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Identification of the degradation of railway ballast under a concrete sleeper

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ABSTRACT

The identification of railway ballast degradation under a concrete sleeper is investigated following the Bayesian approach. The rail-sleeper-ballast system is modeled as a Timoshenko beam with two added masses on an elastic foundation. In the model updating process, the stiffness of the ballast foundation is assumed to be continuous along the sleeper by using a polynomial. The accuracy of the identified ballast stiffness distribution strongly depends on the level of modeling error and measurement noise. In the proposed method, Bayesian probabilistic approach is adopted to explicitly address the uncertainties associated with measured modal parameters. A numerical case study is considered in this paper to verify the proposed ballast damage detection method, and the analysis results are very encouraging showing that it is possible to use the measured vibration data of the in-situ sleeper for the purpose of ballast degradation identification.

1. INTRODUCTION

There are many methods developed for monitoring the functional condition of the rails in the railway track systems. However, the non-destructive evaluation of railway ballast under the sleepers is still relying on visual inspection. It must be pointed out that visual inspection is only effective to observe ballast damage on the surface, while ballast damage under the sleeper cannot be observed. The main function of the ballast is to support the sleeper and keep it in position. If the ballast is damaged, the stiffness that can be provided in supporting the sleeper is reduced. As a result, the vibration characteristic of the rail-sleeper-ballast system is altered. Therefore, it is possible to detect the damage of ballast under the sleeper based on the measured data of the in-situ sleeper.

Timoshenko beam element was first used to model the concrete sleeper by Grassie (1995), a simple two-dimensional dynamic model was presented to calculate

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the dynamic characteristic parameters of the system. The proposed method was experimentally verified using 12 types of sleepers. In 2008, Dahlberg used a vibrating Timoshenko beam on an elastic foundation to analytically model the dynamic behavior of an in-situ concrete sleeper. Dahlberg observed that the effect of ballast is significant for lower modes. This finding is consistent with the analysis results in references (Lam 2010, 2012a). Lam (2012b) conducted impact hammer tests on a segment of full-scale ballasted track. It was concluded that the use of vibration data to detect damage of railway ballast is feasible. It should be noted that these studies were based on some discrete models, which means the ballast foundation were divided into several regions with equal size, the ballast stiffness in a region is constant. Since the ballast stiffness variation is continuous along the sleeper, the discontinuous “jumps” of ballast stiffness at the interfaces between two regions are “artificial”. One of objectives of this paper is to extend the work of references (Lam 2012b; Hu 2012) to overcome this difficulty. In this paper, a class of models is developed by assuming that the distribution of ballast stiffness along the sleeper follows a polynomial. Uncertainty induced by the measurement noise and modeling error is one of the most difficult problems in vibration-based damage detection. Thus another new development in this paper is to employ the Bayesian model class selection (Beck 2004) and the Bayesian model updating (Vanik 2000) methods in ballast damage detection instead of following the deterministic approach.

The structure of this paper is as follows. In section 2, the proposed methodology is introduced together with the relevant theoretical backgrounds, such as the modeling of the rail-sleeper-ballast system, the Bayesian probabilistic framework and the Bayesian model class selection. In section 3, the numerical case study is presented, which verifies and demonstrates the proposed methodology. The conclusions and discussions are given at the end of this paper.

2. PROPOSED METHODOLOGY AND BACKGROUND THEORIES

2.1 The rail-sleeper-ballast system model

The main components of a ballasted track are rails, sleepers and ballast. In this paper, the rail-sleeper-ballast system is modeled as a Timoshenko beam with two additional masses on an elastic foundation with ballast stiffness k_b as shown in Fig. 1. According to references (Lam 2012b; Hu 2012), the two rails on the sleeper can be modeled as two different additional masses which were represented by $m_L = \theta_L m_r$ and $m_R = \theta_R m_r$, where m_r is the nominal value of rail mass and θ_L and θ_R are the two corresponding dimensionless scaling factors. Due to the aging problem, the Young’s modulus of the concrete sleeper is not certain, so another dimensionless scaling factor θ_E is utilized to scale it in the model updating process. These three dimensionless scaling factors, θ_L, θ_R and θ_E, are considered as uncertain model parameters in model updating process. The reference (Lam 2012b) proposed to model the ballast with six regions of equal length, in each region, the ballast stiffness is uniform, thus Lam employed six non-dimensional scaling factors to scale the corresponding stiffness value. This idea is simple but resulting in a relatively large level of modeling error, as the ballast stiffness along the sleeper should be continuous. This paper proposes to use polynomial in approximating the continuous variation of the ballast stiffness distribution. A 2-degree polynomial has 3 more uncertain model parameters (i.e., $c_1x^2 + c_2x + c_3$), and an N-degree polynomial has N+1 more model parameters. Therefore, for a N-degree polynomial model, a total of N+4 uncertain model parameters are considered in the model updating process, which are grouped into a model uncertain parameter vector $\boldsymbol{\theta} = [\theta_E, \theta_L, \theta_R, \theta_{C1}, \ldots \theta_{CN+1}]$. Finite element method is employed in the analysis, and the sleeper is divided into 48 elements in this study.

The classical beam theories are studied in the book (Krenk 2001). The formulations of Timoshenko beam on an elastic foundation is modified from the Euler beam formulations by considering the shear effect, the element stiffness matrix $\mathbf{k}$ is shown below:

$$\mathbf{k} = \frac{EI}{(1+\varphi)L^3}\begin{bmatrix} 12\psi_1 & -6\psi_3 L & -12\psi_2 & -6\psi_4 L \\ -6\psi_3 L & (4+\varphi)\psi_5 L^2 & 6\psi_4 L & (2-\varphi)\psi_6 L^2 \\ -12\psi_2 & 6\psi_4 L & 12\psi_1 & 6\psi_3 L \\ -6\psi_4 L & (2-\varphi)\psi_6 L^2 & 6\psi_3 L & (4+\varphi)\psi_5 L^2 \end{bmatrix} \quad (1)$$

where E and I are the elastic modulus and second moment of area of the element, respectively. Effect of shear flexibility is described by the non-dimensional parameter φ, it is expressed as:

$$\varphi = \frac{12EI}{GA_0L^2} \quad (2)$$

where G is shear modulus, A_0 is the corresponding shear area, which is smaller than the full cross-section area. The coefficients ψ_i for $i = 1$ to 6 can be expressed as:

$$\psi_1 = \frac{1}{3}(\lambda L)^2 \psi\left[\sinh(\lambda L)\cosh(\lambda L) + \sin(\lambda L)\cos(\lambda L)\right] \quad (3a)$$

$$\psi_2 = \frac{1}{3}(\lambda L)^2 \psi\left[\sin(\lambda L)\cosh(\lambda L) + \sinh(\lambda L)\cos(\lambda L)\right] \quad (3b)$$

$$\psi_3 = \frac{1}{3}(\lambda L)\psi\left[\sinh^2(\lambda L) + \sin^2(\lambda L)\right] \quad (3c)$$

$$\psi_4 = \frac{2}{3}(\lambda L)\psi \sin(\lambda L)\sinh(\lambda L) \quad (3d)$$

$$\psi_5 = \frac{1}{2}\psi\left[\sinh(\lambda L)\cosh(\lambda L) - \sin(\lambda L)\cos(\lambda L)\right] \quad (3e)$$

$$\psi_6 = \psi\left[\sin(\lambda L)\cosh(\lambda L) - \sinh(\lambda L)\cos(\lambda L)\right] \quad (3f)$$

where

$$\psi = \frac{\lambda L}{\sinh^2(\lambda L) - \sin^2(\lambda L)} \quad (4)$$

and

$$\lambda = \left(\frac{k_b}{4EI} \right)^{\frac{1}{4}} \tag{5}$$

Consistent mass matrix is employed in this study (Przemieniecki 1985), the local element mass matrix $\mathbf{m}$ is given by:

$$\mathbf{m} = \frac{\rho AL}{420} \begin{bmatrix} 156 & 22L & 54 & -13L \\ 22L & 4L^2 & 13L & -3L^2 \\ 54 & 13L & 156 & -22L \\ -13L & -3L^2 & -22L & 4L^2 \end{bmatrix} \tag{6}$$

where A, ρ are cross-section area and density of the element, respectively.

System stiffness and mass matrix are assembled by the local element stiffness and mass matrix. Modal parameters, such as natural frequencies and mode shapes, of the system can be calculated by solving the corresponding eigenvalue problem.

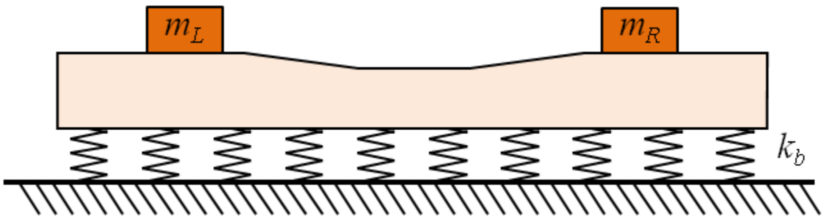

Fig. 1 Modeling of rail-sleeper-ballast system

2.2 Bayesian model updating and model class selection methods

In most deterministic approach, model updating problem is treated as a minimization problem to minimize the discrepancy between the measured and calculated modal parameters (Lam 2012b; Hu 2012; Ruotolo 1997). The objective function in the minimization problem usually consists of two parts, one is the difference in natural frequencies and the other is the difference in mode shapes. Two different weighting factors should be assigned to these two terms to show their relative importance in the model updating process. Nearly no commonly accepted method can be used to decide these two weighting factors under the deterministic approach. This paper proposes to follow the Bayesian probabilistic framework (Katafygiotis 2000) to address this problem. This framework targets in calculating the posterior PDF of the model parameters ($\boldsymbol{\theta}$ in this paper) for a given set of modal data and a given class of models. One can obtain the most probable model $\boldsymbol{\theta}$ by maximizing the posterior PDF which is equivalent to minimize the measure-of-fit function (Vanik 2000):

$$J(\boldsymbol{\theta}) = \sum_{r=1}^{N_m}\sum_{n=1}^{N_s}\left[\frac{\left(\hat{\omega}_r^2(n) - \omega_r^2(\boldsymbol{\theta})\right)^2}{\varepsilon_r^2} + \frac{\boldsymbol{\varphi}_r^T\boldsymbol{\Gamma}^T\left(\mathbf{I} - \hat{\boldsymbol{\psi}}_r(n)\hat{\boldsymbol{\psi}}_r^T(n)\right)\boldsymbol{\Gamma}\boldsymbol{\varphi}_r}{\delta_r^2\left\|\boldsymbol{\Gamma}\boldsymbol{\varphi}_r\right\|^2}\right] \tag{7}$$

where N_m is the number of interest modes, N_s is the number of data sets, ε_r is the standard deviation of the squared circular frequencies obtained from the measured data, $\hat{\omega}_r(n)$ is the r-th mode measured natural frequency in the n-th set of data in rad/s, $\omega_r(\boldsymbol{\theta})$ is the r-th mode calculated natural frequency for a given $\boldsymbol{\theta}$, $\boldsymbol{\varphi}_r$ is the r-th mode calculated mode shape. Note that $\boldsymbol{\Gamma}\boldsymbol{\varphi}_r$ and $\hat{\boldsymbol{\psi}}_r$ are normalized to have unit norm. The selection matrix $\boldsymbol{\Gamma}$ consists of only 1 and 0 that picks the observed degrees of freedom from the model-predicted mode shapes to match the measured ones, $\|\bullet\|$ is the Euclidean norm. The uncertainty of the measured mode shapes δ_r is:

$$\delta_r^{\ 2} = \frac{1}{N_s}\sum_{n=1}^{N_s}\frac{\left\|\hat{\boldsymbol{\psi}}_r(n) - \overline{\boldsymbol{\psi}}_r\right\|^2}{\left\|\hat{\boldsymbol{\psi}}_r(n)\right\|^2} \tag{8}$$

where $\hat{\boldsymbol{\psi}}_r(n)$ is the r-th mode shape of the n-th set of measurement and $\overline{\boldsymbol{\psi}}_r$ is the average mode shape for the r-th mode from the N_s sets of data.

The smallest value of J in Eq. (7) means the "best" fitting between the measured and calculated data, the corresponding $\boldsymbol{\theta}$ is treated as the "most plausible" model.

For a rail-sleeper-ballast system, many model classes can be considered. For the purpose of damage detection, the identification of the "best" model class is essential. From the probability theory (Cox 1961), the probability of a class of models conditional on the set of dynamic data is required to determine the plausible model class. Due to the limitation of space, only the most important formula is presented in this paper, which is the evidence of a model class conditional on a given set of data:

$$p\left(\boldsymbol{D}\middle|\boldsymbol{M}_j\right) \approx p\left(\boldsymbol{D}\,|\,\boldsymbol{\theta}_j^*, \boldsymbol{M}_j\right)(2\pi)^{\frac{N_j}{2}}\,p\left(\boldsymbol{\theta}_j^*\,|\,\boldsymbol{M}_j\right)\left|\boldsymbol{H}_j\left(\boldsymbol{\theta}_j^*\right)\right|^{-\frac{1}{2}} \quad \text{for } j = 1, 2, \ldots N_M \tag{9}$$

where $\boldsymbol{\theta}_j^*$ represents the most probable model parameters in the model class $\boldsymbol{M}_j$, and N_j is the number of uncertain parameters in $\boldsymbol{\theta}_j^*$. And $\boldsymbol{H}_j\left(\boldsymbol{\theta}_j^*\right)$ is the Hessian matrix of the function $g(\boldsymbol{\theta})$ (see Eq. (10)) evaluated at $\boldsymbol{\theta}_j^*$, it can be calculated by finite difference method at the most probable model $\boldsymbol{\theta}_j^*$, the function $g(\boldsymbol{\theta})$ is given by:

$$g\left(\boldsymbol{\theta}_j\right) = -\ln\left\lfloor p\left(\boldsymbol{\theta}_j\,|\,\boldsymbol{M}_j\right)p\left(\boldsymbol{D}\middle|\boldsymbol{\theta}_j, \boldsymbol{M}_j\right)\right\rfloor \tag{10}$$

3. NUMERICAL CASE STUDY

A concrete sleeper with length 2.42m and width 0.28m is employed to demonstrate the proposed methodology, which is modeled as a Timoshenko beam with two masses on an elastic foundation. The nominal value of Young's modulus of the concrete sleeper takes the value of 38 × 10^9 N/m^2, the equivalent ballast stiffness and the additional mass are 108 × 10^7 N/m^2 and 42 kg, respectively. The density of concrete sleeper is assumed to be 2200 kg/m^3.

3.1 Simulation of measured data

The six equal ballast regions model (Lam 2012b) is used to simulate the measured data. In order to verify the proposed ballast damage detection method, damage is artificially introduced to the system. The damage is simulated on the left hand-side of the sleeper and along 2/3 of the length of the sleeper, the stiffness reduction is 30%, thus the ideal ballast stiffness distribution is [0.7 0.7 0.7 0.7 1 1] multiplying the nominal value of ballast stiffness. To simulate the variation in ballast stiffness along the sleeper in real situation, 1% random fluctuation in the ballast stiffness is introduced. The modal parameters can be easily calculated by solving the eigenvalue problem though finite element method. The natural frequencies and mode shapes of the first five modes were considered in the model updating process. To consider the effect of measurement noise, 1% RMS white noise is added to the calculated modal parameters to simulate the "measured" modal parameters. Ten sets of data are considered to form multiple sets of measurements.

3.2 Bayesian model updating and model class selection

After obtaining the measured data, Bayesian model updating can be used to find the most plausible model for a given model class. Since there is no idea about the degree of polynomial should be used, Bayesian model class selection method is used by calculating the evidences of a series of model classes based on the given data. In this case study, 1 to 7-degree polynomial are considered, they form seven model classes, which are represented by $\boldsymbol{M}_j$, for $j = 1,\ldots,7$. The evidences of these seven model classes were calculated and summarized in Table 1. Since the numerical values of the evidences are very large, the logarithm is used to avoid numerical problem.

Table 1 Evidence of different classes of models

Class of models	Logarithm of Evidence	Logarithm of Likelihood factor	Logarithm of Ockham
$\boldsymbol{M}_1$	-4294.5	-4245.3	-49.2
$\boldsymbol{M}_2$	1917.9	1978.1	-60.2
$\boldsymbol{M}_3$	1929.8	2001.7	-71.9
$\boldsymbol{M}_4$	2725.1	2809.6	-84.5
$\boldsymbol{M}_5$	2739.4	2837.1	-97.7
$\boldsymbol{M}_6$	**2829.4**	2941.1	-111.7
$\boldsymbol{M}_7$	2816.8	2943.4	-126.6

From Table 1, we can easily observe that the 6-degree polynomial model with the largest logarithm of evidence (2829.4) is the most probable model class. The logarithm of evidence (2739.4) of 5-degree polynomial model and logarithm of evidence (2816.8) of 7-degree polynomial model are both smaller than 2829.4.

3.3 Ballast degradation *detection*

Bayesian model updating was employed to obtain the set of "most plausible" uncertain parameters, which are listed in Table 2. $\hat{\theta}_E$ is the scaling factor of the Young's modulus, $\hat{\theta}_L$ and $\hat{\theta}_R$ are the scaling factors for the left and right additional masses, and $\hat{\theta}_{C1}$ to $\hat{\theta}_{C7}$ are the coefficients of the 6-degree polynomial, which is used to approximate the ballast stiffness distribution.

Table 2 Updated model parameters of 6-degree polynomial model

$\hat{\theta}_E$	$\hat{\theta}_L$	$\hat{\theta}_R$	$\hat{\theta}_{C1}$	$\hat{\theta}_{C2}$	$\hat{\theta}_{C3}$	$\hat{\theta}_{C4}$	$\hat{\theta}_{C5}$	$\hat{\theta}_{C6}$	$\hat{\theta}_{C7}$
1.0001	1.0003	0.9999	0.2807	-2.2405	6.6102	-8.8515	5.3963	-1.2719	0.7634

Table 2 states that the Young's modulus and two additional masses are identified with high accuracy, the scaling factors are close to unity. With the identified "optimal" model, the natural frequencies and mode shapes are calculated and presented in Fig. 2 together with the "measured" modal parameters. It is clear that the matching between the "measured" and updated mode shapes is very good.

Fig. 3 shows the identified ballast stiffness distribution together with the true ballast stiffness distribution under the sleeper. The red solid line represents the simulated ballast distribution, while the red dashed line is the identified one. It truly points out that the ballast along 2/3 length of the sleeper on left-hand side is damaged and the extent is about 30%. It can be concluded that the proposed method successfully identifies the ballast degradation in this case study.

4. CONCLUDING REMARKS

This paper presents the use of Bayesian model class selection and Bayesian model updating for the detection of the ballast degradation utilizing measured vibration data of the in-situ sleeper. The numerical case study results are very encouraging showing that the proposed methodology is feasible. This method can estimate not only the damage location but also the damage extent in the presence of measurement noise. The result also shows that polynomials can be used to represent the continuous ballast stiffness distribution for the purpose of ballast damage detection. However, there is a difficulty to be solved before this method can be put into real application. It is not easy to tell whether the sleeper or the ballast is damaged by considering only the changes in modal parameters. In the proposed method, it is assumed that there is no damage on the sleeper. However, the concrete sleeper may be damaged, because the damage on sleeper also can alter the modal parameters of the system (Lam 2011), this may introduce the complexity of ballast damage detection. Further research is required in addressing this difficulty.

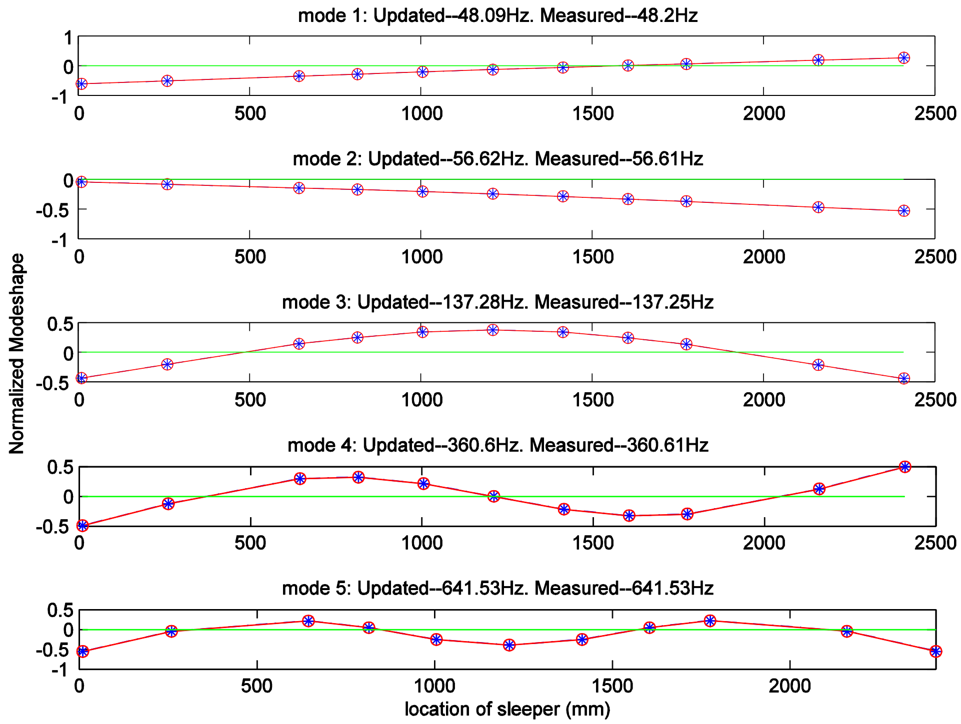

Fig. 2 Matching between the “measured” and model-predicted mode shapes

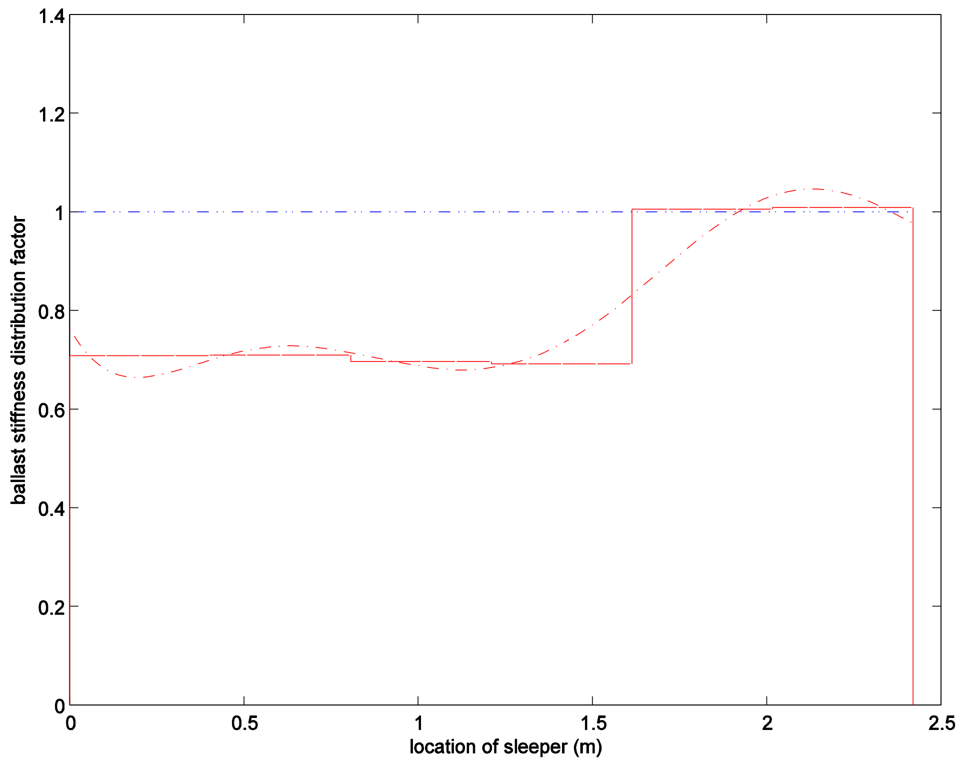

Fig. 3 Ballast degradation identification results

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