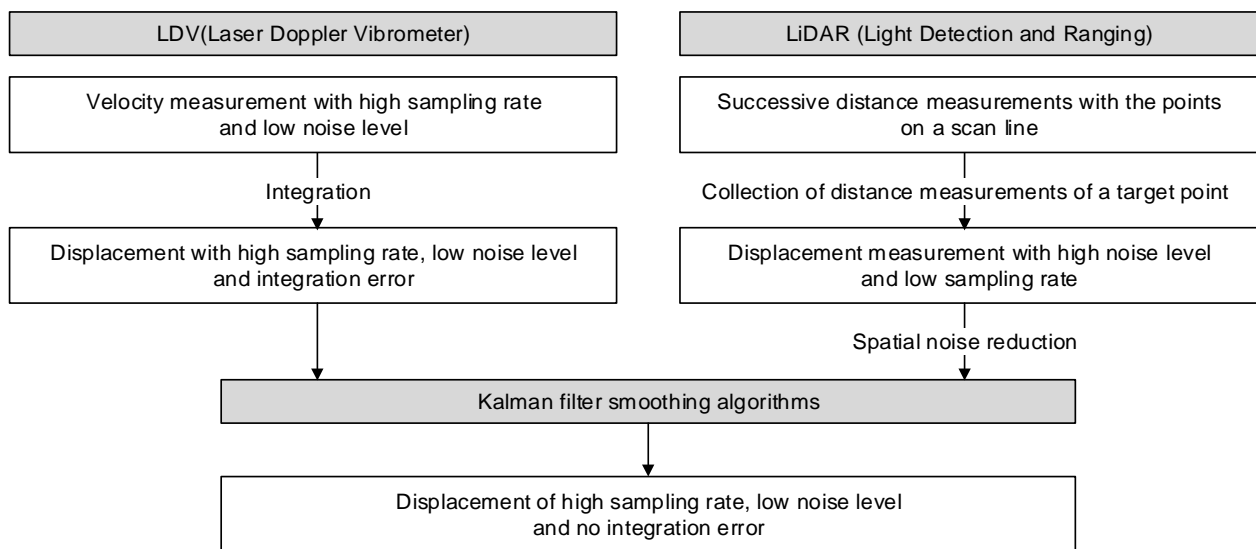


Fig. 1 Flow chart of the proposed method

Although LDV has a critical limitation in measuring dynamic displacement, velocity measurement of LDV is free from the limitation, since the velocity itself is not affected by incorrect calculation by unwrapping process. In analog type velocity decoder, frequency of interfered laser beam is directly converted into velocity in frequency-to-voltage decoder. In digital type velocity decoder, velocity is calculated from the derivatives of phase obtained by arctangent method, but it does not require unwrapping process.

## 2.2 Displacement calculation from LiDAR measurement

Originally, LiDAR is intended for 3D terrestrial scanning. It measures the time-of-flight of emitted laser pulses and transformed the arrival time to distance. The laser pulses sweeps 3D space by rotating its body horizontally and the multi-facet mirror vertically. However, for measuring dynamic displacement, the body rotation should be avoided, since laser pulse need to sweep a target measurement point repetitively. Moreover, the facet mirror rotates fast (100 revolution per second), but the body rotates very slowly (1 revolution in 8 seconds).

For this reason, the proposed method utilizes line scan mode of LiDAR device. Line scan mode enables laser pulses of LiDAR to sweep along a line by fixing the body rotation and rotating the multi-facet mirror only. The scan data contains three

measurements – distance from the multi-facet mirror and scan points along with time stamp and the vertical angle of multi-facet mirror, i.e., the vertical angle of emitted laser pulses. Hence, dynamic displacement can easily be obtained using some simple trigonometric manipulation and signal processing. Fig. 2 describes brief procedure for obtaining dynamic displacement from the scan data of LiDAR.

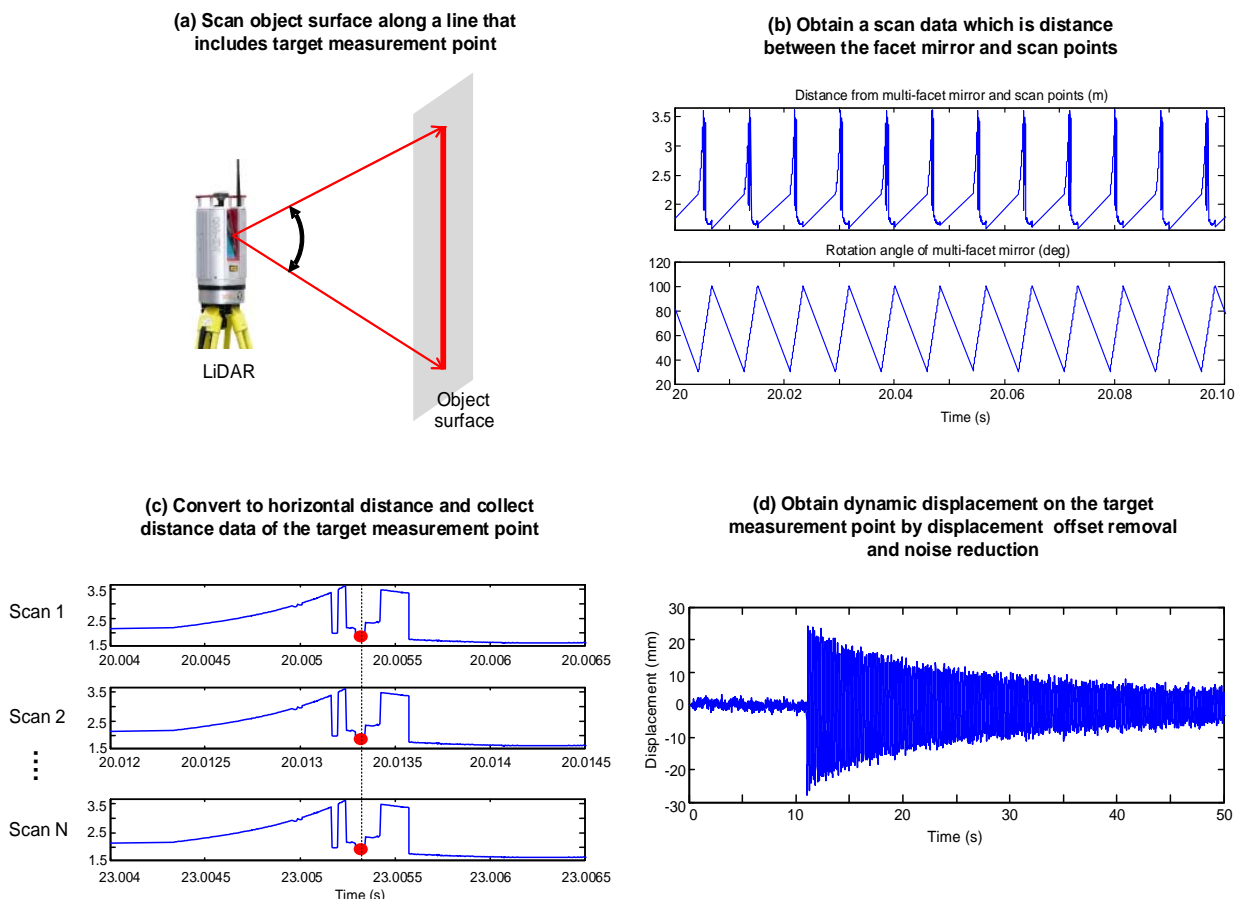


Fig. 2 Dynamic displacement measurement procedure using LiDAR scan data

### 2.3 Kalman filter smoothing techniques for multi-rate data fusion

There are some data fusion algorithms that can be applied to combining two sensor records to remove drawbacks of the measurement of a sensor and estimate more accurate displacement. For example, Moschas and Stiros (2011) complements the drawbacks of GPS sensor by applying high frequency components of acceleration measured by accelerometers. Also, Hong et al.(2013) proposed a method to estimate displacement from acceleration based on a optimization problem, in which intermittent displacement measurements act as constraint of the problem.

In the proposed method, Kalman filter is adopted for multi-rate data fusion. Kalman

filter is known as an optimal state estimator (Kalman 1960) and widely used for various purposes such as noise reduction. One of the most important advantages of Kalman filter in data fusion is that, unlike the aforementioned methods, noise in the sensor measurements is fully considered. Hence, it is much more appropriate to complement the high noise components in LiDAR displacement in the study. Kim et al. (2011) propose a Kalman filter model based on error dynamics, in which the bias error and total error in the measurement are estimated.

The displacement estimation quality can be further enhanced by Kalman filter smoothing (Simon 2006), which is adopted in this study. The estimated displacement by Kalman filter tends to be discontinuous since it corrects accumulated integration error using measured displacement value. Furthermore, Kalman filter cannot estimate displacement precisely in the beginning of the signal since sufficient time steps are needed for the convergence of Kalman gain value. However, these two demerits can be effectively removed by adopting Kalman filter smoothing. There are three algorithms for Kalman filter smoothing – fixed interval, fixed point and fixed lag smoothing. Although fixed interval smoothing is widely used in the enhancement of estimation quality, this algorithm is not able to do estimation in real time, since it averages forward and backward Kalman filter estimation. On the other hands, fixed point and fixed lag smoothing algorithms, which is adopted in the proposed method, can estimate states in near real time.

Both of the smoothing algorithms reduce the uncertainty of estimation and enhance estimation quality by estimating a state using some future measurements. Suppose we have  $k$  measurements and want to estimate the displacement at timestep  $k - N$ . Also, let us define  $x_{a,k}$  is the estimation of  $x$  at timestep  $a$  when  $k$  measurements are made. In fixed lag smoothing, the state vector is defined as

$$\mathbf{x} = \{x_{k,k} \quad x_{k-1,k} \quad x_{k-2,k} \quad \cdots \quad x_{k-N,k}\}^T \quad (1)$$

and the Kalman filter smoothing model is constructed as

$$\begin{bmatrix} x_{k+1}^- \\ x_{k,k+1} \\ x_{k-1,k+1} \\ \vdots \\ x_{k-N,k+1} \end{bmatrix} = \begin{bmatrix} A_k & 0 & \cdots & 0 & 0 \\ I & 0 & \cdots & 0 & 0 \\ 0 & I & \cdots & 0 & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & 0 & \cdots & I & 0 \end{bmatrix} \begin{bmatrix} x_k^- \\ x_{k-1,k} \\ \vdots \\ x_{k-N,k} \\ x_{k-N-1,k} \end{bmatrix} + \begin{bmatrix} I \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} w_k \quad (2)$$

$$y_{k+1} = [H_{k+1} \quad 0 \quad \cdots \quad 0] \begin{bmatrix} x_{k+1}^- \\ x_{k,k+1} \\ x_{k-1,k+1} \\ \vdots \\ x_{k-N,k+1} \end{bmatrix} + v_k$$

where  $x_k^-$  is prior estimate of  $x_k$  and is equivalent to  $x_{k,k}$ , and  $A_k$  is a matrix that describes the process of the integration of LDV velocity and the accumulated integration error in the Kalman filter model, and is defined as

$$A_k = \begin{bmatrix} 1 & \Delta t \\ 0 & 1 \end{bmatrix} \quad (3)$$

where  $\Delta t$  is the interval of the timesteps. Also,  $w_k$  and  $v_k$  is the noise of LDV velocity

and LiDAR displacement measurements respectively, and  $y_{k+1}$  is LiDAR displacement measurement.  $H_k$  is a matrix which express the dynamics of LiDAR, and is defined as  $[1 \ 0]$  in this case.

For fixed point smoothing, the state vector is defined in a simpler way,

$$\mathbf{x} = \begin{Bmatrix} x_k^- \\ x_{j,k} \end{Bmatrix} = \begin{Bmatrix} x_{k,k} \\ x_{j,k} \end{Bmatrix} \quad (4)$$

and the Kalman filter smoothing model is constructed as

$$\begin{aligned} \begin{bmatrix} x_{k+1}^- \\ x_{j,k+1} \end{bmatrix} &= \begin{bmatrix} A_k & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} x_k^- \\ x_{j,k} \end{bmatrix} + \begin{bmatrix} I \\ 0 \end{bmatrix} w_k \\ y_{k+1} &= [H_{k+1} \quad 0] \begin{bmatrix} x_{k+1}^- \\ x_{j,k+1} \end{bmatrix} + v_{k+1} \end{aligned} \quad (5)$$

As shown in eq. (1) and (2), fixed lag smoothing algorithm requires larger size matrices of greater computational burden as more future measurements are considered, but estimate  $x_{k-N,k+1}$  without any iteration. On the other hand, fixed point smoothing requires iteration to estimate  $x_{k-N,k+1}$ , but the matrix size does not increase when the number of the considered future measurements increases.

### 3. LAB SCALE TEST

To demonstrate the performance of the proposed method, a series of lab scale test is performed using a steel cantilever beam, whose height is 1m and thickness is 6mm, as shown in Fig. 3. Impact hammer is used for the excitation of the beam, and the displacement and velocity are measured at the top of the beam.

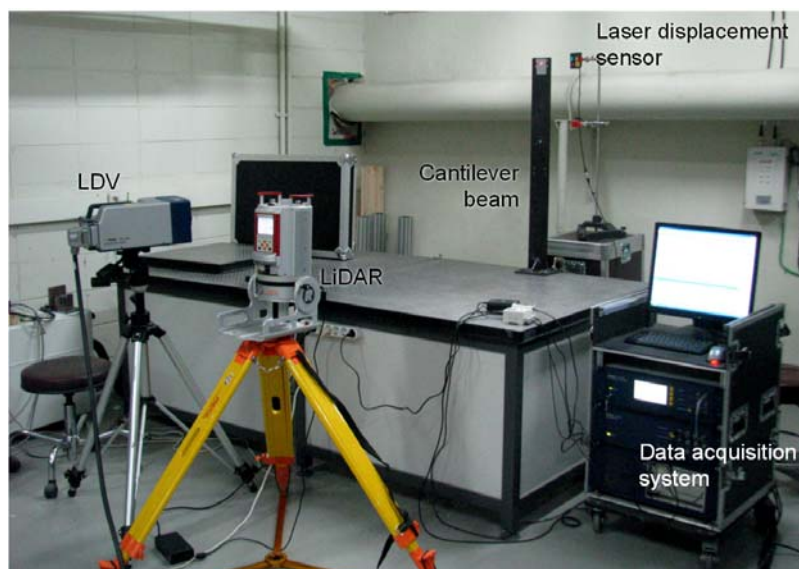


Fig. 3 Test setup

The velocity is measured by LDV (PSV-400, Polytec), which is placed 1.6m away from the beam, and the displacement is measured by LiDAR (VZ-400, RIEGL). Also, a laser displacement sensor (CD4-350, Optex-FA) is installed to measure reference displacement to compare with the estimation results of the proposed method. The LDV velocity and the reference displacement is sampled at 1024Hz, whereas the LiDAR displacement is sampled at 108Hz.

The measured velocity and displacements are shown in Fig. 4. As mentioned before, the displacement measured by LiDAR (Fig. 4(b)) suffers from high level of noise. The velocity measured by LDV is converted to displacement through integration (Fig. 4(c)), but the resultant displacement is distorted due to the integration error, which is caused by the accumulation of sensor bias.

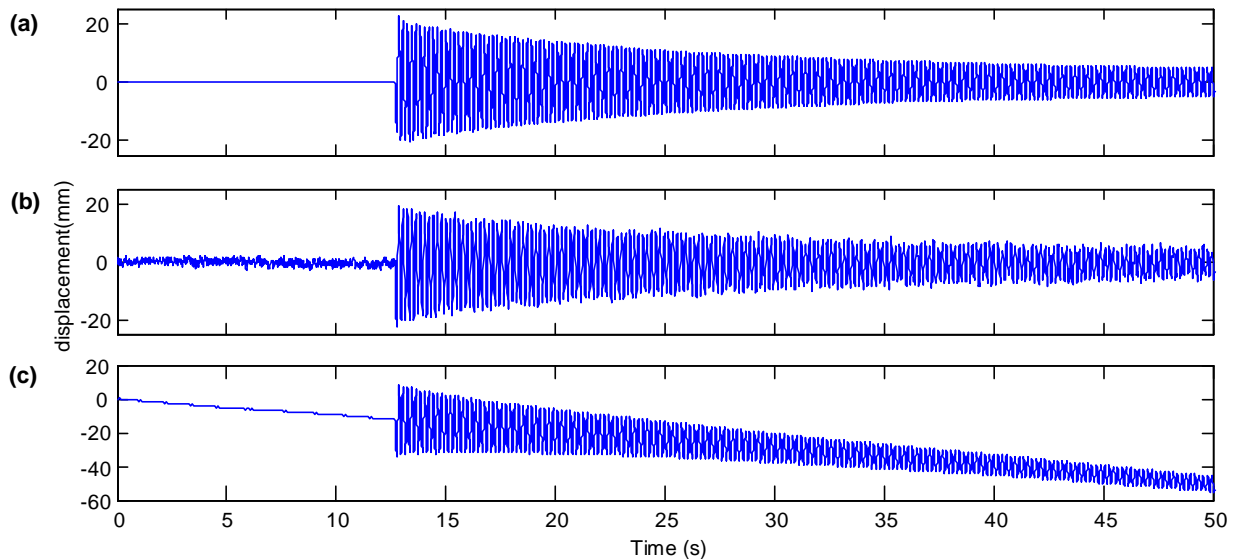


Fig. 4 (a) displacement measured by laser displacement sensor; (b) displacement measured by LiDAR; and (c) integration of velocity measured by LDV

The estimation results of the proposed method are shown in Fig. 5. Figs. 5(a)-(d) are the estimated displacements using fixed point smoothing, and (e)-(h) are the result from fixed lag smoothing. As shown in the figures, the displacements estimated by two smoothing algorithms are identical; however, computational time of fixed lag smoothing is 8.4% faster than fixed point smoothing, since standard Kalman filtering is additionally performed for initial value setup for iteration at each time step in fixed point smoothing.

Fig. 5 illustrates that the estimation quality is more improved as more future measurements are used in the estimation algorithm. This is also shown in Fig. 6, which illustrates the estimated bias error in the LDV velocity. The unstable regions seen on the very beginning of the estimations are rapidly suppressed as the time delay increases. As a result, the estimated displacement with 0.5 sec delay, which corresponds to 640 future measurements, shows a close agreement with the reference displacement measured by laser displacement sensor.

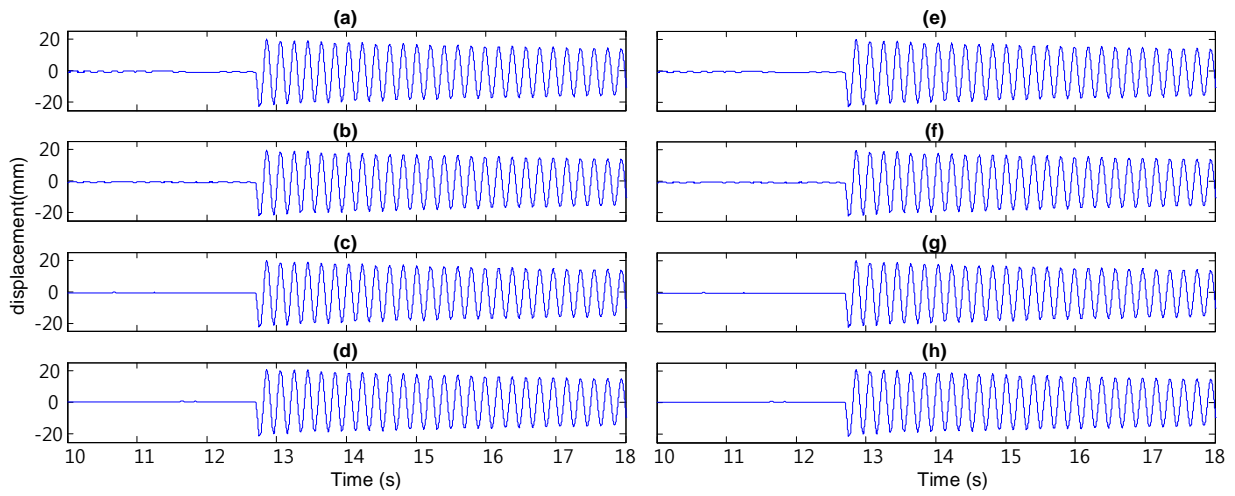


Fig. 5 Estimation result (closed up from 10 to 18 sec) of fixed point smoothing with time delay of (a) 0 sec, (b) 0.1 sec, (c) 0.2 sec, (d) 0.5 sec, and fixed lag smoothing with time delay of (e) 0 sec, (f) 0.1 sec, (g) 0.2 sec, (h) 0.5 sec

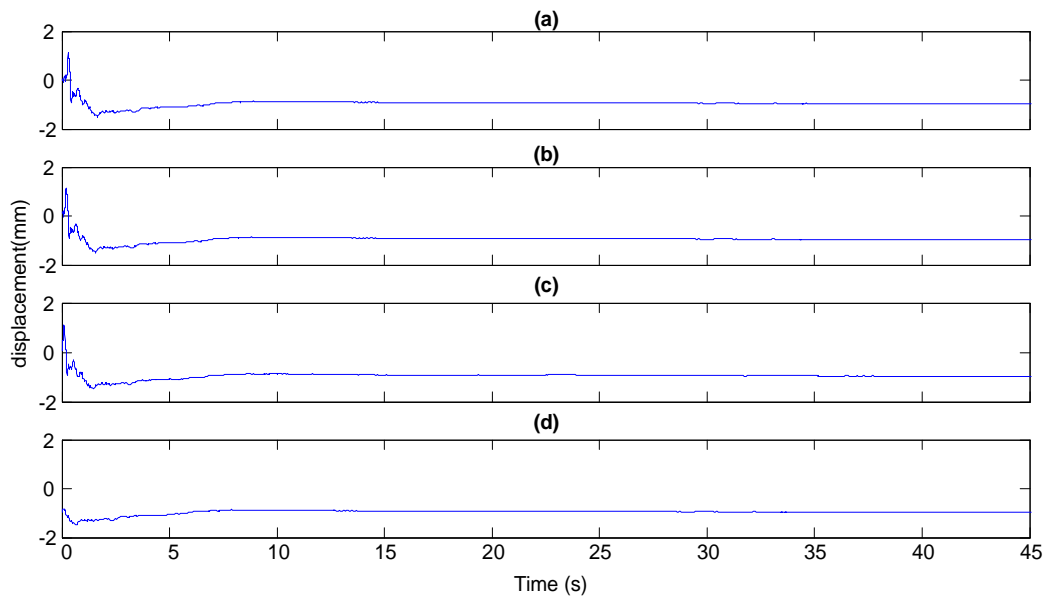


Fig. 6 Estimated integration error using Kalman filter smoothing with time delay of (a) 0 sec, (b) 0.1 sec, (c) 0.2 sec, (d) 0.5 sec

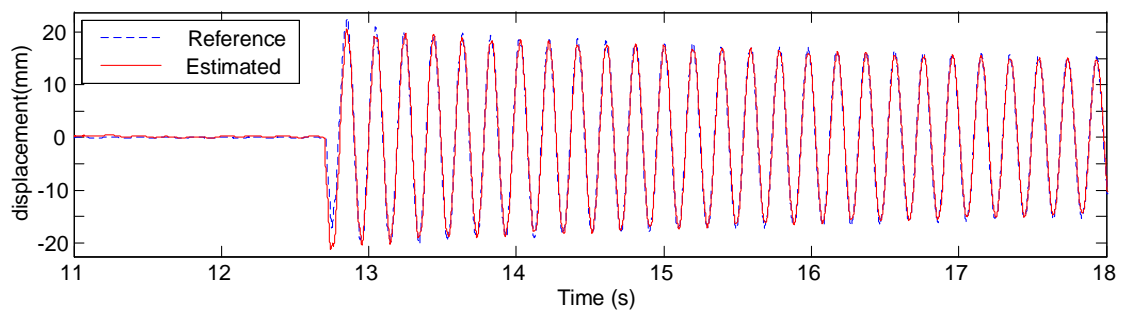


Fig. 7 Comparison of estimation result (0.5 sec delay) and reference displacement measurement



## 4. CONCLUSIONS

This paper proposed a novel displacement measurement method based on the multi-rate data fusion of LDV velocity and LiDAR displacement using Kalman filter smoothing techniques. The Kalman filter based approach enables the method to combine the advantages of LDV and LiDAR effectively and estimate displacement with high sampling rate, low noise level and no integration error.

The displacement estimation method is expected to be applied to various areas instead of conventional sensors such as LVDTs and accelerometers. Furthermore, the method can be further refined by adopting an effective noise reduction algorithm of LiDAR displacement and introducing nonlinear approach on sensor bias estimation.

## ACKNOWLEDGEMENTS

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