

## Optimum structural configuration of irregular buildings 2. Inelastic systems

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### ABSTRACT

Multi-story inelastic buildings, classified by Eurocode 8 (EC8-2004) as irregular buildings, are presenting a practically translational response under a ground motion when (i) they are detailed as planar structures under a code load and (ii) the first mode center of rigidity ( $m_1$ -CR) lies on the axis passing through the centers of floor masses (mass axis). The concept and the usefulness of this point in elastic multistory structures is presented in the companion paper but it retains its value in structures composed by inelastic bents, since the coincidence of its location with the mass axis determines the optimum arrangement of the lateral load resisting elements, in terms of enabling the system to sustain minimum torsional response in the case of a strong ground motion. Inelastic building structures, detailed as above may be regarded as torsionally balanced multistory systems and this is demonstrated in eight story buildings, composed by dissimilar bents, under the ground motions of Kobe 1995 (component KJM000) and Friuli 1976 (component Tolmezzo E-W).

### 1. INTRODUCTION

The seismic vulnerability of eccentric building structures has been demonstrated in many strong earthquakes and it has proved to be a serious cause of severe damage. In elastic single-story buildings with rigid floor diaphragms (decks), this behaviour is due to the eccentricity of the centre of rigidity (CR) from the centre of mass (CM). The properties of CR arose from the consideration of the response of these structures under a static loading: any lateral load passing through CR causes only a translation of the floor deck and any torque applied on the deck causes a rotation about CR. Because of this dual property, sometimes CR is also defined as the 'center of twist' and in practice this point is determined as the center of element stiffnesses. The response of such systems has been extensively analyzed in the past (e.g. Tso and Dempsey,

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1980; Dempsey and Tso, 1982) and it has been demonstrated that unexpected torsional motions may magnify the displacement profile of some structural elements in the perimeter of the building and lead to member failure and possibly to building's collapse. In the case of a unidirectional ground motion, an optimal, simply translational response, may be obtained when a coincidence of CR with CM is achieved. Such systems, with elastoplastic elements, having a strength distribution proportional to the stiffness distribution (usually called torsionally balanced models), present also a purely translational inelastic response under strong ground excitations and for this reason they are used as 'reference' models in relevant studies (e.g. Correnza et al, 1994, 1995; Chandler et al, 1996; Wong and Tso 1994). This behaviour is attained because yielding is initiated at the same instant for all elements and the element force balance about CM is preserved into the inelastic phase, leading to a translational response throughout the ground shaking.

The response of multi-story structures is more complicated than that of single-story systems. At first, in the elastic phase, there is not a clear definition of a vertical axis (usually referred to as 'elastic axis') with properties similar to those of CR in single story systems. Modern codes, although contain extensive criteria about the regularity of building structures, are less specific about the 'center of stiffness' of multi-story buildings. For example EC8-2004 (clause 4.2.3.2) describes this point in frames and systems of slender walls as the center of the moments of inertia of the cross-sections of the vertical elements. Reviewing the literature (a short description is presented below) it may be seen that such a definition applies only in buildings in which the lateral resistance is provided by a single type of bents (e.g. flexural shear walls uniform over the height) or in the general case of proportionate structures (the rare class of buildings where the ratios between the stiffnesses of the various bents are constant over the height of the structure). The lateral resistance of real buildings is usually provided by a combination of different types of bents, (frames, walls, coupled walls, wall-frame assemblies) since such a structural configuration responds more effectively during a strong ground motion (Paulay and Priestley 1992).

The absence of an acceptable definition of the 'elastic axis' has led to many investigations about the issue of establishing a set of points located at the floor levels of a multi-story building with properties similar to those of CR of single-story systems. Since the oscillatory response of single story systems is due to the distance between CM and CR (usually referred to as static eccentricity), early studies on elastic multi-story systems (e.g. Poole 1977; Humar 1984; Smith and Vezina 1985; Jiang et al, 1986) have led to different definitions about the magnitude of this eccentricity at the various floor levels. Cheung and Tso (1986) proposed the 'rigidity centers (CRs)' for structural applications. These are the points that when a given distribution of lateral loading passes through them only translational movement of the floors will occur. However, apart from the proportionate structures these points are load dependant and their space distribution is very irregular, even in uniform structures composed of different types of bents. In another study by the same authors (Tso and Cheung 1986) it was demonstrated that it is possible to determine another set of floor points which do not undergo any translational displacement when the structure is subject to applied torques only. These are defined as 'centers of twist' (CTs) but unlike single-story systems they can not be in general identified as CRs. Acknowledging the cumbersome

analytical process required to determine the location of CRs in multi-story buildings, Goel and Chopra (1993) propose an indirect procedure of three static analyses, which satisfy the code torsional provisions without the need to first determine the location of CRs. However, the irregular space distribution of these points (CRs at different levels may be found on either side of the centroids of the corresponding floors and probably at a distance outside the practical size of a floor deck), makes simply unrealistic any attempt to prove that a building with the centers of masses located at CRs constitutes a system of minimum rotational response when its bents are deformed beyond their elastic limits. The need to determine a vertical axis (at least in uniform over the height building structures) which can be identified as a minimum torsion axis has led to many approximations. Makarios and Anastassiadis (1998a,b) introduced the 'axis of optimum torsion' with promising results (Makarios 2005, 2008; Makarios et al 2006). This axis can be determined by means of an indirect static analysis by applying a set of floor torques equal in magnitude to the lateral forces at the same floors. An alternative mathematical procedure was proposed by Marino and Rossi (2004). In recent years the rotational response of multistory asymmetric structures has received major attention (a qualitative overview is presented by De Stefano and Pintucchi, 2008) and an alternative strategy for controlling this response in multistory structures designed to withstand ground motions into the inelastic region is presented by Aziminejad et al (2008) and Aziminejad and Moghadam (2009). In these studies the problem of element strength distribution on the rotational response of the structure is studied by using a proper configuration of the centers of mass, strength and stiffness according to the findings obtained from single story systems with elements having strength dependant stiffness (Myslimaj and Tso, 2002, 2004). Interesting results are also highlighted by Lucchini et al (2008). In the studied cases of shear type 3-story inelastic buildings it was shown that deep into the nonlinear range the storey shears producing the maximum floor displacement demands in all the different lateral resisting elements of the building are located at the centers of resistance. In the companion paper it is shown that when the mass axis of eccentric elastic buildings is passing through  $m_1$ -CR, the seismic response is practically translational, which means that this point can be considered as the 'center of stiffness'. The present definition  $m_1$ -CR is based on author's earlier papers (Georgoussis 2009, 2010, 2012) modified in such a way to reflect the response of buildings with dissimilar bents with a higher degree of accuracy. The usefulness of this point in the post-elastic range is examined in the present study.

The aim of the present work is dual: (i) to demonstrate that inelastic multi-story systems, classified as irregular buildings according to EC8-2004, with the mass axis passing through  $m_1$ -CR, constitute torsionally balanced systems when the element strength assignment is obtained from a planar static analysis under a set of lateral forces simulating a 'seismic loading', and (ii) that the definition of the 'center of stiffness' according to the aforementioned code is unsuccessful in the sense that the coincidence of this point with the mass axis is not implying a translational response. It is shown that 8-story building models, composed by different types of resisting bents (moment resisting frames and structural walls) with a strength assignment obtained from a planar static analysis under a code (triangular) lateral loading, present minimum rotational response under ground excitations, when the mass axis is passing through  $m_1$ -CR. Any other arrangement of the resisting bents produces an increased torsional response in

terms of both top rotations and base torques. Therefore, a structural design that is based on the criterion of having  $m_1$ -CR as close as possible to the mass axis enforces the building to undergo a low or moderate torsional response during a strong motion. It is obvious that such a 'structural property' can be easily attained by the practicing engineer during the preliminary stage of a structural application.

In the models examined inelasticity in frames was assumed to occur by allowing plastic hinges at the ends of the beams and at the foot of the ground floor columns, while in the structural walls, inelasticity was taken into account by means of plastic hinges at their bases. Two characteristic ground motions (Kobe 1995, component KJM000 and Friuli 1976, component Tolmezzo E-W), selected from the strong ground motion database of the Pacific Earthquake Engineering Research (PEER) Center (<http://peer.berkeley.edu>) and scaled to a PGA=0.5g, are used to compare rotations and base torques of the models examined.

## 2. CONSTRUCTING A TORSIONALLY BALANCED MULTI-STORY BUILDING

Consider a multi-story mono-symmetric building model, with an orthogonal floor plan (Fig. 1(a)) which is subjected to a ground motion along the y-axis of asymmetry. The resisting bents of this model structure are aligned along the principal axes x and y, the origin of which coincides with the center of mass. As outlined in the companion paper, to construct a torsionally balanced system, the first requirement is to have a translational response in the elastic range, that is, when all the resisting elements are stressed below their limits. In common building systems, composed by dissimilar types of resisting bents, this can be achieved with remarkable accuracy when the vertical mass axis is passing through  $m_1$ -CR. Because of the symmetry in x-direction,  $m_1$ -CR lies on the horizontal principal axis, but the determination of its x-coordinate requires the calculation of the first mode effective frequencies of the resisting  $j$ -bents aligned along the y-direction. Assuming that the first mode effective mass of the symmetrical counterpart structure (Fig. 1(b)) is equal to  $M_{y1}^*$ , the aforementioned effective frequencies can be evaluated from the properties of the planar multi-story subsystems which (i) have the same floor masses as the real structure and (ii) their lateral resistance is provided uniquely by the corresponding  $j$ -bent. Therefore if the first mode frequency and effective mass of the  $j$ -bent (subsystem) are equal to  $\omega_{j1}$  and  $M_{j1}^*$ , the effective frequencies of the resisting bents along the y-direction, are given as

$$\bar{\omega}_{j1} = \omega_{j1} \sqrt{\frac{M_{j1}^*}{M_{y1}^*}} \quad (1)$$

It is evident that when the lateral stiffness of a building structure is composed by similar types of bents (e.g. flexural shear walls), their effective frequencies  $\bar{\omega}_{j1}$  are respectively equal to  $\omega_{j1}$  ( $j=1,2,\dots$ ).

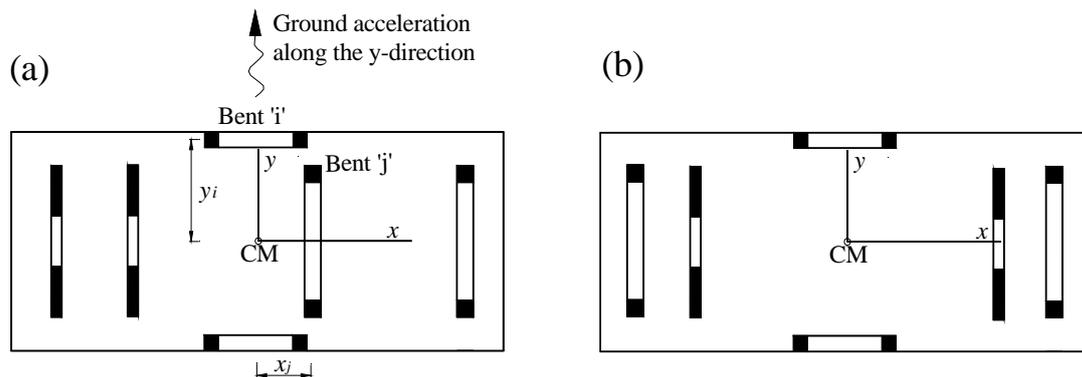


Fig. 1 (a) floor plan of typical unbalanced system; (b) the symmetrical counterpart structure

Denoting with  $x_j$  the location of the  $j$ -bent in the assumed reference system, the  $x$ -coordinate of  $m_1$ -CR is given by the formula

$$e = \frac{\sum(x_j \bar{\omega}_{j1}^2)}{\sum(\bar{\omega}_{j1}^2)} \quad (2)$$

Taking into account the findings of the companion paper that in the elastic phase a virtually translational response is obtained when the mass axis is passing through  $m_1$ -CR, a strength assignment derived from a planar static analysis, under a horizontal loading that has the shape of the first mode of vibration (or even the simpler 'code triangular' distribution), could produce a similar response into the inelastic region as a result of the almost concurrent yield initiation in the resisting bents in the direction of the ground motion. Obviously, this static analysis is equivalent to the analysis of the symmetrical counterpart structure of Fig. 1(b) under the same loading passing through the mass axis. The numerical data that follow aim in demonstrating that in common buildings, where the strength assignment is implemented as above, any arrangement of the resisting bents which results in a location of  $m_1$ -CR in a clear distance from the mass axis constitutes a model structure of increased rotational response, while, on the other hand, an optimum (minimum) torsional response is obtained when the distance of  $m_1$ -CR from the mass axis is practically negligible.

### 3. SYSTEMS ANALYZED

The 8-story model structure described in the companion paper (Fig. 2), is examined under the ground motions of Kobe 1995 (component KJM000) and Friuli 1976 (component Tolmezzo E-W), shown in Fig. 3 and scaled to a PGA=0.5g (unidirectional excitations along the y-axis). All resisting elements (bents) are assumed to have only in-plane stiffness and their strength assignment is based on a planar static analysis under an external lateral loading with floor forces having the shape of the 'inverted triangle' and summing to a base (design) shear equal to  $V_d=2400\text{kN}$  (approximately

equal to 25% of the total weight of the structure, since the mass per floor is  $m=120\text{kNs}^2/\text{m}$ ). More specifically, allowing for plastic hinges at the bases of walls WA and WB and detailing frame FR according to the strong column-weak beam philosophy (that is, allowing plastic hinges at the ends of the beams and at the foot of the ground floor columns), this static analysis provides the following data: (i) the bending (yield) capacity at the plastic hinges at the base of walls WA and WB are respectively equal to 25025 and 13475 kNm and, (ii) the bending (yield) capacity of the plastic hinges at the ends of the beams of FR (from the top downwards) is equal to 519, 609, 594, 583, 544, 475, 368, and 222 kNm respectively, while the corresponding capacity at the plastic hinges at the base of the ground columns of FR equals 383 kNm. The strength of walls Wx, aligned in the y-direction, is the same as WB. As in the companion paper, Wall WA and frame FR are assumed to be located at a fixed positions, the first on the left of CM in a distance equal to 4m and the second on the right of CM at a distance of 6m, while wall WB is taking all the possible locations along the axis of symmetry.

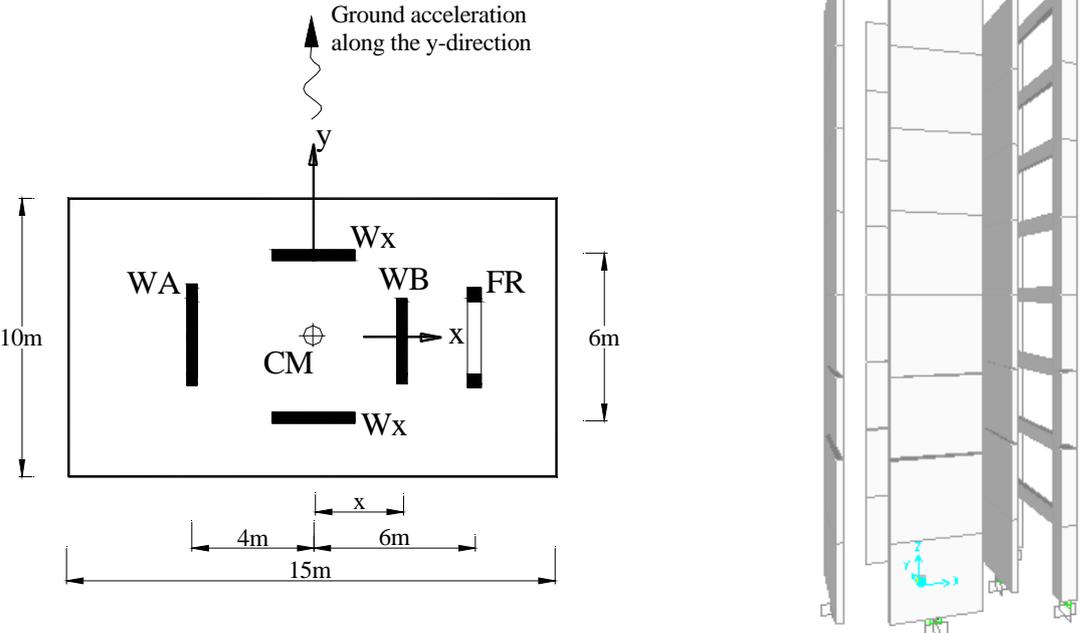


Fig. 2 The 8-story model structure (plan and perspective)

All analyses were performed by means of the program SAP2000-V11, using inelastic link elements at the assumed locations of plastic hinges. The moment-rotation relationships of these elements were assumed bilinear with a post-yielding stiffness ratio of the generalized load-deformation curve, equal to 4%. The nonlinear response history analyses were performed using the numerical implicit Wilson- $\theta$  time integration method, with the parameter  $\theta$  taken equal to 1.4.

As the purpose of these analyses is to compare the torsional response of structural assemblies, which are composed by the same inelastic bents but at different locations of wall WB, it is worth mentioning here that minimum elastic rotational response is expected when  $m_1$ -CR lies on the vertical mass axis. This means that the x-coordinate of this point, as given by Eq. (2), should be zero, resulting in an 'optimal' location of WB (as demonstrated in the companion paper) equal to  $\bar{x} = 0.55$ .

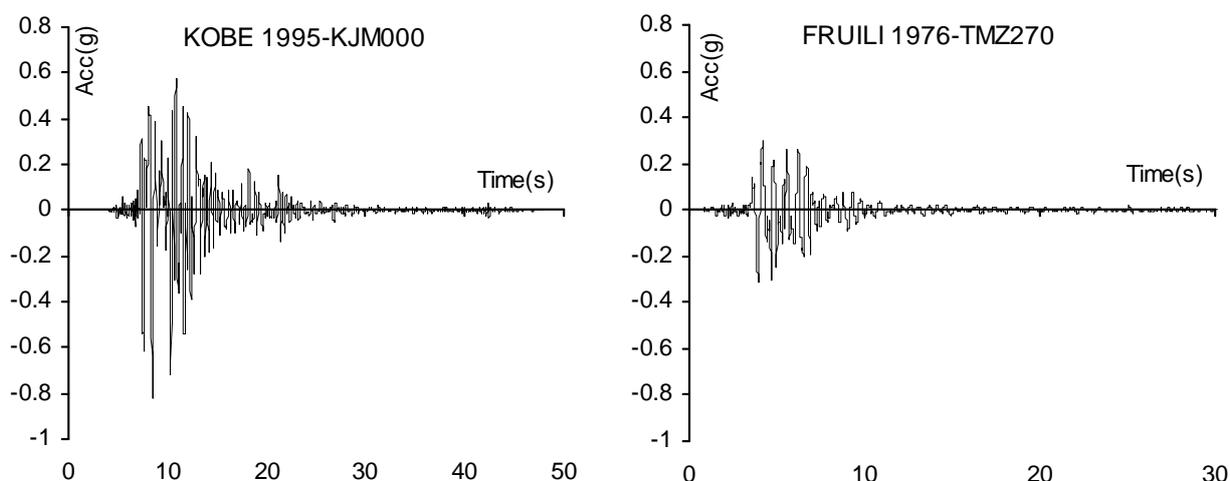


Fig. 3 Ground motions considered (unscaled records)

#### 4. ELASTIC AND INELASTIC RESPONSES UNDER STRONG GROUND MOTIONS

The torsional response of the model structure under the assumed Kobe and Friuli excitations for any possible location of wall WB (denoted by the normalized coordinate  $\bar{x} = x/r$ ), is shown in Fig. 4 and 5. Three response parameters, obtained by time history analyses assuming a 5% damping ratio, are shown for both the elastic and inelastic systems: top rotations,  $\theta$ , normalized base shears and normalized base torques. The red lines represent the peak elastic response (top rotations:  $\theta_e$ , are shown by dashed lines, normalized base shears:  $\bar{V}_e = V_e/V_d$  by solid lines and normalized base torques:  $\bar{T}_e = T_e/rV_d$  by dotted lines) and the corresponding black lines represent the peak inelastic behavior ( $\theta_{in}$ ,  $\bar{V}_{in} = V_{in}/V_d$ ,  $\bar{T}_{in} = T_{in}/rV_d$ ). For both the assumed ground excitations, minimum rotational response ( $\theta_e$  and  $\bar{T}_e$ ) of the elastic system appears when the wall WB approaches the coordinate  $\bar{x} = 0.5$ , which is very close to the value  $\bar{x} = 0.55$ , predicted by equating Eq. (2) to zero.

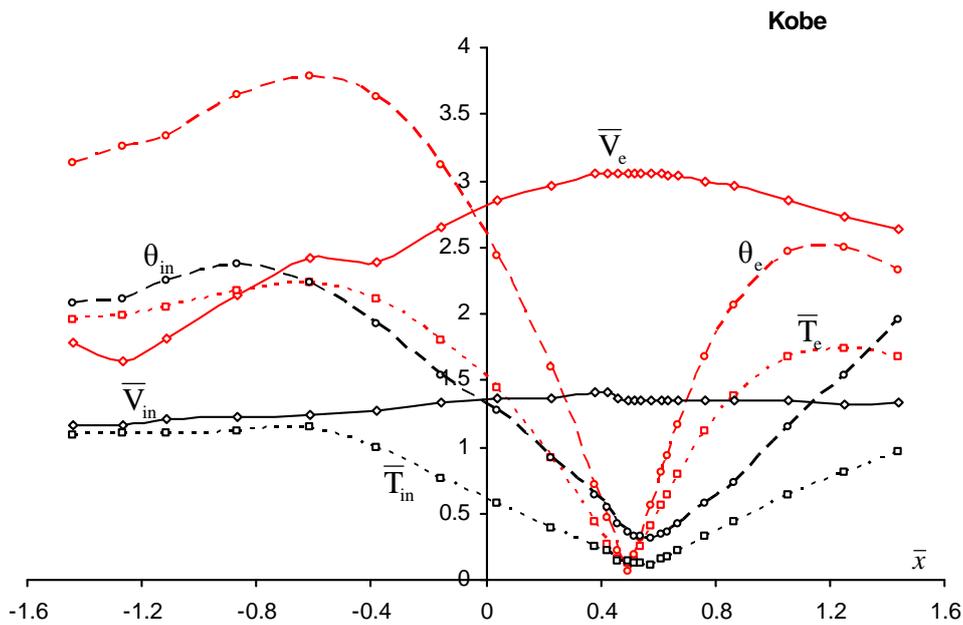


Fig. 4 Top rotations ( $\times 10^{-2}$ , rads) and normalized base shears and torques of the assumed models under the Kobe 1995- KJM000 ground motion

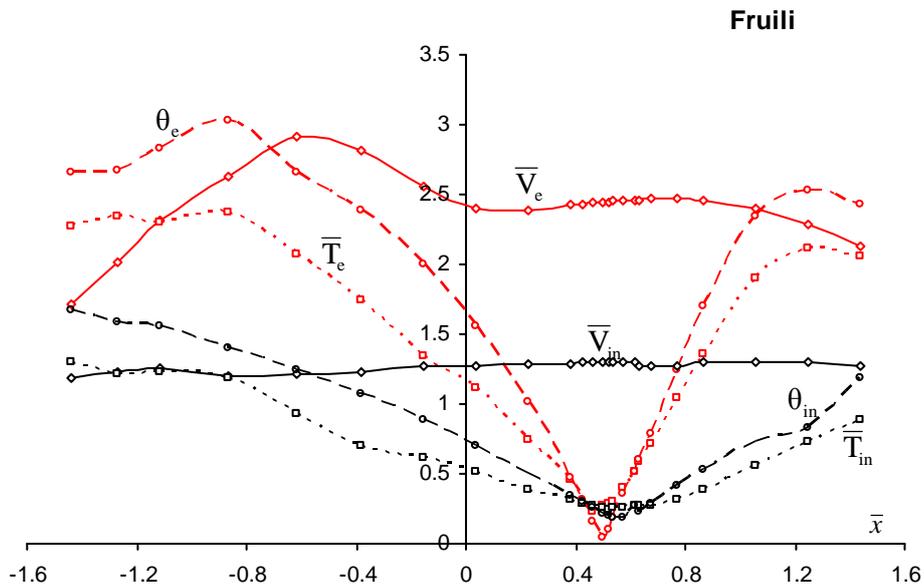


Fig. 5 Top rotations ( $\times 10^{-2}$ , rads) and normalized base shears and torques of the assumed models under the Friuli 1976-TMZ270 ground motion.

This difference, although negligible from the engineering point of view (the actual distance between the two 'locations' of WB is about 0.26m), is attributed to the fact that the point  $m_1$ -CR is determined from the first mode frequencies of the component bents, neglecting the higher mode frequencies. In Fig. 4, it is noticeable that the configuration

of minimum torsional elastic response corresponds to the peak value of base shear, but this is not the case in Fig. 5. In general, the results of these elastic analyses are in agreement with those presented in the companion paper, which were obtained by means of response spectrum analyses.

The results of the inelastic analyses (black lines in Figs. 4 and 5) are, in general terms, lower but similar to the elastic ones and the minimum torsional response appears for a location of WB almost equal to  $\bar{x} = 0.55$ . Since the same value is obtained by equating Eq. (2) to zero, it can be said that for the particular example structure the aforementioned equation predicts more accurately the location of FR which produces minimum rotational response in the inelastic phase. The rotational responses,  $\theta_{in}$ , of the inelastic systems are, in general, lower than those of the elastic ones and the variation of  $\theta_{in}$  and  $\bar{T}_{in}$  are smoother than that of the elastic systems. The peak base shear  $\bar{V}_{in}$  is practically constant as it is determined by a deformation profile in which all bents are well into the inelastic region in the same direction. The less sensitive variation of  $\theta_{in}$  and  $\bar{T}_{in}$  is attributed to the fact that yielding detunes the torsional coupling, resulting in an inelastic response that has the tendency to be more translational and less rotational (Goel and Chopra, 1991). It is also worth mentioning here that the right edge element, when minimum rotation is obtained, in both the elastic and inelastic systems, is the frame FR, which is a bent quite different from a wall. Yielding in a frame may be initiated in any of the assumed locations of the plastic hinges (in beams or the ground columns), but this bent becomes fully plastic after the formation of all the plastic hinges. This means that full plasticity occurs with a time lag after initial yielding, while in a wall full plasticity is concurrent with initial yielding.

As mentioned in the companion paper, according to EC8-2004 (Clause 4.2.3.2), most of the structural configurations examined herewith should be considered as irregular buildings. This applies to the configuration ( $\bar{x} = 0.55$ ) which produces minimum rotational response. Therefore, the concept of  $m_1$ -CR may be applied with the confidence in buildings classified by the aforesaid code as irregular structures. On the other hand the definition of this code about the center of stiffness, as the center of the moments of inertia of the cross sections of the vertical members, seems unsuccessful. According to this definition, this point coincides with CM when the wall WB is located at far right side of the deck ( $\bar{x} = 1.44$ ), but at this position the corresponding elastic and inelastic models present an excessive rotational behavior.

## 5. CONCLUSIONS

The inelastic rotational response of mixed-bent-type multistory building structures is affected by the yielding behavior of the resisting elements. Large torsional moments are generated when one element yields and the other remains into the elastic phase. On the other hand, a small time lag in the initiation of yielding among the various elements implies a reduced rotational response and further, an almost in-phase element yielding produces minimum torsional response. Since in the elastic state, a virtually translational

response is obtained when the mass axis passes through  $m_1$ -CR or within a close distance from this point, this response is preserved into the inelastic phase when the element strength assignment is obtained from a planar static analysis under a set of lateral forces simulating the distribution of the inertia forces of a translational response. To perform such a static analysis, a simple distribution of lateral forces over the height according to the 'inverted triangle' rule is adequate for low or medium height structures. Such a strength assignment satisfies the criterion of concurrent yielding in the resisting elements, when the elastic behavior is practically translational. This is demonstrated in typical 8-story monosymmetric building models, composed of dissimilar types of resisting bents (a moment resisting frame and two structural walls) and analyzed under the Kobe and Friuli excitations. The behavior of these models is investigated in a parametric form: one of the walls and the frame are at fixed locations while the other wall was assumed to take any possible position along the axis of symmetry. According to EC8-2004 (Clause 4.2.3.2), most of the structural configurations examined herewith should be considered as irregular buildings and also the configuration which presented the minimum rotational response. This means that even in irregular buildings the response may be essentially translational when the mass axis passes through  $m_1$ -CR. These findings may be found useful in structural applications: since the determination of  $m_1$ -CR is very simple, the designer, from the beginning of a structural design, may create an element arrangement with the mass axis passing close, as possible, to  $m_1$ -CR.

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