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Reliability Assessment of double-wall containment under missile impact

*Baha M. A. Khateeb[1], Nadeem A. Siddiqui[2], Tarek H. Almusallam[3], and Husain Abbas[4]

[1), 2) 3), 4)] *Department of Civil Engineering, King Saud University, Riyadh, Saudi Arabia*
[2)] *nadeem@ksu.edu.sa*

ABSTRACT

Effectiveness of single or double-wall containment structures against a possible strike of projectiles, missiles or airplanes is well researched. However, how the uncertainties involved in the various design parameters influence the reliability of the containment is not very well known. In the present study, a simple methodology based on Monte Carlo Simulation (MCS) technique is presented to study the reliability of double-wall containment structures against the impact of external hard missiles on outer RC wall for varying impact velocities.

In order to illustrate the proposed methodology, an idealized double-wall containment structure and a hard missile were chosen. The probability of failure and the reliability indices of the selected double-wall containment structure were obtained for different striking velocities of the missile, and safety of the containment was correlated with the ballistic limit of the outer RC wall. The results of the study show that under given uncertainties the selected double-wall containment is “safe enough” against the impact of selected missile if the missile nominal impact velocity is less than 0.65 times the containment outer wall nominal ballistic limit (V_{BL}). However, when impact velocity is more than $0.90V_{BL}$, failure probability is quite high. A number of sensitivity studies for this idealized double-wall containment were also carried out to obtain the results of field interest.

1. INTRODUCTION

Despite the existence of a large number of containment structures all over the world, there are only a few containment structures whose detailed quantitative safety assessments were carried out against the impact of high velocity missiles, projectiles or even commercial planes. Such quantitative safety and reliability assessments are necessary in order to incorporate uncertainties involved in various design parameters related to the target (containment) and the impacting objects. Neglecting these uncertainties or considering them just in terms of qualitative safety factors may sometimes lead to a structure with substantially high probability of failure. As the

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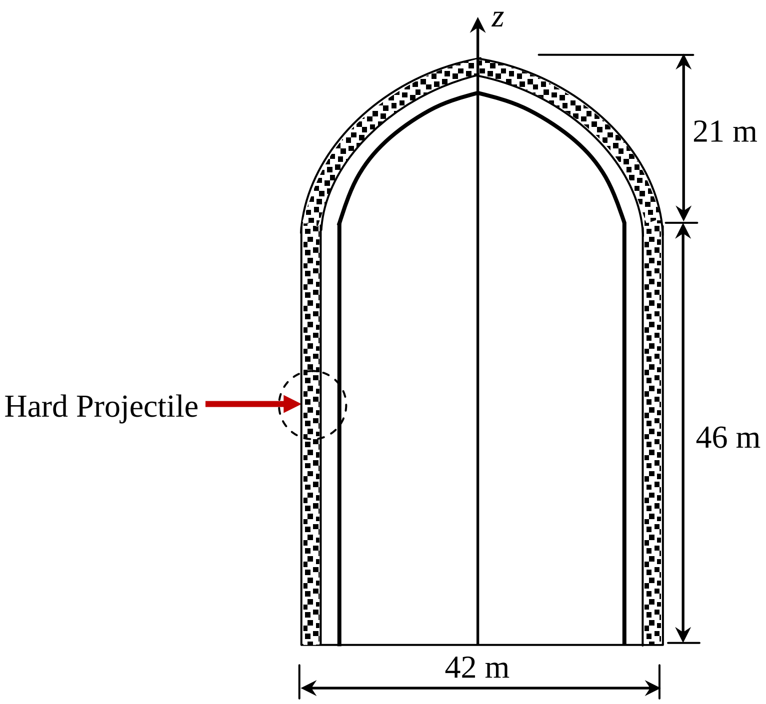
[1] M.Sc. Student
[2] Associate Professor
[3] Professor
[4] Professor

consequences of containment structures failure are far severe than even the collapse of a 100 storey commercial building, it is very much necessary to propose a simple methodology for the reliability assessment of such containment structures against the impact of projectiles, missiles etc. To propose a simple methodology for the reliability assessment of a double-wall containment structure (Fig. 1) against the impact of high velocity non-deforming missiles is the subject of the current investigation.

In the past, some limited works are reported on the reliability analysis of structures subjected to impact loads of projectiles, missiles and airplanes on concrete structures. Choudhury et al. (2002) presented a methodology for the reliability analysis of a buried concrete target against normal missile impact. Siddiqui et al. (2003) presented a methodology for detailed reliability analysis of nuclear containment without metallic liners against aircraft crash. Pandey (1997) presented a quantitative reliability-based approach to evaluate the containment integrity in terms of the condition of bonded prestressing systems. Han and Ang (1998) suggested serviceability design load factors, and carried out the reliability assessment of RC containment structures. Penmetsa (2005) presented a methodology for the system reliability analysis that can determine the probability of the destruction of buried concrete targets using deep penetration weapons. Siddiqui et al. (2009) carried out the reliability assessment of concrete targets subjected to impact forces due to striking missiles. They identified various design parameters which can be judiciously chosen to achieve the desired reliability for concrete targets subjected to impact forces.



Vertical sectional view and impact location

Fig. 1: Schematic representation of double wall containment structure

A detailed review of the literature on the reliability analysis of containment structures against possible impact of projectiles, missiles etc. show that very limited studies are available on the reliability assessment of double-wall containment structures against the impact of missiles. The aim of the present study is to present a simple methodology for the reliability assessment of double-wall containment structures against the impact of external non-deformable/hard missiles.

2. LIMIT STATE FUNCTION

The reliability analysis requires a limit state function which is a mathematical representation of a particular mode of failure. This function assumes a negative or zero value at the failure and a positive value when the structure is safe against that possible mode of failure.

In order to derive the limit state function for the reliability analysis of a double-wall containment, a hard (non-deformable) sharp nose missile is assumed to normally impact the outer RC wall with a velocity that it perforates the concrete wall and then hits the inner steel wall with its residual impact energy (or residual velocity). The double-wall containment is assumed to fail when the inner steel wall also gets perforated.

The failure of the steel wall/plate is assumed to occur when the missile impact energy exceeds perforation energy of the steel wall. Keeping above points in view, if the perforation energy of the steel wall is E_{perf} and the missile kinetic energy E_{proj} then the limit state function can be expressed as:

$$g(\underline{x}) = E_{perf} - E_{proj} = E_{perf} - \frac{1}{2} M V_*^2 \tag{1}$$

where, E_{proj}= Missile energy; *M* = mass of the missile; V_* = the residual velocity of the missile with which it hits the steel wall. The residual velocity is the velocity of impacting missile after the perforation of the outer concrete wall.

From the above equation, it is obvious that the failure of the steel wall will occur if perforation energy (E_{perf}) of the wall is equal to or less than the residual kinetic energy $\left(= \frac{1}{2} M V_*^2 \right)$ of the impacting missile.

The energy required to perforate a steel wall was obtained by the empirical equation originally proposed by Thomson (Thomson 1954, Corbett et al. 1996) for steel plates. This equation can be expressed as:

$$E_{perf} = \pi d^2 t \left(0.125Y + 0.0625 \rho_s C_E \left(\frac{V_* d}{h} \right)^2 \right) \tag{2}$$

Where, E_{perf} = perforation energy; *d* = diameter of the aft body of missile; *t* = thickness of steel wall; *Y* = yield stress of the steel wall; ρ_s = density of the steel wall; *h* = nose length of the missile; C_E = Constant = 1 for a conically-tipped missile and C_E = 1.86 for an ogival-headed missile. Sodha and Jain (1958) subsequently corrected the analysis for the ogival-headed missile, giving a new value of CE = 0.62 (Corbett et al. 1996).

If the concrete wall is perforated, the velocity after perforation is called the residual velocity (V_*), which can be computed by (Chen et al. 2004, 2008):

$$V_* = V_0 - V_{BL} \quad \text{for } \chi \le \chi_c \tag{3a}$$

$$V_* = \sqrt{\left(V_0^2 - V_{BL}^2\right)} \text{ for } \chi > \chi_c \tag{3b}$$

where, V_{BL} = ballistic limit velocity, which is defined as the minimum velocity for the perforation of RC wall. The ballistic limit equations were formulated by Chen et al. (2008) for an RC target.

In the above equation, χ is dimensionless thickness of concrete such that $\chi = \frac{H}{d}$ and H_{BL}^{*} = thickness of conical plug at the ballistic limit; χ_c = dimensionless critical thickness of concrete target which can be estimated using expressions given by Chen et al. (2008).

In the above limit state Eq. (1), the variables $V_0, d, f_c^{'}, \rho_c, f_y, h, M, \rho_s, Y, H, t, p_s$ are implicitly or explicitly involved. These variables have significant inherent uncertainties, and due to this reason, they are considered as random variables in the subsequent reliability analysis. Arranging these variables in vector form leads to

$$\underline{x} = \left(V_0, d, f_c^{'}, \rho_c, f_y, h, M, \rho_s, Y, H, t, p_s\right) \tag{4}$$

Here $\underline{x}$ is the vector of basic random variables.

Having derived the limit state functions, the next step is the assessment of probability of failure (also called risk) and reliability (measured in terms of reliability index *β*) of the double-wall containment structure against the normal impact of the missile. For this purpose, Monte Carlo Simulation technique (Nowak and Collins 2000) has been employed.

3. DISCUSSION OF RESULTS

Employing the data presented in Table 1 and using the Monte Carlo simulation technique, the reliability analysis of the containment was carried out for different impact velocities taken as a percentage of the nominal ballistic limit of the outer RC wall (= 290 m/s). Fig. 2 shows the variation of nominal residual velocity and residual kinetic energy of the missile with the striking velocity.

Table 1: Random variables considered for the reliability analysis of the double-wall containment structure.

Random Variable	Nominal	Bias factor	COV	Distribution	Reference
Concrete wall					
Concrete strength, $f_c^{'}$ (MPa)	40	0.9	0.10	Lognormal	Penmesta 2005
Reinforcement ratio, p_s (%)	1.1	0.9	0.10	Normal	Assumed
Uni-axial tensile strength of reinforcing bars, f_y (MPa)	420	0.9	0.10	Lognormal	Assumed
Thickness of concrete target, *H* (m)	1.2	1.0	0.05	Normal	Siddiqui et al. 2002
Concrete density, ρ_c (kg/ m^3)	2440	0.95	0.10	Lognormal	Choudhury et al. 2002
Steel wall					
Yield strength of the steel wall, *Y* (MPa)	420	0.95	0.05	Normal	Assumed
Steel density, ρ_s (kg/m^3)	7850	0.95	0.10	Lognormal	Assumed
Thickness of steel wall, *t* (mm)	150	1.00	0.03	Normal	Assumed
Projectile					
Nose length of the projectile, *h* (mm)	495	1.00	0.025	Normal	Penmesta 2005
Diameter of the projectile, *d* (mm)	165	1.05	0.05	Normal	Penmesta 2005
Mass of the projectile, *M* (kg)	182	1.10	0.05	Lognormal	Penmesta 2005
Impact velocity, V_0 (m/s)	Variable	1.00	0.10	Extreme Type -1	Choudhury et al. 2002

COV: Coefficient of variation

Fig. 3 shows that under given uncertainties, if the impact velocity is less than 65% of the nominal ballistic limit (i.e. 0.65V_{BL}), the containment is sufficiently reliable as for this impact velocity the reliability index is above 3. Any important structure with reliability index above 3.0 is generally considered sufficiently reliable (Siddiqui et al. 2003, Choudhury et al. 2002). When impact velocity is more than 90% of the ballistic limit (i.e. 0.90VBL), failure probability is quite high as the reliability index is less than 1.0 which is an indication that the containment is not safe as desired. In other words, the present double-wall containment is safe enough against the impact of the missile if the missile nominal impact velocity is less than 0.65 times the nominal ballistic limit (V_{BL}) of containment outer concrete wall. It is worth mentioning that although the nominal impact velocity is less than the nominal ballistic limit of outer wall, there is a finite probability of steel wall penetration. This is due to the fact that when impact velocity was simulated using the statistical values and probability distributions shown in Table 1, there were a number of simulated velocities which were above the nominal ballistic limit value. This caused substantial probability of failure of the steel wall even though nominal impact velocity was less than ballistic limit (V_{BL}) of the outer RC wall. Fig. 3 illustrates the variation of reliability index with the impact velocity expressed in terms of the percentage of outer concrete wall ballistic limit. This figure shows that when the impact velocity is close to 0.65VBL, $(\beta - \beta_D)^2$ is close to zero for the desired reliability index of 3.0 to 3.5. This is an indication that the containment is "safe enough" if the striking velocity of the impacting missile is less than $0.65V_{BL}$.

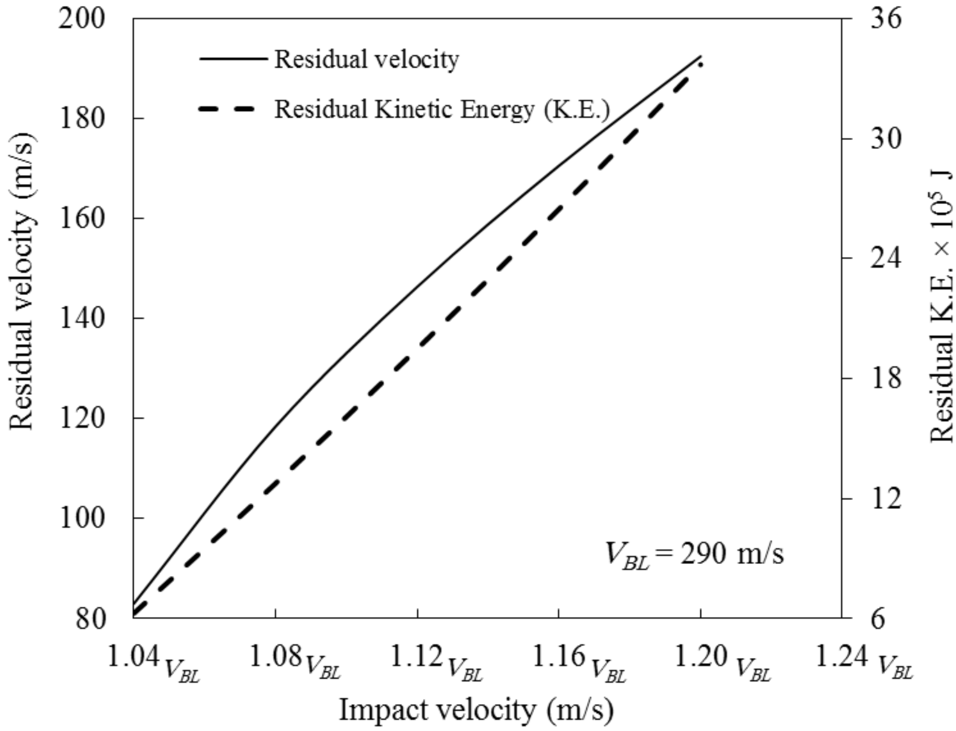


Fig. 2 Variation of nominal residual velocity and residual kinetic energy of the projectile with impact velocity

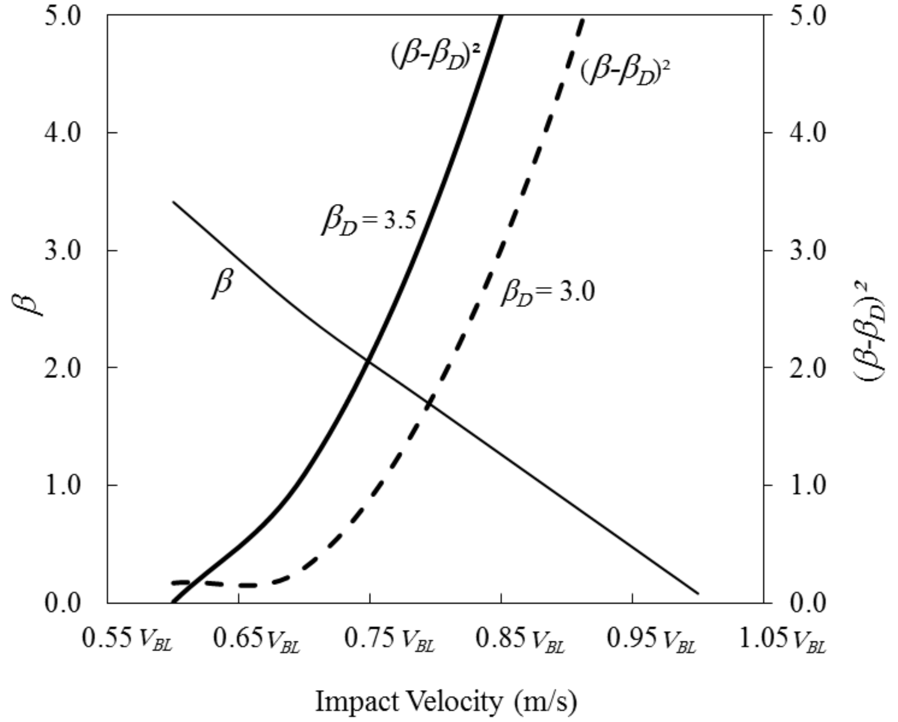


Fig. 3 Variation of containment reliability with impact velocity

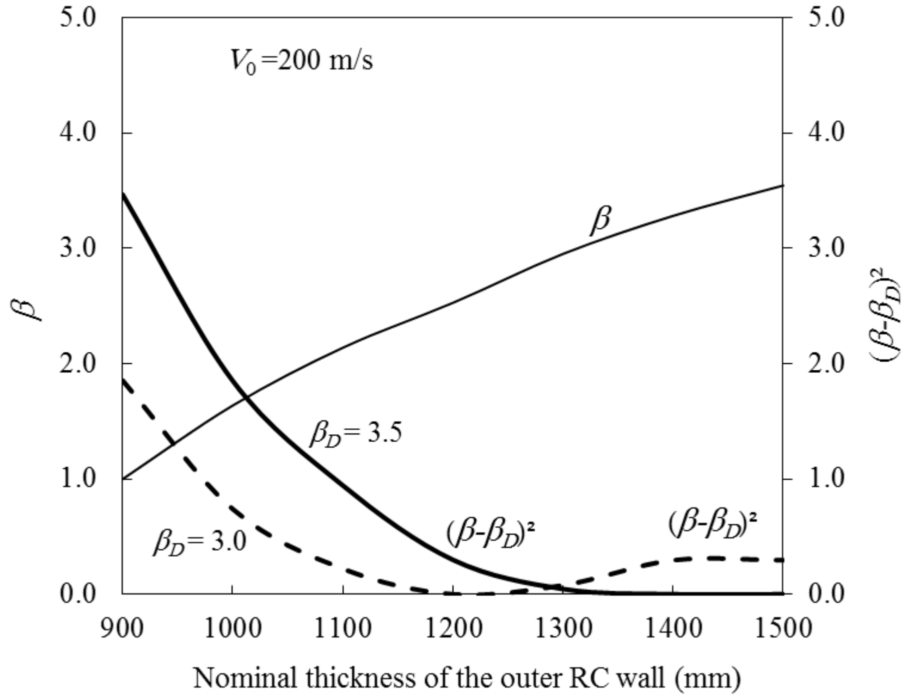


Fig. 4 Variation of the containment reliability with the thickness of the outer RC wall

3.1 Sensitivity Study

In the present section, a few sensitivity analyses were carried out to obtain the results of practical interest. For the sensitivity study of a variable, its nominal value was varied to study its effect on failure probability and reliability index of the double-wall containment. The impact velocity of the missile was taken as 200 m/s and all other variables were taken same as shown in Table 1. The sensitivity study was carried out to study the influence of the variables on the containment reliability.

Effect of the concrete wall thickness Figure 4 shows that as the thickness of the outer concrete wall increases, reliability of the containment also increases. This is an expected trend as the concrete thickness increases the residual velocity of the missile decreases which reduces the failure probability of the inner steel wall. Fig. 4 also shows that a little change in the thickness of the concrete wall can alter the reliability substantially. This figure clearly illustrates that an increase of 200 mm in the concrete thickness can increase the containment reliability index approximately by 1. This change in the reliability of the containment is due to the change in the residual kinetic energy of the missile.

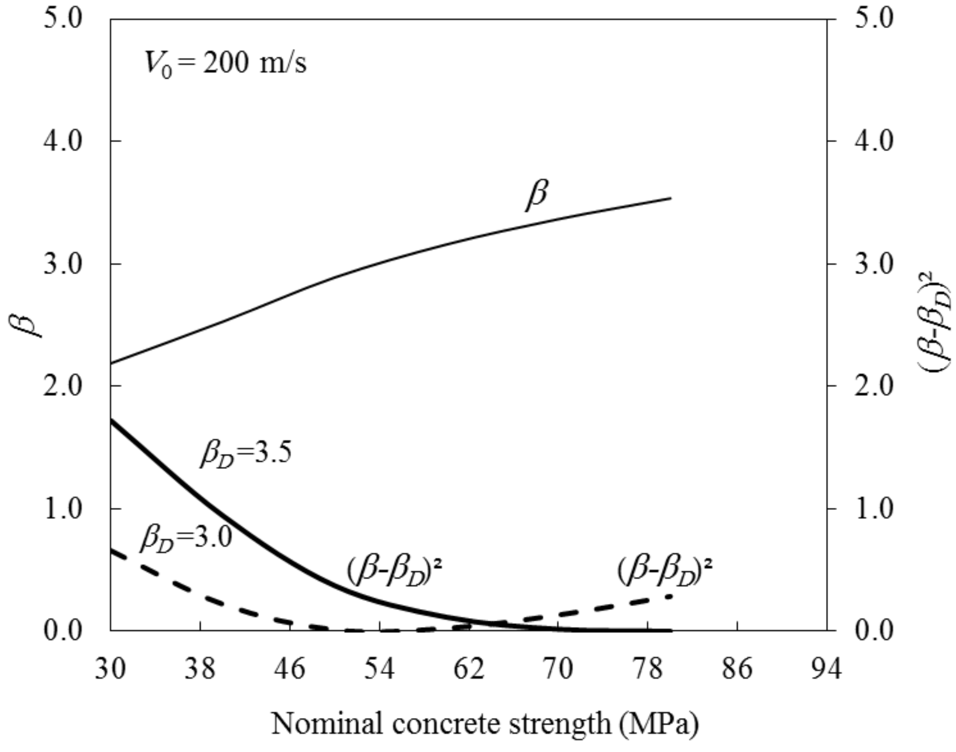


Fig. 5 Variation of the containment reliability with the concrete strength

Effect of the concrete strength Figure 5 shows that as the concrete strength increases, reliability of the containment increases. This is due to the fact that an increase in the concrete strength will decrease the residual velocity of the missile which will increase the overall reliability of the containment. Figure 5 also shows that for the desired reliability index of 3.0, the strength of the concrete should not be less than 54 MPa and for achieving the desired reliability of 3.5, the strength of concrete should be about 70 MPa. In other words, a minimum concrete strength of 54 MPa is desirable for achieving the desired safety of the containment against the impact of the present missile.

4. CONCLUSIONS

Following are the conclusions which can be drawn from the present numerical study:

- The present double-wall containment is safe enough against the impact of the projectile if the projectile nominal impact velocity is less than 65% of the nominal ballistic limit V_{BL} of the containment outer-wall.
- An increase of only 10 m/s (~4%) in the residual velocity causes about 30-40% increase in the residual kinetic energy of the projectile. Owing to this reason, a little change in the impact velocity may cause a phenomenal change in the containment reliability.
- As expected, with an increase in the impact velocity of the projectile there is a sharp decrease in the reliability of the double-wall containment.
- A change in the concrete wall thickness can alter the reliability substantially. An increase of 200 mm in the concrete thickness can increase the containment reliability index approximately by 1.
- As the concrete strength increases, reliability of the containment increases. In order to achieve the desired reliability index of 3.0, the strength of the concrete should not be less than 54 MPa, and for achieving the desired reliability of 3.5, the strength of the concrete should be about 70 MPa.

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