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A general criterion for geomechanics: extension to jointed rocks masses

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ABSTRACT

A general smooth and convex yield function had been proposed , able to model the particular behavior of porous materials, particularly rock materials, that are characterized by a linear or parabolic Mohr's envelope, and a particular shape in the deviatoric plane. These characteristics are defined by two functions: the equation of the criterion in the meridian plane and the extension ratio, which are integrated in a general equation ensuring convexity and smoothness of the yield function, whatever the characteristic functions. In this paper, further developments of the criterion are made to encompass to modelize the behavior of damaged, weathered or heavily fractured rocks. We identify the functions that allow to develop a smooth version of the generalized Hœk-Brown criterion. So it can be use do predict the behavior of rock masses, relying on identification on intact core sample, and taking into account observations made by geologist and field engineers through the Geological Strength Index of Hoek.

1. INTRODUCTION

Estimating the resistance and deformations problems is a challenging problem for rock mechanics engineers and geologist. The rock mass differs radically from the values measured in laboratories on core sample. A first reason is that rock masses can be heavily jointed and disturbed. A second that that the surfaces can be weathered, filled with fines and clays. The generalised Hœk-Brown criterion is an extension of the Hœk-Brown criterion (1980), developed to be used for jointed rock masses, or very poor quality rock masses. This criterion has been found very practical in the filed, as it can be deduced from the Hœk-Brown parameters measured on intact specimen and from the Geological Strength Index (GSI). Both criteria are expressed of the extremal stress, and thus present sharp corners in the stress space. As failure envelop tend to be smooth surfaces, a smooth version of Hœk-Brown has been developed (Maïolino, 2005). We

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present in this paper how this smooth criterion can be extended do the case of jointed and low quality rock masses to developp a smooth generalized Hœk-Brown criterion.

Stress sign convention: Traction stresses are positive, and the principal stresses ordered as follow : $\sigma_I \geq \sigma_{II} \geq \sigma_{III}$

2. THE HŒ K-BROWN CRITERION

The Hœk-Brown criterion for intact rock pieces is defined by:

$$f(\underline{\underline{\sigma}}) = (\sigma_I - \sigma_{III}) - \sigma_{ci}\sqrt{1 - m_i \frac{\sigma_I}{\sigma_{ci}}} \quad (1)$$

Where m_i is the value of the Hœk-Brown constant (values between 5 and 30) for intact rock and σ_{ci} is the unixial compressive strength of intact rock pieces.

The failure criterion has been modified to take into account various field situations, to very poor quality rock masses. In this paper, we consider the criterion and its parameters, as defined in the last major revision (Hoek et al., 2002) :

$$f(\underline{\underline{\sigma}}) = (\sigma_I - \sigma_{III}) - \sigma_{ci}\left(s - m_b \frac{\sigma_I}{\sigma_{ci}}\right)^a \quad (2)$$

Where m_b is the value of the Hœk-Brown constant for the rock mass, and $s(0 < s \leq 1)$ and $a(0.5 \leq a \leq 1)$are constants which depend upon the rock mass characteristics.

A new classification of rock mass the Geological Strength Index has been developed (note ranging from 0 to 100 (intact rock mass)). It can be used to rely the parameters of the original Hœk-Brown criterion identified on intact rock samples and the field properties of the rock mass :

$$m_b = m_i \exp\left(\frac{GSI - 100}{28 - 14D}\right) \quad (3)$$

$$s = \exp\left(\frac{GSI - 100}{9 - 3D}\right) \quad (4)$$

$$a = \frac{1}{2} + \frac{1}{6}\left(\exp\left(-\frac{GSI}{15}\right) - \exp\left(\frac{-20}{3}\right)\right) \quad (5)$$

Where D is a disturbance factor which depends upon the nature of the excavation methods and the nature of the engineering problem. It varies from 0 for undisturbed rock masses to 1 for very disturbed rock masses.

3. POLAR DECOMPOSITION OF A YIELD SURFACE

For a given mean stress ($\sigma_m = Tr\underline{\underline{\sigma}}/3$),the yield surface can be reduced to its cross-sectional shape on the deviatoric plane, or π -plane. A yield surface can be represented in a unique manner by the mean stress and the deviatoric invariants ($J_2 = Tr(\underline{\underline{s}}^2)/2$, $J_3 = Tr(\underline{\underline{s}}^3)/3$), with $\underline{\underline{s}} = \underline{\underline{\sigma}} - \sigma_m \underline{\underline{1}}$,but it is more practical to replace the third invariant by the Lode angle θ, to work in the π plane.

$$-\frac{\pi}{6} \le \theta = \frac{1}{3}\arcsin\left(-\frac{3\sqrt{3}}{2}\frac{J_3}{\sqrt{J_2}^3}\right) \le \frac{\pi}{6} \tag{6}$$

The set $\left(\sqrt{J_2}, \theta\right)$ define polar coordinates on one sixth of the deviatoric plane, which is sufficient for an isotropic criterion. Zienkiewicz and Pande (1975), using the fact that a yield surface can be reduced to its polar expression, provided tools to study the regularity, the sensitivity to the extension and the convexity of a criterion starting from the shape function $g_p(\theta)$

$$\sqrt{J_2} = \sigma^+ g_p(\theta) \tag{7}$$

The deviatoric radius : $\sigma^+(\sigma_m) = \sqrt{J_2}_{/\theta=\pi/6}$ $\sigma^+(\sigma_m) = \sqrt{J_2}_{/\theta=\pi/6}$, gives the yield function in the meridional plane $\left(\sigma_m, \sqrt{J_2}\right)$, for $\theta = \pi/6$. This value of the Lode angle corresponds to a classical triaxial test, or compression triaxial test $\left(\sigma_I = \sigma_{II} > \sigma_{III}\right)$.

The function $g_p(\theta)$ is the shape function of the yield surface in the deviatoric plane. It is normalized $\left(g_p(\pi/6) = 1\right)$ and gives directly the value of the extension ratio $g_p(-\pi/6) = L_S$.

The extension ratio L_S has a physical meaning and can be determined from experiment. The condition $\theta = -\pi/6$ corresponds to extension triaxial tests $(\sigma_I > \sigma_{II} = \sigma_{III})$, which can be performed with the same triaxial cell as compression triaxial test.

$$L_S = \frac{\sqrt{J_2}\left(\theta = -\frac{\pi}{6}\right)}{\sqrt{J_2}\left(\theta = \frac{\pi}{6}\right)} = \frac{\left(\sigma_I - \sigma_{III}\right)\left(extension\right)}{\left(\sigma_I - \sigma_{III}\right)\left(compression\right)} \tag{8}$$

Physically, this means that for the same mean stress, the yield value of $\sqrt{J_2}$ would be lower in extension than in compression. The value of L_S is directly linked to the deviatoric shape of a yield surface. While this value can be independent from the mean stress (Mohr-Coulomb), some rocks present a shape of their yield surface changing from triangular (low confinement) to circular (high confinement) (Kim and Lade, 1984) i.e, L_S increases from 0.5 to 1.

4. POLAR DECOMPOSITION OF HŒK-BROWN

When studying the Hœk-Brown criterion, it can be useful to normalise some quantities like mean stress(Carranza-Torres and Fairhurst, 1999)

$$P_i = \frac{1}{m_i^2} - \frac{\sigma_m}{m_i \sigma_{ci}} \tag{9}$$

We can thus determine the characteristic functions(deviatoric radius and extension ratio) of the Hœk-Brown criterion :

$$\sigma^+ = \frac{m_i \sigma_{ci}}{2\sqrt{3}} \frac{\sqrt{1+36P_i}-1}{3} \tag{10}$$

$$L_S = 2\frac{\sqrt{1+9P_i}-1}{\sqrt{1+36P_i}-1} \tag{11}$$

Whereas the Hœk-Brown criterion encompass the particular nature of rocks, that is a parabolic intrinsic curve, his shape present edges, whereas the envelop of failure of rocks tends to be smooth. The variations of L_S with mean stress reflects physical properties of rocks in triaxial condition.

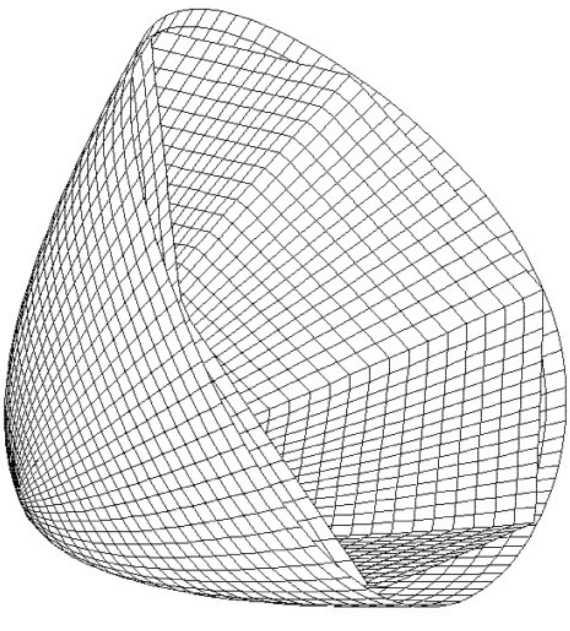

Fig 1 Hœk-Brown criterion and smooth version

5. GENERAL YIELD FUNCTION

The following function allows to develop a smooth version of Hœk-Brown (Maïolino, 2005), using the deviatoric radius and extension ratio of the Hœk-Brown criterion :

$$f\left(\underline{\underline{\sigma}}\right)=\frac{3}{2}\sqrt{3}\left(1-L_S\right)J_3+\left(L_S^2+1-L_S\right)\sigma^+J_2-\sigma^{+3}L_S^2 \tag{12}$$

6. DETERMINATION OF THE DEVIATORIC RADIUS OF GENERALISED HŒK-BROWN

We normalise the mean stress :

$$P_i=\frac{s}{m_b^2}-\frac{\sigma_m}{m_b\sigma_{ci}} \tag{13}$$

For a given value of a, σ^+ is the root of the following equation :

$$\sqrt{3}\frac{\sigma^+}{\sigma_{ci}}-\left(P_i m_b^2-\frac{2\sqrt{3}m_b}{3}\frac{\sigma^+}{\sigma_{ci}}\right)^a=0 \tag{14}$$

It is impossible to get an explicit expression of σ^+ except for a = 0.5(Hœk-Brown) or a=1.

$$\sigma^+_{|a=1}=\frac{m_b\sigma_{ci}}{6\sqrt{3}}\frac{18P_i m_b}{m_b+3} \tag{15}$$

So we have replaced σ^+ using the function $r(a,m_b,P_i)$

$$\sigma^+=\frac{m_b\sigma_{ci}}{6\sqrt{3}}r(a,m_b,P_i)\ \sigma^+= \tag{16}$$

So that Eq.(15) can be replaced by the following equation :

$$\left(\frac{m_b r}{6}\right)^{\frac{1}{a}}-m_b^2\left(P_i-\frac{r}{18}\right)=0 \tag{17}$$

So the roots $\tilde{r}$ have been computed, $a\in[0.5,1],m_b\in[1,35],P_i\in[0,1]$. Finally the following expression of r is proposed :

$$r(a,m_b,\mathrm{P}_i)=\frac{nc(a)m_b}{m_b+d(a)}\left(\left(1+36\mathrm{P}_i\right)^{p(a)}-1\right) \tag{18}$$

First a is fixed so that $nc(a)$, $d(a)$ end $p(a)$ can be computed. Then each one is studied as a function of a. The following functions are identified :

$$\begin{aligned} d(a) &= \frac{6\left(a-\frac{1}{2}\right)}{d_2 a^2 + d_1 a + \zeta} \\ nc(a) &= 0.05 a^{-\zeta} + 0.45 \\ p(a) &= -2a^2 + 4a - 1 \end{aligned} \tag{19}$$

With the following constants:

$$\begin{aligned} \zeta &= \frac{\ln 11}{\ln 2} \approx 3.46 \\ d_2 &= 5.7829 \\ d_1 &= -8.2424 \end{aligned} \tag{20}$$

The quality of Eq.(18) is excellent as the values of R^2 is of 0.9957. Comparison has been made upon 87500 computed values of $\tilde{r}$.

7. DETERMINATION OF THE EXTENSION RATIO

To determinate the value of the extension ratio, we will first have to identify $\sqrt{J_2}$ in extension $(\theta = -\pi/6)$, that we will name σ^-. Once again the function can be expressed directly only for two boundary cases : the Hœk-Brown criterion ($a = 0.5$) or when $a = 1$:

$$\sigma^-_{|a=\frac{1}{2}} = \frac{m_b \sigma_{ci}\left(\sqrt{1+9P_i} - 1\right)}{3\sqrt{3}} \tag{21}$$

$$\sigma^-_{|a=1} = \frac{m_b \sigma_{ci}}{3\sqrt{3}} \frac{9 m_b P_i}{2m_b + 3} \tag{22}$$

The identification was performed like for the deviatoric radius. First we determine numerically the root of the following equation ($a \in [0.5,1], m_b \in [1,35], P_i \in [0,1]$) :

$$\left(\frac{m_b r^e}{6}\right)^{\frac{1}{a}} - m_b^2\left(P_i - \frac{r^e}{9}\right) = 0 \tag{23}$$

We assume that r^e can be expressed as follow :

$$r^e(a, m_b, P_i) = \frac{nc^e(a) m_b}{m_b + d^e(a)}\left(\left(1 + 9\mathrm{P}_i\right)^{p^e(a)} - 1\right) \tag{24}$$

We identify the following functions of a:

$$
\begin{aligned}
d^e(a) &= \frac{6\left(a-\frac{1}{2}\right)}{d_2^e a^2 + d_1^e a + d_0^e} \\
nc^e(a) &= nc_1 a^{-nc_p} + nc_0 \\
p^e(a) &= 1-\frac{1}{14}\left(\frac{1}{a^3}-1\right)
\end{aligned}
\tag{25}
$$

With the following constants :

$$
\begin{aligned}
d_2^e &= 5.7205 \\
d_1^e &= -6.6491 \\
d_0^e &= 2.9286 \\
nc_1^e &= 0.02755 \\
nc_p^e &= 5.219 \\
nc_0^e &= 0.973
\end{aligned}
$$

We have $R^2 = 0.9968$.

We can thus propose the following extension ratio for a generalised Hœk-Brown:

$$
L_S = \frac{\sigma^-}{\sigma^+} = \frac{nc^e(a)\left(m_b + d(a)\right)}{nc(a)\left(m_b + d^e(a)\right)} \frac{\left(1+9P_i\right)^{p^e(a)}-1}{\left(1+36P_i\right)^{p(a)}-1} \tag{26}
$$

8. CONCLUSION

We have proposed a smooth version of generalised Hœk-Brown. It was not possible to obtain literal expression of the characteristic functions, so we have to perform numerically. The proposed functions have an excellent correlation with values computed directly from the generalized Hœk-Brown criterion. One should remember that a, m_b and also σ_{ci} are constant. So, even if the functions we present seem to require a lot of computations at first glance, in fact all of these values are constant for a given rock mass, so that in a numerical code, they are computed only once, and the only real variable is P_i. So in a finite element code, the deviatoric radius and the extension ratio are function of only one variable that is mean stress.

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