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Transient MR fluid flows in valve mode using a non-convex constitutive relation

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ABSTRACT

In the current paper, a non-convex constitutive relation based on phenomenological phase transition theory is employed to construct a differential model for the MR fluid flow in valve mode. The dynamical velocity distributions of the MR flow are simulated for two different sinusoidal loading conditions based on the proposed differential model. The plug flow predicted both by Bingham plastic model and Herschel-Bulkley model are well captured by the proposed model. It is shown that the proposed differential model can well predict the dynamical behaviors of MR fluids in the valve mode, including the transient behaviors for the plug flow boundaries when the pressure gradient is reversed.

1. INTRODUCTION

Magnetorheological (MR) fluids are mainly dispersions of particles made of soft magnetic materials in carrier oil. The unique characteristics of MR fluids are derived from the ability to switch reversibly between liquid-like and solid-like states in a fraction of millisecond when an external magnetic field is applied, and the strength of MR fluids to resist shear forces is field dependent. Such characteristics are attractive in many engineering applications, such as clutches, valves, brakes, damping devices, pumps, etc. However, it is a challenging task to model the dynamics of MR devices, because of the inherent nonlinear dynamics of MR fluids.
Many MR devices work in valve mode of MR fluids and the dynamic behaviour of MR fluids flowing through rectangular or circular pipe were widely investigated in recently years (Li 2003, Shaju 2008, Grunwald 2008, and Gedik 2012). To model the dynamical velocity distributions of MR fluids in the valve channel, a suitable constitutive model for the MR fluids is essential. Several models have been developed to mimic the characters of MR fluids in the past two decades. Among the extensive investigations, Bingham plastic model has been widely used to describe the behaviour of MR fluids

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(Nishiyama 2002, Hong 2007). In Bingham model, the fluid is assumed to be a Newtonian fluid with a constant plastic viscosity which is defined as the measured shear stress versus shear strain rate. Though this model is mathematically simple and has a clear physic concept, for cases where fluid experiences post-yield shear thinning or shear thickening, the assumption of constant plastic viscosity is not valid. Herschel-Bulkley model was proposed to capture the post-yield shear thinning or thickening behaviour (Widjaja 2008 and Zhang 2008).Nevertheless, similar to Bingham plastic model, the model cannot predict the hysteretic behaviour at low shear strain rate and the transient behaviors when the loading is dynamical, which is of vital importance for design and control analysis of MR devices.

In the current paper, a non-convex constitutive model based on phase-transition theory is employed to analyze the dynamical behaviour of MR fluids through two fixed plates. The dynamical velocity distributions of MR fluid and the plugs in the fluid are numerically simulated for two different sinusoidal loadings. For each loading, several velocity distributions of the flow at different stages are presented and discussed.

2. THE NON-CONVEX CONSTITUTIVE RELATION

The Bingham plastic model is widely used to describe the stress-strain rate relation, and was formulated as Eq. 1:

$$\tau = \tau_y + \eta_0 \dot{\varepsilon}, \quad |\tau| \ge \tau_y \tag{1}$$

Where τ_y is the yield stress induced by the magnetic field, η_0 is the viscosity of the fluids, and ε is the shear strain. In this model, the plastic viscosity η_0 is simply defined as the slope of the measured shear stress versus shear strain rate data, and would increase without limit when the shear stress increases. Obviously, this is physically impossible and the shear stress cannot increase without limitation.

In order to overcome such drawback of the Bingham plastic model, the Herschel-Bulkley model was proposed to describe the post-yield shear thinning and thickening behaviour, and was formulated as Eq.2:

$$\tau = \tau_y + K|\dot{\varepsilon}|^n, \ |\tau| \ge \tau_y \tag{2}$$

Where τ_y is the yield stress, *K* is plastic viscosity of MR fluids, and *n* is flow behaviour index. Compared with the Bingham plastic model, Herschel-Bulkley model is a more flexible representation of MR postyield behavior and the shear thinning behavior can be captured with appropriately chosen model parameters *K* and *n*.

Though both Bingham plastic model and Herschel-Bulkley model can describe the behaviour of MR fluids to some extent, the relation of shear stress and shear strain is a one-on-one relation, and the unique hysteretic dynamics is sill not captured. In the current paper, a non-convex constitutive relation is employed to investigate the behaviour of MR fluids, and the model is formulated as Eq.3:

$$\dot{\varepsilon} = \alpha\tau + \beta\tau^3 + \gamma\tau^5 \tag{3}$$

Where ε is the shear strain; α, β, and γ are material-specific model parameters, which are also dependent on the strength of the applied magnetic field. A typical curve of such a non-convex relation is as shown in Fig. 1.

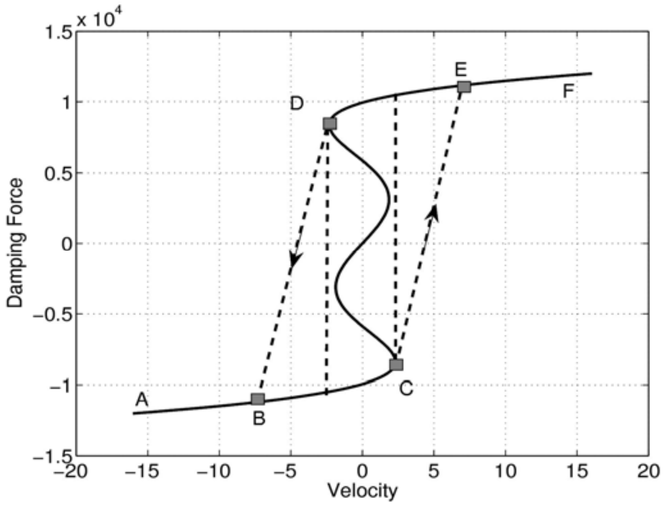

Fig. 1 The nonlinear constitutive relation involving hysteretic behaviour

Due to the non-convexity of the constitutive relation, bifurcations will be induced in the dynamics of MR fluids. The bifurcation points are *C* and *D*, and hysteresis loop will be resulted, as outlined by points *BCED* connected by dashed lines with arrows in the figure. The hysteretic dynamics of MR fluids can be modeled successfully by modeling the switching processes among these bifurcation points. The details about bifurcation and hysteresis loops were discussed by Wang (Wang 2006).

3. DYNAMICAL VELOCITY DISTRIBUTIONS

In order to develop MR devices working in valve mode, the behaviour of MR fluids flowing through two parallel plates are widely examined. In most cases, Bingham plastic model or Herschel-Bulkley model were employed as the constitutive relationship, and MR fluids were assumed working in the post-yield region. With such an assumption, the quasi-steady flow at constant loading conditions was investigated, and then a plug with constant thickness was demonstrated. However, none can capture the dynamical behaviour of MR fluids under dynamical loading conditions, because of the intrinsic limitation of constitutive models mentioned in the above section. To model the dynamics of MR fluids, the constitutive model should be able to capture the behaviour of both pre-yield and post-yield regions. Furthermore, the hysteretic dynamics should also be embedded in the constitutive relationship, which can be done by employing the proposed non-convex constitutive relationship of MR fluids.
In the current paper, the dynamical behaviours of MR fluids flowing through two parallel plates, as shown in Fig.2, are investigated under two sinusoidal loadings. In the present

study, the upper and bottom plates act as magnetic poles, and a linear pressure gradient is introduced to create the flow in valve mode. An external magnetic field *H* is applied and the field strength is assumed to be uniformly distributed.

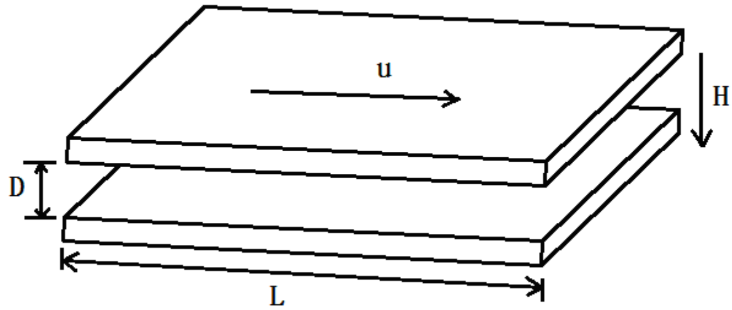

Fig. 2 A two parallel plate model

The flow between the plates is one-dimensional and should satisfy the principle of momentum; the governing equation is presented below as Eq. 4.

$$\rho\frac{\partial u}{\partial t}-\frac{\partial \tau}{\partial y}=-\frac{\partial p}{\partial x} \tag{4}$$

Where ρ is the mass density, τ is the shear stress and $\partial p/\partial x$ is the pressure gradient. The non-convex constitutive relationship can be reformulated as Eq.5.

$$\frac{\partial u}{\partial y}=\beta\tau+\beta\tau^3+\gamma\tau^5 \tag{5}$$

By solving Eq.4 and Eq.5 simultaneously with the nonslip boundary conditions u(0)=0 and u(d)=0, the velocity distributions can be well simulated.

One can easily see from Eq.4 and Eq.5 that the system is modeled by two simultaneous first order differential equations and the dynamical velocity distributions is regarded as the response of a nonlinear dynamical system to external loadings. The velocity distribution is determined by the pressure gradient $\partial p/\partial x$ and the values of model parameters α , β and γ.

To validate the differential model given by Eq.5 for the dynamical velocity distributions of MR fluids in valve model, a set of appropriate chosen coefficients with hysteresis embedded are employed for numerical experiment. In this study, model parameters used for simulations are as following: α=8.007, β =–21.50, γ=21.22, which were estimated from the experimental data of the dynamics of a MR damper with hysteretic behaviour (Wang 2006). The mass density ρ is set to be 1.The pressure loadings are $\partial p/\partial x= -\sin(2\pi t)$, and $\partial p/\partial x= -\sin(4\pi t)$, respectively. The simulated results are as shown in Fig. 3 and Fig 4, respectively.

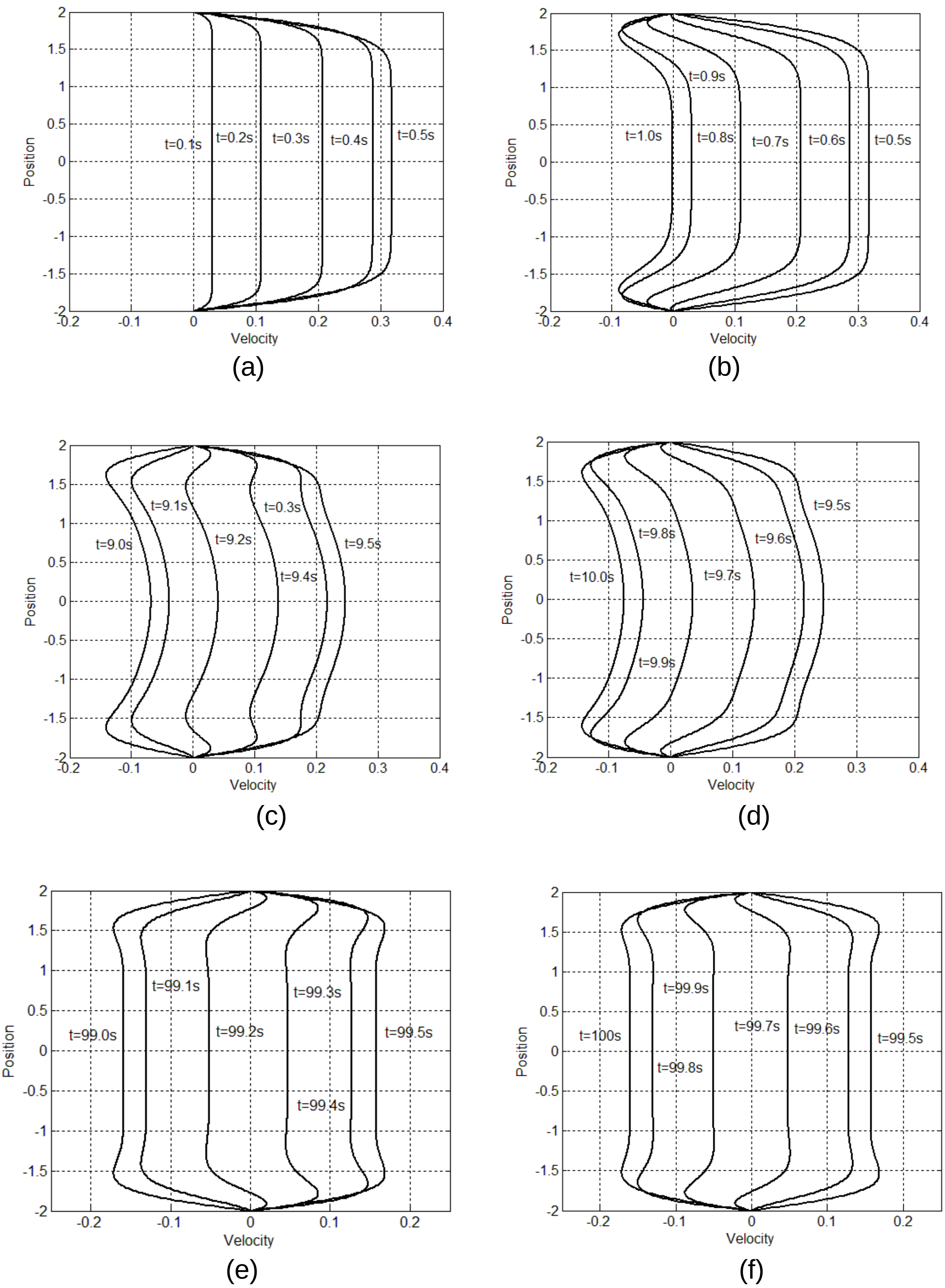

Fig. 3 The dynamical velocity distributions when $\partial p/\partial x = -\sin(2\pi t)$, (a)-(b) the 1st period, (c)-(d) the 10th period, (e)-(f) the 100th period

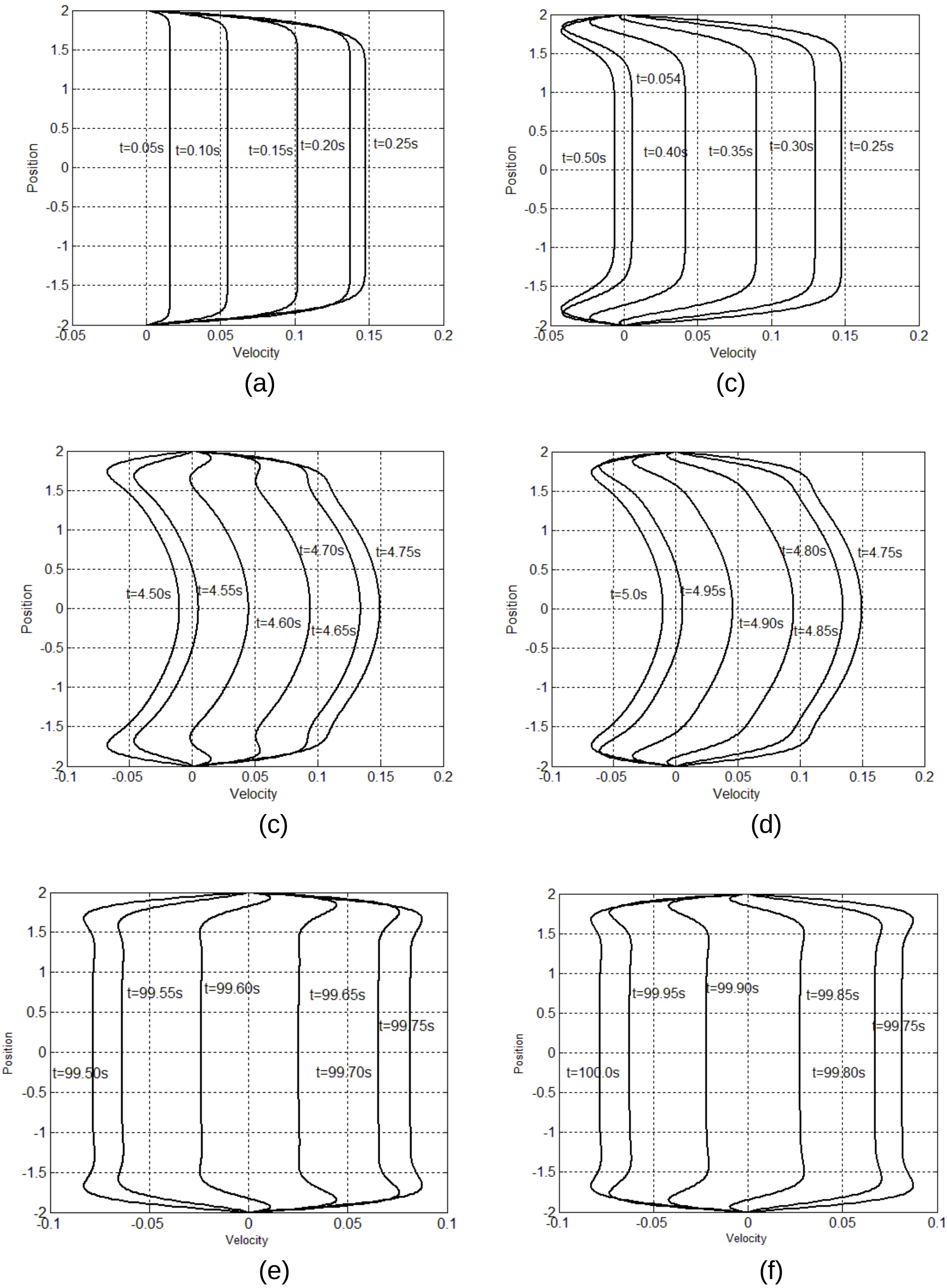

Fig. 4 The dynamical velocity distributions when $\partial p/\partial x = -\sin(4\pi t)$, (a)-(b) the 1st period, (c)-(d) the 10th period, (e)-(f) the 200th period

The model is a real dynamical one, because the derivative of velocity with respect to time is explicitly involved in Eq. 4; therefore, the transient behaviors of the flow are naturally captured. From Fig. 3 and Fig. 4, one can see that a plug flow predicted both by Bingham plastic model and Herschel-Bulkley model are captured by the proposed differential model. Nevertheless, the thickness of the plug flow is not stable at dynamical loadings, especially at early periods. This can be explained by the fact that thickness of the plug flow is determined by the relative motion between the outmost particle layers. For the first half of the first period, the plug is relatively thick, but the plug thickness would decrease when the pressure gradient $\partial p/\partial x$ is reversed, because the relative motion is much greater. The simulated velocity distributions are not symmetrical at the beginning due to the inertial effect; however, after a transit process when the inertial effect becomes much weaker, the simulated velocity distributions would reach a symmetric state, as shown in bottom of Fig. 3 and Fig.4. Hence the dynamical behaviors in both stable and transit stage can be well predicted by the proposed model.

CONCLUSIONS

In the current paper, the dynamical velocity distributions of MR fluids working in valve mode are investigated mathematically. Details of the simulated results for two sinusoidal loadings are presented. It is shown that the proposed differential model can well predict the dynamical behaviors of MR fluids in the valve mode, including the transient behavior when the fluids are subjected to sinusoidal loadings, which is important for system design and control analysis in real applications where transient behavior is important.

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