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Failure process of carbon fiber composites

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ABSTRACT

Some research results of failure behaviour of carbon fiber composites are presented. The solution of material instability on the basis of fiber kinking theory is adopted for the treatment of the failure process. The micromechanical modeling adopting the FETM-approach is used for numerical analysis of the problem. Some numerical and experimental results with actual applications are submitted in order to demonstrate the efficiency of the approaches suggested.

1. INTRODUCTION

A multilevel approach for the local micromechanics analysis of carbon nanotube based composite materials is suggested. Carbon nanotubes are seen as graphene sheets rolled into hollow cylinders composed of hexagonal carbon cells. The hexagonal cell is repeated periodically and binds each carbon atom to three neighbouring atoms with covalent bonds, creating one of the strongest chemical bonds today with impressive mechanical properties.

A long-standing difficulty in designing of carbon fiber composites is the formulation of a consistent theory that describes their failure behavior under non-uniform stress fields. As problem appears there the discrepancy between the four-point bend and simple tensile test data. The bend specimens fail at higher strain compared with the tensile specimens. When the bend and tensile data are analyzed using classical linear elastic theory the bend stress at any strain prior to failure of a tensile test specimen is 20 - 35 % higher than corresponding uniaxial tensile stress. Such strength discrepan-cy remains unresolved even when corrections are made for the nonlinearity of the stress-strain curves. By attempts to explain such discrepancy only very limited suc-cess has been achieved with failure theories, including the Weibull`s statistical model and the fracture mechanics approach. Similar experiences also appeared by the application of linear fracture mechanics or couple-stress theory.

The carbon fiber composites adopted in structural engineering are made of typical components listed as:

1. carbon fibers, with strength and elasticity moduli in scope 2.2 – 5.7 GPa and 300 – 700 GPa, respectively,
2. aramide fibers, with strength 3.5 GPa and elasticity moduli in scope 80-185 GPa.

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The carbon fiber composites consist of micromechanical fibers and surface resin skin. The calculation on the micromechanical level takes into account the behaviour of single fiber in interaction with another fibers and surface skin. In present time are available new types of fiber composites equipped with surface skin on the basis of advanced ceramics or metals having high strength and load-bearing capacity as well as increased temperature resistance and fatigue reliability.

The material instability appearing in the failure process of carbon fiber composites is treated by the fiber kinking theory using the analysis on the micro-mechanical level. In this paper the following is submitted:

1. fiber kinking approach for the failure analysis of carbon fiber composites,
2. mathematical formulation of governing equations for numerical treatment of the problem,
3. numerical and experimental assessment with structural applications.

2. ANALYSIS

In carbon fiber composites the transformation strains and other field quantities appearing in elastic moduli are the periodic functions of space, time and temperature. The periodicity is exploited in an effort to obtain accurate estimates for the transfor-mation strains used to approximate mechanical properties of such composites.

The Washizu´s variation principle is adopted in order to include the initial stress and strain components into analysis. The stress in the carbon fiber microelement at the beginning of time and temperature increments studied is considered as initial stress with thermal strain increments. The variation principle under consideration is then written in the terms of time rate quantities given by

$$I = \{\int_V [S_{ij}\, \varepsilon_{ij} + 0.5\, W_{ij}\, u_{ki}\, u_{kj} - (\varepsilon^o_{ij} + 0.5\, \varepsilon'_{ij})\, S_{ij}]\, dV - \int_{A1} r_i\, u_i\, dA1 -$$
$$- \int_{A2} s_i\, (u_i - w_i)\, dA2\}(dt)^2 + \{\int_V W_{ij}\, \varepsilon_{ij}\, dV - \int_{A1} r_i\, u_i\, dA1 - \int_{A2} p_i\, (u_i - w_i)\, dA2\}dt\,, \qquad (1)$$

where W_{ij} and S_{ij} are the Piola-Kirchhoff stress tensors for initial stress and strain rate states, respectively, p_i and s_i are the Lagrangian surface traction and its time rate quantity, respectively, r_i and $r_i^{(.)}$ are prescribed on surface area $A1$ and w_i on area $A2$ whereas V is the volume bounded by the surface area $A=A1+A2$. The total strain rate ε_{ij} is composed of the initial strain rate ε^o_{ij} and ε'_{ij}, corresponding to the stress rate S_{ij}. To evaluate the thermal strain rate the thermal expansion coefficient at tempera-ture T is $\alpha(T)$ and at temperature $T+dT$ is $\alpha(T+dT)$. By expanding $\alpha(T+dT)$ into Taylor series the average thermal strain rate is obtained. The governing equation is

$$\mu\, \eta(w_t) + (\lambda + \mu)\, grad(div\, w_t) + f = \rho\, \partial^2 w_t/\partial t^2\,, \qquad (2)$$

where λ and μ are Lame´s constants, the mass density is ρ, corresponding Laplace operator is η, the body force vector is f and the vector of displacements is w_t (Tesar 1988 and 1993).

In the terms of derivatives of displacements w_t the governing equation is given by

$$c_2\, w_t + (c_1^2 - c_2^2)\, w_t + f_i/\rho = a_t \ , \tag{3}$$

with propagation velocities for dilatation displacements

$$c_1 = \sqrt{[(\lambda + 2\mu)/\rho]} \ , \tag{4}$$

and shear displacements

$$c_2 = \sqrt{(\mu/\rho)} \ . \tag{5}$$

Strain and stress components are given by

$$\varepsilon_{ij} = (w_{i,j} + w_{j,i})/2 \tag{6}$$

and

$$\sigma_{ij} = \lambda\, \varepsilon_{kk}\, \delta_{ij} + 2\, \mu\, \varepsilon_{ij} \ , \ i,j = 1, 2, 3 \ , \tag{7}$$

with Kronecker delta function δ_{ij} .

3. FAILURE PROCESS

The kinking of microscopic fiber bundles focuses the attention on this type of material instability when dealing with failure process of carbon fiber composites. The material buckling of carbon fibers under static or dynamic compression and flexure is assumed as a possible mode of failure as well as the item influencing the above bend/tensile discrepancy. The local short-wave imperfections as well as general long-wave imperfections along the length of fibers (Simo 1990 and Budiansky 1983) are to be taken into account.

Elastic plane strain deformation in compressed part of the carbon fiber composite gives displacements w(x,y) in normal direction to the fibers, governed by

$$(1 - \sigma/G)\, \partial^2 w/\partial x^2 + E_T/G\, \partial^2 w/\partial y^2 = (\sigma/G)\, \partial^2 w_o/\partial x^2 \ , \tag{8}$$

where $w_o(x,y)$ is an initial displacement pattern. Taking into account half-plane $y \geq 0$ the effect of short-wave imperfections is given by

$$w_o = \delta_D(x)\, \delta_D(y) \ , \tag{9}$$

where δ_D is the Dirac delta function.

A long-wave imperfection concentrated at the edge y = 0 is given by

$$w_o = -\,|\,x\,|\ \ \delta(y) \ . \tag{10}$$

The deviations from ideal fiber alignment due to fiber spacing irregularities induce the patterns of angular misalignment (elastic distortion) that arrange themselves into inclined domains. Such rotations induce the plastic kinking into similarly inclined kink bands. The failure follows there after the start of plastic deformation with kinking failure stress σ_S.

The consequent correlations between σ_S and kink angle β for long wave imperfections (Fig. 1) are given by

$$\tan \beta = \pm \sqrt{[(1 - \sigma_S/G)/(E_T/G)]} \quad (11)$$

and for short wave imperfections by

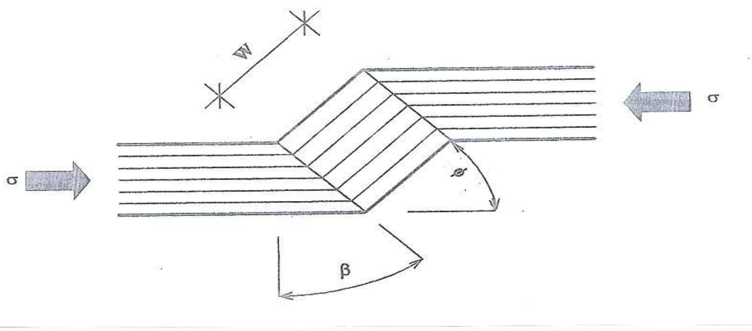

Fig. 1 Kinking of fiber bundles

$$\tan \beta = \pm (\sqrt{2} - 1) \sqrt{[(1-\sigma_S/G)/(E_T/G)]}\,. \quad (12)$$

To find rational estimates for the kink width *W* (Fig. 1) the single fiber dimensions are to be taken into account. The carbon fiber diameter *d* is the meaningful size in the problem. The kink width is clearly delineated by bending in the fibers so that the local fiber bending resistance must be considered explicitly. The fibers are assumed to undergo inextensional bending until they break. At the same time, the elastic strain in the matrix can be neglected with respect to its plastic strains, i.e., the matrix is assumed to be rigid-plastic. Smoothing of the fibers then states a simple couple-stress which gives no fiber rotations outside a band. The rotation $\phi(x+y \tan \beta)$ within the band is governed by

$$d^2\phi/dX^2 + \sigma/\tau_r = 1\ , \quad (13)$$

with

$$\tau_r = (\tau_y^2 + \sigma_{Ty}^2 \tan^2 \beta) \quad \text{and} \quad X = (4x/d)\,[\tau_r/(c\,E)]^{1/2} \quad (14)$$

and with *c* as volume concentration of carbon fibers.

By treatment of Eq. (13), taking account of the condition that the rotations vanish at the ends of the kinking domain, the evolution of the plastic kinking is studied. The carbon fiber breaking is presumed to occur when a critical tensible strain ε_F is rea-ched in combined compression and flexural action at the points of the maximum curvature. For the case of perfectly brittle fibers then holds the formula for the kink width given by

$$W/d = \pi/4\ [(2\ \tau_{ry})/(c\ E)]^{-1/3}\ . \qquad (15)$$

The equation holds for perfectly straight fibers. However, the studies that take initial misalignment into account have shown that W can substantially differ from the per-fect fiber case. An idea appears there to adopt the micromechanical simulation of single fibers by micro strings in order to study the problem. The resistance of such string fibers contains elastic, plastic, visco-elastic and visco-plastic parameters pos-sibly appearing. The approach for such model is submitted below.

For physical interpretation of above definitions the internal and left-hand external displacements of the string micro-element are denoted by w_a and w_b. The internal displacement vector w_i is eliminated beforehand, giving the stiffness matrix by

$$K(\omega) = \begin{bmatrix} K_{aa} & K_{ab} \\ K_{ba} & K_{bb} \end{bmatrix}\ , \qquad (16)$$

and the deformation vector by

$$w = \begin{bmatrix} w_a \\ w_b \end{bmatrix}\ . \qquad (17)$$

Corresponding force vectors are given by

$$n_a = K_{aa}\ w_a + K_{ab}\ w_b\ , \qquad (18)$$
$$n_b = -K_{ba}\ w_a - K_{bb}\ w_b\ . \qquad (19)$$

The state vector v is defined as combination of displacements and internal forces gi-ven by

$$v = [w, n]^T\ . \qquad (20)$$

The state vector at the boundaries a and b is given by

$$v_b = S\ v_a\ , \qquad (21)$$

with corresponding transfer matrix S

$$S = \begin{bmatrix} S_{aa} & S_{ab} \\ S_{ba} & S_{bb} \end{bmatrix}\ , \qquad (22)$$

and with

$$S_{aa} = -K_{ab}^{-1}\ K_{aa}\ ,\ \ S_{ab} = K_{ab}^{-1}\ ,\ \ S_{ba} = -K_{ba} + K_{bb}\ K_{ab}^{-1}\ K_{aa}\ ,\ S_{bb} = -K_{bb}\ K_{ab}^{-1}\ . \qquad (23)$$

The damping parameters are contained in the isothermal bulk modulus K_I given by

$$K_I = K_o(1 + i\ \eta_B) \qquad (24)$$

and appearing in the stiffness terms of the transfer matrix *S*. In Eq. (24) η_B is the damping factor.

The heterogeneity is an essential ingredient of the thermoelastic dissipation in the body of the carbon fiber composite studied. The adiabatic bulk modulus K_A is related with isothermal modulus K_I by

$$K_A = K_I\,[\,1 + K_I\,\gamma^2\,T/C_V\,] \quad , \tag{25}$$

where γ is volumetric thermal expansion coefficient, C_V is the heat capacity at the constant volume and *T* is absolute temperature.

The increased stiffness under adiabatic conditions is due to the fact that compression produces heating and therefore more pressure is needed to produce a given volumetric strain compared with isothermal conditions. In carbon fiber composite the adiabatic heating induce the non-uniform temperatures under oscillatory loading and the heat will then flow among the constituents. The consequent phase difference between stress and strain leads to energy dissipation in each stress cycle.

The velocity *v* versus stress σ is given by

$$v = \rho/G + \sigma/\eta \quad , \tag{26}$$

with ρ as time differentiation of σ. Considering the substitution $\upsilon = G/\eta$ and taking into account the initial conditions $v(0) = 0$ and $\sigma(0) = 0$ for $t = 0$, the average stress is given by

$$\sigma_a = G \int e^{-\upsilon(t)}\, a(t)\,\mathrm{d}t \quad , \tag{27}$$

with acceleration *a(t).*

With the spectral function $N(\upsilon)$, specifying the density of υ and G in the micro-element carbon fiber spring studied, the resulting stress is given by

$$\sigma_R = G_o\,v + \int N(\upsilon)\mathrm{d}\upsilon \int e^{-\upsilon\, t}\, a(t)\, dt \quad . \tag{28}$$

Adopting the value

$$G_s = \int N(\upsilon)\, d\upsilon \quad , \tag{29}$$

as well as the function of relaxation

$$\Psi(t) = G_s(1 - G_s^{-1} \int N(\upsilon)\, e^{-\upsilon t}\, \mathrm{d}\upsilon)/(G_o+G_s) \quad , \tag{30}$$

then resulting stress is modified into

$$\sigma_R(t) = (G_o+G_s)\,[v(t) - \int a(t)\,\Psi(t)\,\mathrm{d}t] \quad . \tag{31}$$

For the approach of $N(\upsilon)$ the Dirac function given by $\delta(x)=0$ for $x\neq 0$ and by $\int \delta(x)\,\mathrm{d}x = 1$ is adopted. Because for each function $f(x)$ there holds

$$\int f(x)\ \delta(x - x_o)\ \mathrm{d}x = f(x_o) \quad , \tag{32}$$

the approach may be done by replacing function $\upsilon\ e^{-\upsilon t}$ by substitution $t^{-2}\ \delta(\upsilon - t^{-1})$. The approximation for $N(\upsilon)$ is given by

$$N(\upsilon) \approx \upsilon^{-1}\ \Psi(1/\upsilon) \quad . \tag{33}$$

The complex modulus of elasticity, appearing as function of frequency ω, is given by

$$E = G_o + i\omega \int N(\upsilon)/(\upsilon + i\omega)\ \mathrm{d}\upsilon \quad . \tag{34}$$

The division into real and imaginary components yields

$$E_1 = G_o + \int \omega^2/(\omega^2 + \upsilon^2)\ \ N(\upsilon)\ \mathrm{d}\upsilon \tag{35}$$

$$E_2 = \int \omega\upsilon/(\omega^2 + \upsilon^2)\ \ N(\upsilon)\ \mathrm{d}\upsilon \quad , \tag{36}$$

with G_o valid for ω=0 .

Instead of infinite number of micro-elements the above approach allows the modeling of the carbon fiber composite by string elements with modulus $G(\omega)$ and stiffness $\eta(\omega)$, both appearing as functions of the frequency ω. The complex modulus of elasticity is given by

$$E = G(\omega) + \eta(\omega)\ \omega i \tag{37}$$

and is implemented into transfer matrix S.

The calculation run of the FETM-wave approach (Finite Element versus Transfer Matrix Methods), using the above matrix S, is adopted with updated variability of micro-mechanical mesh size in space, time and temperature (Tesar 1988). The ultimate analysis of carbon fiber composite is given by

1. Micro-mechanical modeling of the material and macro-mechanical modeling of the structure in space, time and temperature.
2. Updated calculation of stress and strain in space, time and temperature.
3. Automatic comparison with ultimate strength of the elements adopted.
4. Initiation of cracks in micro-mechanical elements trespassing the ultimate strength.
5. Updated calculation of the crack distribution in space, time and temperature until total failure of structure.

The regime of the crack initiation and distribution is rather complex. One or several cracks develop and propagate slowly along the critical regions of the carbon fiber material studied (Tesar 2002). In the case of the shear the cracks turn inside of the body in a direction being quasi-perpendicular to the tension.

4. VERIFICATION

Numerical and experimental assessment of the standard carbon fiber specimen (Fig. 2) was made first of all. The specimen was subjected to tension until the failure and the results were compared with failure strength of the same specimen made of steel.

The experimental facility adopted and the failure are in Figs. 3 and 4, respectively. The results obtained have stated that carbon fiber specimen has eight times higher tensile strength compared with the steel equivalent.

The comparison of numerical and experimental results obtained shows good correspondency of both approaches (Fig. 5).

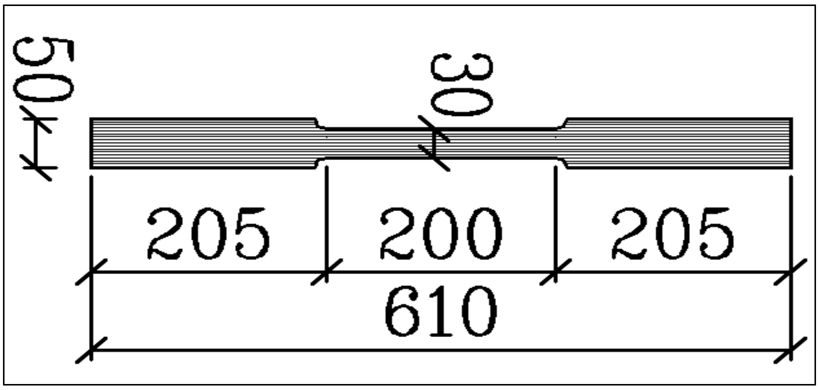

Fig. 2 Standard carbon fiber specimen with thickness 1.4 mm

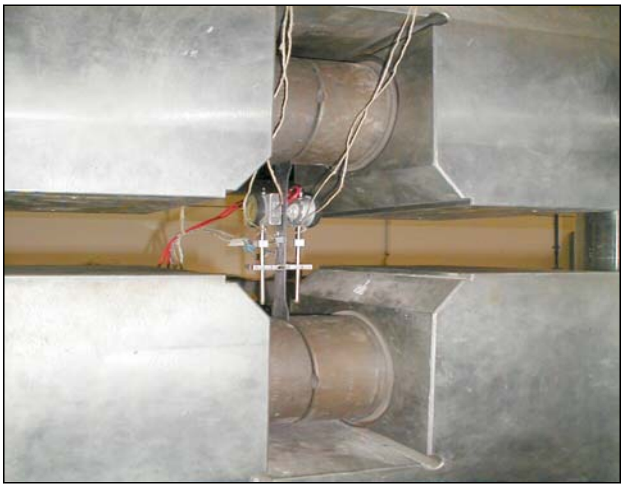

Fig. 3 Experimental facility for tensile testing of standard carbon fiber specimen studied

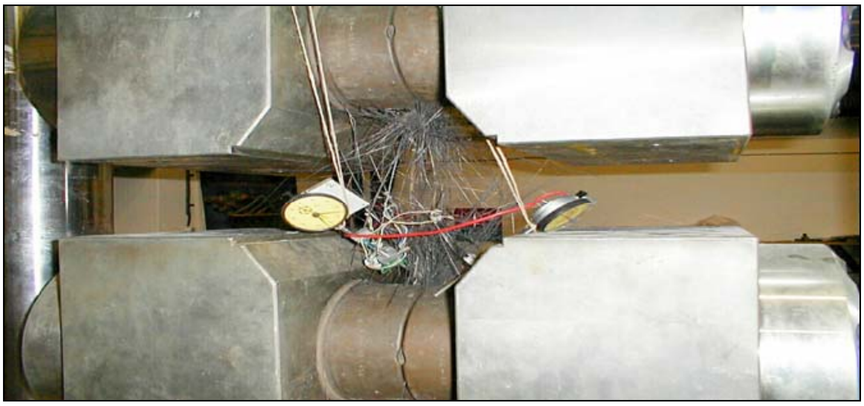

Fig. 4 Explosive failure of the standard carbon fiber specimen tested

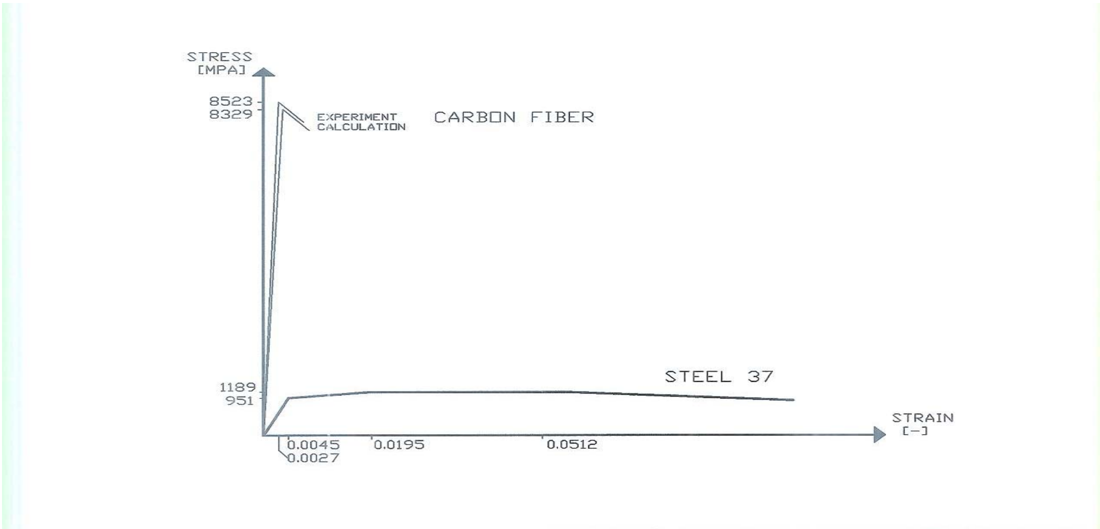

Fig. 5 Stress-straIn curves of standard specimens studied

5. APPLICATION

An ultimate flutter response of slender suspension bridge as shown in Fig. 6 is studied. The span of the bridge is 100 m. The main girder of the bridge is made of laminated wood combined with carbon fiber reinforcements. The carbon fiber composite is adopted for the cables. The structural parameters of the bridge are: girder width is 7.9 m, girder height is 4.1 m and mass per m^2 is 1830 kg.

The bridge is subjected to standard laminar and turbulent air flows. Ultimate flutter time response appearing during simultaneous action of flutter eigenvalues given by resonance frequency of the bridge 0.66 Hz and by critical wind velocity 23.6 m/sec was studied. The energy approach for the analysis of nonlinear time response was adopted for calculation of resulting ultimate aeroelastic response of the bridge. The assessment has shown the dominant influence of flutter rotation modes on resulting ultimate response of the bridge. Starting with simultaneous occurrence of both eigenvalues and assuming the discretization of the bridge span into nnodes, the structural time response until the bridge collapse was studied. In scope of the tuned vibration control of the bridge the wind cables were submitted to variable axial tensile forces. The bridge response in time points 300 sec (for tension 0.1 MN), 660 sec (for tension 0.25 MN) and 720 sec (for tension 0.5 MN) after initiation of eigenvalues, is plotted in Fig. 7.

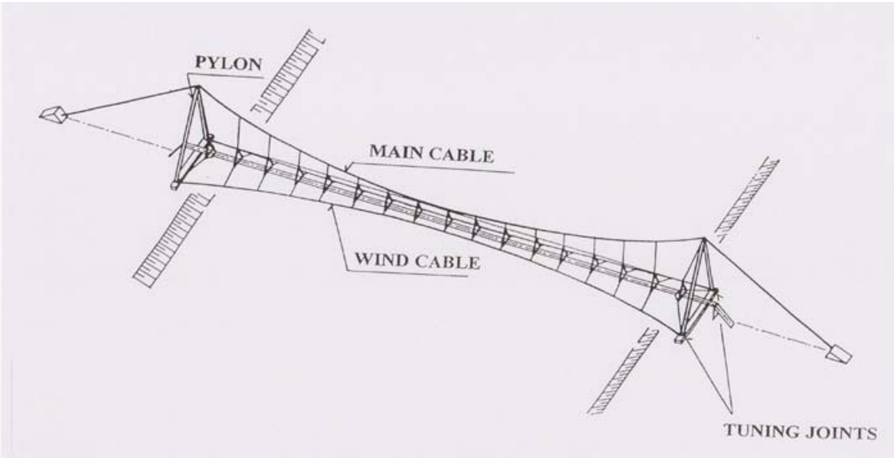

Fig. 6 Slender wood bridge studied

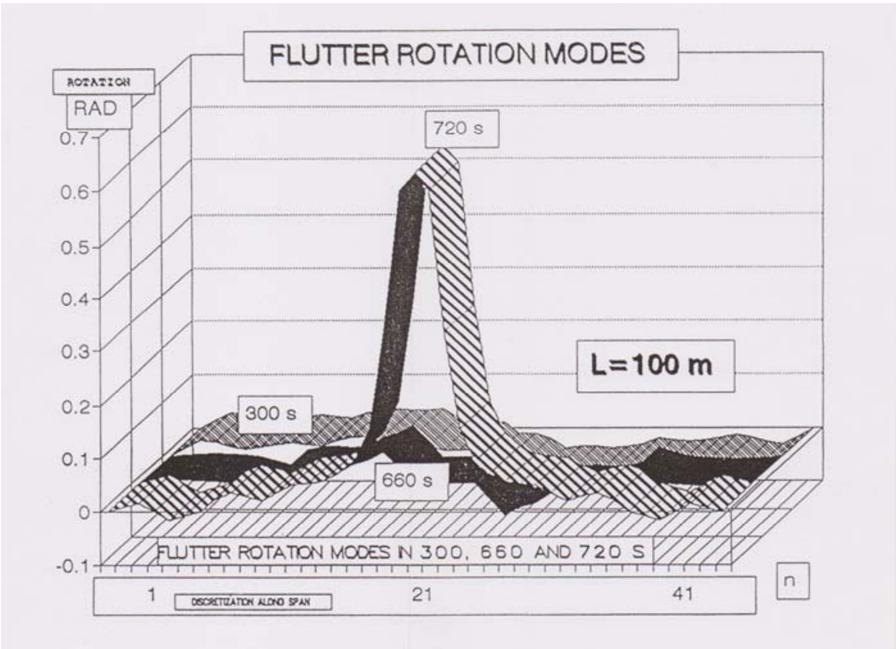

Fig. 7 Ultimate bridge response

6. CONCLUSIONS

The results obtained submit some image on the ultimate response of structures made of carbon fiber composites. The analysis of material instability with adoption of the fiber kinking theory for study of the failure process resulted in the approximation for the treatment of the problem. Micro-mechanical analysis performed is based on the discrete simulation of the problem in space, time and temperature.

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