The 2013 World Congress on
Advances in Structural Engineering and Mechanics (ASEM13)
Jeju, Korea, September 8-12, 2013

On the surface effect formulation for nonlocal elasticity

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ABSTRACT

A nonlocal elasticity is very popular in literature, because it is simple and able to consider the size effect including the surface effect. The most popular one may be Eringen's model which takes the form of differential equations instead of the integral form. The nonlocal effect is simulated by the Laplacian operator for axial and thickness length scales, respectively. The present formulation reveals that the axial length scale is somewhat similar to shear deformation effect, whereas the thickness length scale is somehow related to the surface stress effect. A new formulation for nano-beams is derived to incorporate the effect into the continuum-based nonlocal model.

1. INTRODUCTION

Nano-engineering has given attention to industry as well as academic societies for the last decade. Molecular dynamics and/or quantum mechanics-based simulations may be required to analyze nano-systems for an engineering design purpose. However these simulations require tremendous computational efforts (Kim et. al, 2012). Continuum-based nano-mechanics would be a viable option for an engineering design in practice. Most popular one is a nonlocal elasticity (Eringen, 2002), in which the traditional definition of stress is abandoned.

Nonlocal theories can be classified into three groups; nonlocal elasticity, strain gradient elasticity, and couple stress theory. Among others, a nonlocal elasticity is very popular in literature, since Eringen proposed a simple differential model. Many researches have been conducted by employing the Eringen's model for various existing beam/plate/shell theories. Specifically, for beams, most people take the simple form of

$$t_{11} - c t_{11,11} = \sigma_{11} \tag{1}$$

where t_{11} is a nonlocal stress, σ_{11} is a local stress, and *c* is the length scale parameter. Equation (1) implies that the axial length scale effect is considered only.

In this paper, the original Eringen's model is explored to investigate the length scale effects including the thickness length scale effect that would consider the surface effect.

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2. FORMULATION

After simple manipulation, one can find that Equation (1) is quite similar to that of a Rankine-Timoshenko beam model (see Fig. 1). This clearly indicates that the length scale effect is a sort of shear deformation effect that obviously considers the axial length scale (more precisely the length-to-thickness ratio). The equilibrium equation is the same as that of Euler-Bernoulli beam model, however, the boundary conditions are different, which are similar to those of the asymptotic expansion method (Kim and Wang, 2011).

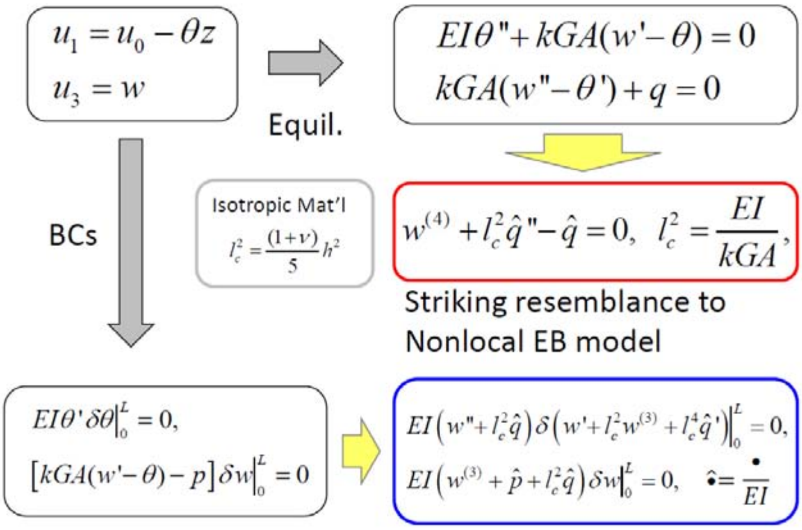

Fig. 1 A nonlocal elasticity with axial length scale is similar to a conventional shear deformable beam theory

On the other hand, the simplified Eringen's nonlocal model for two-dimensional elasticity is given as follows:

$$t_{11} - c\left(t_{11,11} + t_{11,33}\right) = \sigma_{11} \tag{2}$$

which differs from Equation (1) by considering the through-the-thickness derivative. This is illustrated in Fig. 2, where the derivative reveals that there is the surface stress. Otherwise the nonlocal stress is the same as the local stress. When the surface stress is considered, then the bending stiffness can increase or decrease depending on the state of surface stresses. This is different from Equation (1) whose results indicate that the bending stiffness always decrease as long as the nonlocal parameter is positive.

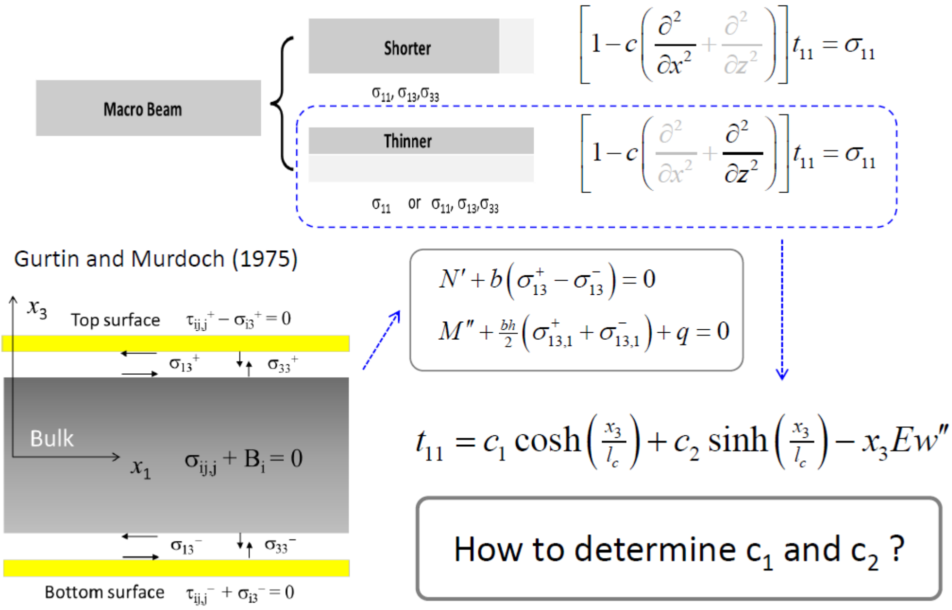

Fig. 2 Schematic of the thickness scale effect from the Laplacian operator shown in nonlocal elasticity

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