The 2013 World Congress on
Advances in Structural Engineering and Mechanics (ASEM13)
Jeju, Korea, September 8-12, 2013

A study on the design guideline of SMA spring with the initial curvature added bi-stable composite

* Jong-Gu Lee[1)], Junghyun Ryu[2)], Seung-Won Kim[3)], Kyu-Jin, Cho[4)], Maenghyo Cho[5)]

[1), 2), 5)] *Smart Structure & Design Laboratory, School of Mechanical & Aerospace Engineering, Seoul National University, Seoul, Korea*
[3), 4)] *Biorobotics Laboratory, School of Mechanical & Aerospace Engineering, Seoul National University, Seoul, Korea*
[5)] *mhcho@snu.ac.kr*

ABSTRACT

In order to suggest the design guideline of SMA spring, which is connected at the bi-stable composites with initial curvature. We calculate the critical moment which triggers off the snap-through of the initial curvature added bi-stable composites, cured unsymmetrical cross-ply laminate composites, by using Rayleigh-Ritz method. As a result, we show that the critical moment changes linearly with respect to the initial curvature. On this basis, we suggest the explicit design guideline of SMA spring.

1. INTRODUCTION

Bi-stable composites, one of energy efficient morphing structures, are attracting the worldwide attention of researchers due to the bi-stability and the instantaneous morphing behavior by snap-through from one stable configuration to another stable configuration. For practical application of the bi-stable composites, many researchers have carried out investigations for the prediction of critical force which triggers off the snap-through and the design of the actuators to induce the snap-through. (Dano 2002) have predicted the snap-through force by Rayleigh-Ritz technique and have design SMA (shape memory alloy) wire to induce the snap-through. (Hufenbach 2003) have designed piezo-ceramic MFC(macro fiber composite) to induce the snap-through.

Something to notice here is that these applications of the bi-stable composites are limited in the bi-stable composites manufactured on the flat tool-plate. For this reason, the final (cured) curvatures corresponding to each stable state are not only same, but the snap-through loads corresponding to each stable are also. However, in the case of the bi-stable composite with the initial curvature induced by curing laminate on the curved tool-plate, the final curvatures corresponding to each stable state are not only different (Ryu 2012), but the snap-through loads corresponding to each stable are also.

[1) 2) 3)] Graduate Student
[4) 5)] Professor

For this reason, the actuator, SMA spring, is required to be designed separately, depending on each stable state. In order to suggest the proper design guideline of SMA spring actuator, the critical moment, which triggers off the snap-through of the bi-stable composites with initial curvature, is calculated using Rayleigh-Ritz method. From the result of analysis, we show that the critical moment changes linearly with respect to the initial curvature. On this basis, we suggest the explicit design guideline of SMA spring actuator.

2. ANALYTICAL MODEL

In order to obtain the deformed shape of the bi-stable composites subject to line moment, moment equilibrium condition was introduced to the total potential energy by means of Lagrangian multiplier. The critical moment was calculated by minimizing the modified total potential energy.

2.1 The bi-stable composites subjected to line moment

By assumption of plane stress, total potential energy considering the moment equilibrium is represented as follows.

$$\Pi^{*}=\int_{-\frac{L}{2}}^{\frac{L}{2}}\int_{-\frac{L}{2}}^{\frac{L}{2}}\int_{-\frac{t}{2}}^{\frac{t}{2}}\frac{1}{2}\overline{Q_{11}^{i}}\left(\varepsilon_{11}^{e}\right)^{2}+\frac{1}{2}\overline{Q_{22}^{i}}\left(\varepsilon_{22}^{e}\right)^{2}+\overline{Q_{12}^{i}}\varepsilon_{11}^{e}\varepsilon_{22}^{e}+\frac{1}{2}\overline{Q_{66}^{i}}\gamma_{12}^{e}dzdxdy+\zeta\left(M_{x}^{\text{int.}}-M_{x}^{ext.}\right) \tag{1}$$

, where, L is the side-length of composite, t is the thickness of bi-stable composite, $\overline{Q_{\alpha\beta}^{i}}$ is the transformed reduced stiffnesses corresponding to the i-th layer.

The elastic strain, $\varepsilon_{11}^{e},\varepsilon_{22}^{e},\gamma_{12}^{e}$ are defined by difference of the total strain and the sum of the thermo-elastic strain and the strain induced by initial curvature as follows.

$$\begin{aligned}\varepsilon_{11}^{e}&=\varepsilon_{11}^{total}-\alpha_{11}\Delta T-z\kappa_{11}^{initial}\\ \varepsilon_{22}^{e}&=\varepsilon_{22}^{total}-\alpha_{22}\Delta T-z\kappa_{22}^{initial}\\ \gamma_{12}^{e}&=\varepsilon_{12}^{total}-\alpha_{12}\Delta T-z\kappa_{12}^{initial}\end{aligned} \tag{2}$$

The Lagrangian multiplier, ζ was introduced to consider the moment equilibrium which was represented by the difference of the internal moment, $M_{x}^{\text{int.}}$ and the external moment, $M_{x}^{ext.}$.

In order to obtain the deformed shape of the bi-stable composite subject to the line moment, first variation of total potential energy should be satisfied with nonlinear algebraic equality condition, that is to say, first order necessary condition should be satisfied. The solution satisfying the nonlinear algebraic equality condition was obtained by means of Newton-Raphson iterative method.

$$\delta\Pi^{*}=\sum_{i=1}^{n}\left(\frac{\partial\Pi^{*}}{\partial a_{i}}\right)\delta a_{i}=0 \tag{3}$$

2.2 Numerical result

In order to obtain the critical line moment, which triggers off the snap-through of the bi-stable composites, external line moment was applied along the edge of the bi-stable composite. In the analysis, the more line moment increased linearly, the more curvature decreased progressively. Eventually, snap-through occurred at the sudden change of curvature, at which the corresponding line moment was defined as the critical line moment triggering off the snap-through.

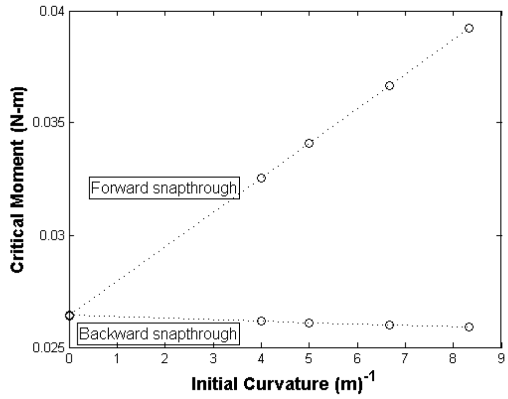

Fig. 1 $M^{critical} - \kappa^{critical}$ curve of bi-stable composite with initial curvature

As a result of analysis, the critical line moment of forward snap-through increased linearly with respect to the initial curvature and the critical line moment of backward snap-through decreased linearly with respect to the initial curvature, as shown in Fig. 1.

2. DESIGN GUIDELINE OF SMA SPRING

On the basis of the aforementioned linearity of the change of critical line moment with respect to the initial curvature, the critical moment and the critical curvature is easily predicted for the bi-stable composites with the other initial curvatures.

In addition, if the spring constant of SMA spring at pure Austenite state and Martensite state are known, SMA spring is also easily designed by following the inequality conditions for inducing the snap-through and for keeping the stable state, as written in Eq.(4),(5)

<Inequality condition for inducing the snap-through>

$$M_{critical}^{forward} < K_A \left[L_{critical}^x - L_{initial}^x \right] h \approx K_A \left[L_{initial}^x \left\{ 1 + \left(\kappa_{cured}^x - \kappa_{critical}^x \right) h \right\} - L_{initial}^x \right] h \quad (4)$$

<Inequality condition for keeping the stable state after snap-through>

$$M_{critical}^{backward} > K_M \left[L_{snapped}^y - L_{initial}^y \right] h \approx K_A \left[L_{initial}^y \left\{ 1 + \left(\kappa_{cured}^y - \kappa_{snapped}^y \right) h \right\} - L_{initial} \right] h \quad (5)$$

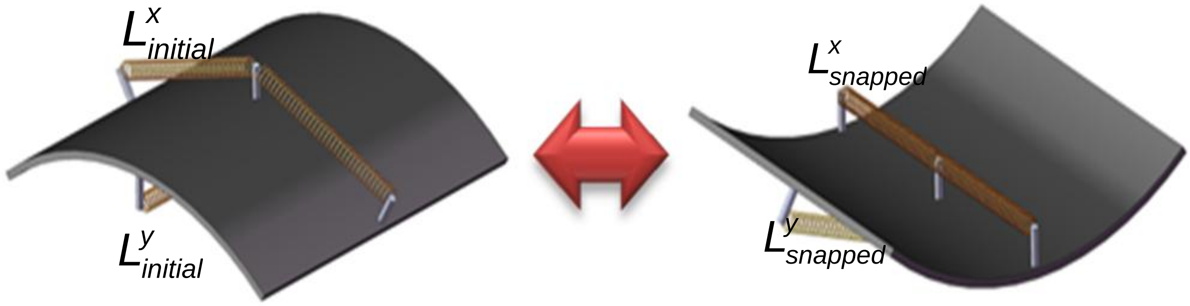

Fig. 2 Schematic of the snap-though of the bi-stable composites with SMA spring

3. CONCLUSION

In this study, by prediction of the critical line moment, which triggers off the snap-through, we showed that the critical line moment changed linearly with respect the initial curvature. On the basis of this result, we suggested the simple design guide line of SMA spring which is connected at the initial curvature added bi-stable composites.

For future work, in order to develop the more general design guide line of SMA spring which is considered in the design parameters of SMA spring; wire diameter, spring diameter, spring index, number of coil, etc. A parametric study on the SMA spring will be carried out.

ACKNOWLEDGMENTS

This work was supported by the National Research Foundation of Korea(NRF) grant funded by the Korea government(MEST) (No. 2012K001368).

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