

## **Numerical simulation of projectile penetrating concrete target using SPH-Lagrange coupled method**

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### **Abstract**

This paper demonstrates the use of two different numerical simulation methods for predicting the penetration and deceleration over time of a high velocity projectile (276 m/s to 456 m/s) penetrating into a high strength concrete target with a confined pressure of 39 MPa. Lagrangian formulation with 3D solid Finite Elements and Smooth Particle Hydrodynamics coupled with Lagrangian Finite Elements (SPH-FE) are chosen for the discretization of the simulation problem. In the SPH-FE method, SPH formulation, a meshless method, is used to model the area of penetration and the remaining part of the structure using finite elements. This helps to avoid mesh problem and also largely reduced the total computational resources compared to only finite elements are used to model penetration problem. Results from both simulation methods are compared to experimental results from Forrestal et al and show good correlation. With the appropriate technique of the SPH-FE method, a robust and efficient numerical solution is provided to solve penetration and perforation simulation problem.

### **1. INTRODUCTION**

The responses of concrete to transient dynamics loading is critical to the for the design of important installation for both military and civil applications. Concrete exhibits a unique response compared to other material. Several penetration technology programs have made successful attempts to measure the deceleration of a projectile, when it impacts on concrete target. This deceleration was measured with the use of measuring instrumentation installed in the projectile and helps researchers to calculate the net force . Forrestal et al and DJ Frew et al (2006,2003) are the different authors that have performed deceleration measurement of different calibre radius head (CRH) and size of projectile impact against targets of different material.

Numerical simulation can also obtain the detail information about the dynamic processes in penetrating testing. Advanced finite element software for explicit dynamics like LS DYNA (Hallquist 2007) offers many material model and complex contact for simulation. There are several constitutive equation for concrete under dynamic loading are available in LS DYNA.

In this paper, 3D numerical simulation of a projectile impact against a high strength concrete is performed. The simulation's results - depth of penetration and deceleration versus time are compared with experimental result from Forrestal et al (2003) for bench mark.

Two methods, Lagrangian formulation with 3D solid Finite Elements and Smooth Particle Hydrodynamics coupled with Lagrangian Finite Elements (SPH-FE) are performed for the same simulation. Comparison are made to exemplify the increasingly popular use of SPH-FE coupling method. This coupling method offers an excellent choice to simulate a penetration event. Using 3D solid elements that modelled the region of interest to simulate a penetration - meaning one body (penetrator) to pass through another (target) - of a target will cause deformable elements to suffer large deformation. Numerical treatment, such as erosion criteria is needed to eliminate largely deformed solid element. While using a meshless method - SPH formulation is capable to handle large deformations without the problems of mesh distortion that occurs in 3D solid elements. Also SPH do not require any hourglass formulation.

## 2. THEORETICAL BACKGROUND

### 2.1 Selection of constitutive model for concrete target

When concrete is subjected to high velocity impact, it undergoes high pressures, large strains, high strain rates and damage. Holmquist et al (1993) and Riedel et al (1999) developed the Holmquist Johnson Cook (HJC) and RHT concrete models respectively that consist of features like, high pressure, strain rate effect, material damage. Hence, this two models are suitable for popular among authors for the numerical simulation of the dynamic responses of concrete structures. Though, both models are almost similar, there are some exceptions. Unlike HJC model, the RHT includes a third invariant of deviatoric stress into the pressure-shear expression which is able to distinguish the compressive and tensile meridians. HJC model only considered the effect of pressure and shear which is on terms of the Von Mises equivalent stress, and is not able to reflect the shear response accurately (2008). Also the tensile behaviour of concrete is simply considered through maximum tensile hydrostatic pressure. Yet, HJC model is still popular for high velocity penetration and/or perforation, because of its lower number of material constants requirements compared to other models.

In the HJC model, the constitutive equation employ the use of a equivalent strength model, accumulated damage model and pressure-volumetric strain.

The normalized equivalent stress is defined as  $\sigma^* = \sigma/f'_c$ .  $\sigma$  and  $f'_c$  are the actual equivalent stress and quasi-static uniaxial compressive strength respectively. The expression of the normalized equivalent stress  $\sigma^*$  is given as follows

$$\sigma^* = [A(1 - D) + BP^{*N}][1 + C \ln \dot{\epsilon}^*] \quad (1)$$

D is the damage parameter between 0 and 1. A, B and C are the normalized cohesive strength, normalized pressure hardening and the strain rate coefficient respectively.  $P^*$  denotes the normalized pressure, shown as  $P^* = P/f'_c$ . N is the pressure hardening exponent and  $\dot{\epsilon}^*$  denotes the dimensionless strain rate. The accumulated damage

model is given as follows

$$D = \sum \frac{\Delta \varepsilon_p + \Delta \mu_p}{\varepsilon_p^f + \mu_p^f} \quad (2)$$

The pressure and volumetric strain relationship due to loading and unloading of the concrete is divided into three response regions. The first region is the linear elastic zone. The second region is the when the material is in the plastic transition state and the concrete interior voids gradually reduce in size as the pressure and plastic volumetric strain increase. The third region defines the relationship for fully dense material. The pressure and volumetric strain is given as

$$P = K_1 \bar{\mu} + K_2 \bar{\mu}^2 + K_3 \bar{\mu}^3 \quad (4)$$

For the steel projectile, simplified Johnson Cook material model is used to model it. The Von Mises flow stress  $\sigma$  is expressed as

$$\sigma = [A + B \bar{\varepsilon}_p^N][1 + C \ln \dot{\varepsilon}^*] \quad (5)$$

The first two brackets are similar with the Johnson Cook model. In the first bracket, A represent the material's yield strength, B and N represent the strain hardening factor and  $\bar{\varepsilon}_p$  denote the effective plastic strain The second bracket represent a strain rate hardening factor, where C and  $\dot{\varepsilon}^*$  is strain rate coefficient and dimensionless strain rate respectively. A third bracket that consist of a homologous temperature is omitted in this equation as the effect of temperature is not great. Failure occurs as element erosion to describe the failure phenomenon of the projectile when the effective plastic strain exceeds the user determined value.

## 2.2 Smooth particle hydrodynamics coupling method

In numerical simulation for structural dynamics, Lagrangian codes is the most widely used for explicit dynamics. In a Lagrange system, the calculation follows the motion of fixed elements of mass (Zukas 2004). Lagrangian formulation with Finite Elements use dynamic grids to follow the distortion of materials. For a particular cell in a Lagrangian point of reference, mass remains constant throughout computation, but the volume is allowed to change due to extension or compression of the material. Lagrangian formulation with Finite Elements exhibit inaccuracies with computations involving large deformations resulting in highly distorted meshes ( Anderson 1987). The disadvantage of Lagrangian formulation with Finite Elements is that the numerical grid can become severely distorted or tangled in an extremely deformed region, which can lead to adverse effects on the integration time step and accuracy. However, these problems can be overcome to a certain extent by applying numerical techniques such as erosion and rezoning (Huang et al 2011). Rezoning is only available for 2D simulation. Erosion technique offers the best solution in a Lagrangian formulation with Finite Elements. Many erosion criteria and limit values to simulate failure of concrete structure has been presented by Luccionia et al (2011).

SPH method uses no underlying grid and has a physical domain discretized with particles (Johnson 1996). The SPH particles are not only interacting mass points but

also interpolation points used to calculate the value of physical variables based on the data from neighbouring SPH particles, scaled by a weighting function. Johnson review (2011) presented a summary work of the use of SPH and commented on the this method that can handle sever distortions better than Lagrangian method. The absence of a mesh and the calculation of interactions among particles based on their separation alone means that large deformations can be computed without difficulty. Hence, SPH method is applicable to the use of impact and penetration which is to simulate material behaviour subjected to severe deformation and distortion. Furthermore, SPH method is compatible with most material law and equation of state in LS DYNA software (Liu 2010).

The discussion of the Lagrangian formulation with Finite Elements and SPH methods is to introduce the formulation of SPH-FE coupling method. This method restrict the use of SPH dicretization in the area of interest, where impact or penetration occurs, away from the penetration of the concrete is discretized using Lagrangian mesh. This solid mesh that is not in the area of interest can be coarsely mesh to save computational work.

2.3 Numerical Analysis

Experiments were performed by Forrestal et al, which launched steel projectiles to penetrate a concrete target with a confined strength of 39 MPa. Each steel projectile had a head shape of CRH 3.0, a body dimension of 76.2 mm in diameter, 530 mm in length and weighs approximately 13 Kg. The striking velocity range from 200- 460m/s (Forrestal 2003). A single channel date recorder structurally mounted within the projectile was to measure the rigid body deceleration of the projectile.

Projectile with an striking velocity of 456m/s and 276 m/s is named as experiment number SNL-00-14/5 and SNL-00-12/4 respectively. Both experimental results are used to compare with the simulation result using the above discussed material constitutive equations and formulation methods to evaluate their respective performance and correlation

The parameters of the experimental work are shown in shown in Table 1. In a paper presented by Frew et al (2006), it was observed that the target diameter effect is not a concerned as penetration depth has negligible changes and only a small decrease in deceleration magnitude. Hence, in this paper , all simulation modelling of the concrete target has a diameter of 0.9 metre and a length of 1.5 metre. This set up differs from the original dimension and is shown in Table 2.

Table 1 Experimental result From Forrestal et al (2003)

Launched number	Projectile Mass (Kg)	Target Diameter (m)	Target Length (m)	Striking Velocity (m/s)	Penetration (m)	CRH	$f'_c$ (MPa)
SNL-00-14/5	12.957	1.83	1.83	456.4	0.94	3.0	39
SNL-00-12/4	12.900	1.83	1.83	275.7	0.38	3.0	39

Table 2 : Parameters for simulation work

Simulation work no.	Projectile Mass (Kg)	Target Diameter	Target Length	Striking Velocity	CRH	$f'_c$ (MPa)
1	12.96	0.9	1.5	276	3.0	39
2	12.96	0.9	1.5	456	3.0	39

Concrete target and projectile adopted the LS DYNA material model \*MAT\_JOHNSON\_HOLMQUIST\_CONCRETE and \*MAT\_JOHNSON\_COOK respectively. The parameters for the concrete target and steel projectile are presented in Table 3 (Johnson 1998) and Table 4 (Johnson 1985) respectively.

Table 3 : Material parameters of the high strength concrete

Material Constant		Pressure-Volumetric Strain Relationship parameters	
Density, $\rho$ (kg/m <sup>3</sup> )	2250	$P_{crush}$ (Mpa)	13.6
Shear Modulus, G (MPa)	13570	$\mu_{crush}$	0.00058
Strength Constants		$K_1$ (GPa)	17.4
A	0.75	$K_2$ (GPa)	38.8
B	1.65	$K_3$ (GPa)	29.8
N	0.76	$P_{lock}$ (GPa)	1.05
C	0.007	$\mu_{lock}$	0.1
$f'_c$ (MPa)	39.0		
$S_{max}$	11.7		

Table 4 : Material parameters for the steel projectile

Material Constant		Strength Constants	
Density, $\rho$ (kg/m <sup>3</sup> )	7840	A (MPa)	910
Young Modulus, G (GPa)	200	B (MPa)	586
Poisson Ratio, $\nu$	0.3	N	0.26
		C	0.014

In this paper, modelling of the penetration event uses 3D model for both methods. Only a quarter of the geometry was modelled to reduce computational time, hence two planes of symmetry are also required.

For the simulation using Lagrangian formulation with Finite Elements, hexahedral 8 nodes solid element meshed both the concrete target and the projectile. In the region of interest where penetration occurs, a densely mesh solid elements of 2 mm covered a diameter of 0.6 metre and a depth of 1.5 metre. Solid element has 1 point integration

with hourglass control. This is shown in Figure 1.

In the SPH-FE coupling method, the region of interest is meshed with smooth particle element and it covers a diameter of 0.6 metre and a depth of 1.2 metre. The remaining region is meshed with hexahedral 8 nodes solid elements. To ensure the interface where the SPH element and Lagrangian solids element to be continuous bond with the used of two formulation, a function in LS DYNA

\*CONTACT\_TIED\_NODES\_TO\_SURFACE\_CONSTRAINED\_OFFSET is used. This is shown in Figure 2.

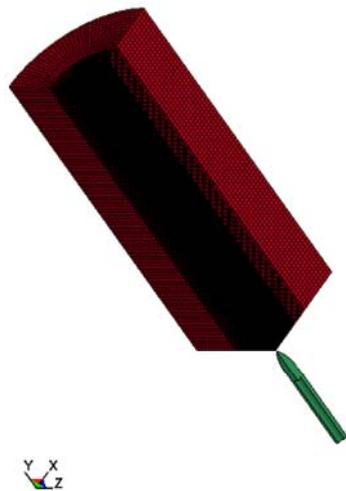


Figure 1 : Modelling using FE method

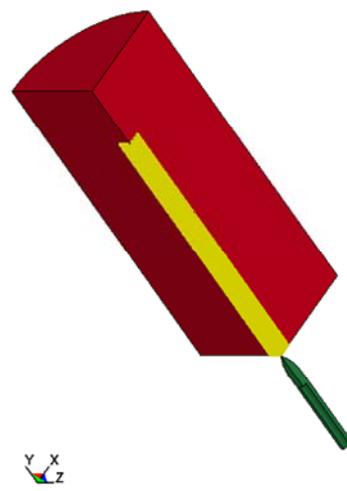


Figure 2 : Modelling using SPH-FE method

### 3. RESULTS AND DISCUSSIONS

Figure 3 and 4 show the deceleration versus time of the steel projectile penetrating into concrete target of confined strength 39 MPa with striking velocity of 456 m/s and 276 m/s respectively. In Figure 3 and 4, they are comparing the simulation results of SPH-FE and FE methods, together with experimental result from Forrestal et al (2003).

The deceleration of the projectile show a penetration resistance of the concrete target. In Figure 3, both FE method and SPH-FE Method show similarity with the experimental result from Forrestal et al (2003), SNL-00-14/5. The trends can be classified in three regions. At the start of the deceleration time graph, the rise time or the start of the penetration of the projectile into the target, both simulation methods are able to reflect the initial step increase of the deceleration within 0.5 milliseconds. The peak deceleration response during the rise time for both methods are almost the same at approximately 13,000 G. The response of the projectile in the middle region show a similar trend of decreasing deceleration. The response starts from a peak value of 13,000 G and decrease to a response of approximately 7000 G. After this middle region, indicate the end of the penetration event, both methods demonstrate the similar

response of the deceleration of the projectile in the experimental result, that it decrease to zero within 0.5 milliseconds. The penetration depth of both simulation methods show a maximum of 5% difference compared to the experimental result as shown in Table 5 .This total event has a duration of approximately 5 millisecond.

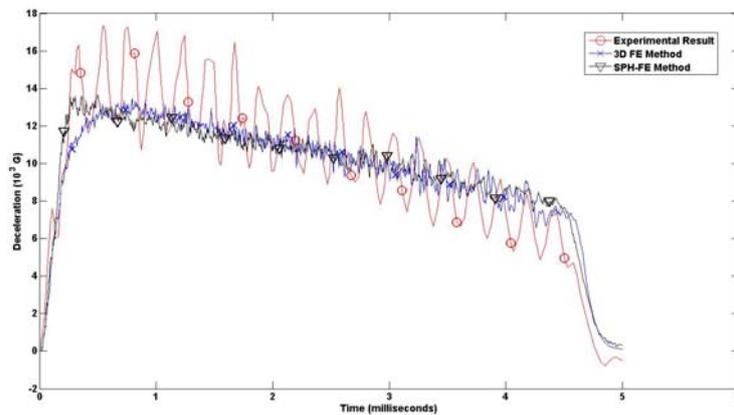


Figure 3 : Deceleration versus time of projectile penetrating into concrete target (Striking velocity of 456m/s, SNL-00-14/5 (Forrestal 2003)). Experimental result versus simulation result

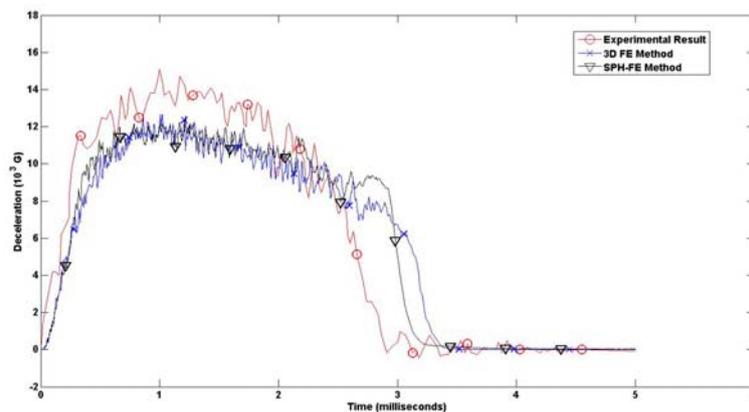


Figure 4 : Deceleration versus time of projectile penetrating into concrete target. (Striking velocity of 276m/s SNL-00-12/4 (Forrestal 2003)). Experimental result versus simulation result

We repeated the analysis of the using two simulation method with a lower striking velocity of 276 m/s penetrating the concrete target. Figure 4 shows the deceleration versus time of the steel projectile penetrating into concrete target. Both SPH-FE and FE method give similar result of the deceleration time curve. However, when compared with the experimental result, both methods showed a lower response at the first peak and also in the subsequent middle region. After this middle region, both methods show

a steep decrease in deceleration within 0.5 milliseconds too. When compared to the simulation result, both SPH-FE and FE methods show a longer duration of the whole event, with FE method more than SPH-FE method by 0.2 milliseconds. In Table 5, FE method and SPH-FE methods shows a difference of 13% and 9% respectively in penetration depth when compared to the experimental result. The difference in results between the simulation and experimental results is larger in the lower striking velocity. SPH-FE method shows a nearer correlation to the experimental result when compared to FE method. The element erosion in the FE method may cause the lower strength response of the target, which in turn causes the lower longer duration and deeper penetration depth.

Table 5 : Penetration depth of projectile from experimental and simulation result

Striking Velocity (m/s)	Penetration Depth (m)		
	Experimental Result (SNL-00-14/5)	3D FE Method	SPH -FE Method
456	0.94	0.984	0.972
Striking Velocity (m/s)	Experimental Result (SNL-00-12/4)	3D FE Method	SPH -FE Method
276	0.38	0.43	0.415

**4. CONCLUSIONS**

Both FE and SPH - FE methods are well suited with the use of material model HJC for the prediction of the depth and deceleration response of a steel projectile penetrating into it. FE methods requires element erosion method in penetration problem and inappropriate selection of parameters will affect the response largely. SPH-FE method solves this problem of not experiencing distortion of elements. In addition, SPH-FE method save more computational power as only region of interest - especially in penetration between two bodies - is discretized with the appropriate particle density.

Other simulation method will be proposed, for example an adaptive technique to switch between FE and SPH. This method maybe more adaptive and it needs more investigation on when the switch should be done and how the variables are transferred from one mesh to the other with minimum energy transfer properties.

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