

## **A Simplified Model of Caisson System for Stability Monitoring**

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### **ABSTRACT**

In this study, a simplified model of caisson system for stability monitoring is presented. First, a simplified model of the caisson system is designed on the basis of the characteristics of existing harbor caisson structures. A mass-spring-dashpot (MSD) model allowing only the sway motion is formulated. In the model, each unit is connected to adjacent ones via springs and dashpots to represent the condition of interlocking mechanisms. Next, the MSD model is evaluated for a 3-D FE model of the caissons and a lab-scale caisson to examine the accuracy of the simplified model's vibration responses.

### **1. INTRODUCTION**

On the demand to support the dynamic analysis of harbor caisson structure, several simplified dynamic models of the caisson-type breakwater were presented with small differences in the recent years (Smirnov and Moroz, 1983; Goda, 1994; Vink, 1997). In those models, a caisson was treated as a rigid body on springs and dashpots which represent an elastic foundation. Interested DOFs of those caisson models are normally three which are vertical, horizontal and rotational movements in the plane normal to the caisson array axis. However, the existing caisson models were proposed based on physical models which did not consider the effects of the longitudinal array structure of the breakwater. For vibration analysis of a real caisson breakwater, the following main issues should be considered: (1) the submerged condition of the coastal structure limits the accessibility for vibration measurement; and (2) the harbor caisson system consists of multiple caisson segments which are normally interconnected with each other by shear-keys to resist against the incident wave force acting perpendicular to the front wall.

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In this study, a simplified model of caisson system for stability monitoring is presented. First, a simplified model of the caisson system is designed on the basis of the characteristics of existing harbor caisson structures. A mass-spring-dashpot (MSD) model allowing only the sway motion is formulated. In the model, each unit is connected to adjacent ones via springs and dashpots to represent the condition of interlocking mechanisms. Next, the MSD model is evaluated for a 3-D FE model of the caissons and a lab-scale caisson to examine the accuracy of the simplified model's vibration responses.

## 2. SIMPLIFIED MODEL OF HARBOR CAISSON

Since the wave action is usually perpendicular to the caisson array axis (i.e., x-direction), the vibration in the impact direction (i.e., y-direction) is relatively larger than other directions (Lee et al., 2012; Yoon et al., 2012). Therefore, only the sway motion of caissons (i.e., y-direction) is taken into account in this study. Based on a few existing simplified models, a planar model of interlocked caissons is proposed as shown in Fig. 1. Caissons are treated as rigid bodies on elastic half-space foundations which can be described via the horizontal springs and dashpots. Springs and dashpots are also simulated between adjacent caissons units to represent the interlocking condition.

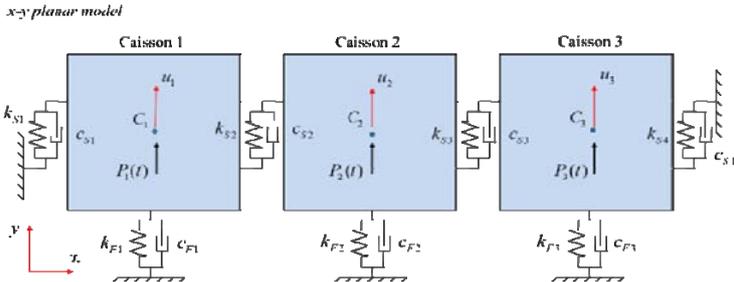


Fig. 1 Dynamic conceptual model of three interlocked caissons

### 2.1 Equations of Motion

Based on the equilibrium conditions of the free-body diagrams of caissons, the sway motion can be formulated in matrix form as:

$$\begin{aligned}
 & \begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix} \begin{Bmatrix} \ddot{u}_1 \\ \ddot{u}_2 \\ \ddot{u}_3 \end{Bmatrix} + \begin{bmatrix} c_{F1} + c_{S1} + c_{S2} & -c_{S3} & 0 \\ -c_{S2} & c_{F1} + c_{S1} + c_{S2} & -c_{S3} \\ 0 & -c_{S3} & c_{F1} + c_{S1} + c_{S2} \end{bmatrix} \begin{Bmatrix} \dot{u}_1 \\ \dot{u}_2 \\ \dot{u}_3 \end{Bmatrix} \\
 & + \begin{bmatrix} k_{F1} + k_{S1} + k_{S2} & -k_{S3} & 0 \\ -k_{S2} & k_{F1} + k_{S1} + k_{S2} & -k_{S3} \\ 0 & -k_{S3} & k_{F1} + k_{S1} + k_{S2} \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix} = \begin{Bmatrix} P_1(t) \\ P_2(t) \\ P_3(t) \end{Bmatrix} \quad (1)
 \end{aligned}$$

where  $m_j$  is the total horizontal mass of the  $j^{th}$  caisson;  $k_{Fj}$  and  $c_{Fj}$  separately represent the horizontal spring and dashpot of the  $j^{th}$  caisson's foundation ( $j=1-3$ );  $k_{Sj}$  and  $c_{Sj}$  respectively represent the horizontal spring and dashpot of the  $k^{th}$  shear-key connection

( $k=1-4$ );  $\ddot{u}_j$ ,  $\dot{u}_j$  and  $u_j$  are the horizontal acceleration, velocity and displacement of the the  $j^{th}$  caisson; and  $P_j(t)$  is the external force placed at the center of gravity of the  $j^{th}$  caisson.

## 2.2 Determination of Structural Parameters

### Mass parameter

The total horizontal mass of the  $j^{th}$  caisson ( $m_j$ ) includes the mass of the caisson itself ( $m_j^{cai}$ ), the horizontal hydrodynamic ( $m_j^{hyd}$ ) and the horizontal geodynamic masses ( $m_j^{geo}$ ) as follows:

$$m_j = m_j^{cai} + m_j^{hyd} + m_j^{geo} \quad (2)$$

For calculating the horizontal hydrodynamic mass, the following equation presented by Oumeraci and Kortenhaus (1994) is used:

$$m_j^{hyd} = 0.543L_j\rho_w H_w^2 \quad (3)$$

in which the quantities  $L_j$  and  $H_w$  represent the  $j^{th}$  caisson's length and the water level; and the quantity  $\rho_w$  is the mass density of sea water. According to Vibration of Soils and Foundations (Richart et al., 1970), the horizontal geodynamic mass can be computed as follows:

$$m_j^{geo} = 0.76\rho_s(B_jL_j/\pi)^{3/2}/(2 - \nu) \quad (4)$$

where  $\rho_s$  and  $\nu$  are respectively the mass density and Poisson's ratio of the foundation soil; and  $B_j$  is the  $j^{th}$  caisson's width

### Stiffness parameter

It is commonly accepted in geotechnical engineering that the horizontal spring constant ( $k_{Fj}$ ) of the elastic foundation is the function of the horizontal modulus of subgrade reaction ( $b$ ) as follows:

$$k_{Fj} = bL_jB_j \quad (5)$$

Normally, caisson segments are designed with the uniform linking capacity, where  $k_{S2} = k_{S3}$ . Since the rest of caisson array is not represented in the planar model, the stiffness of the last shear-keys (i.e.,  $k_{S1}$  and  $k_{S4}$ ) is smaller than that of the middle shear-keys (i.e.,  $k_{S2}$  and  $k_{S3}$ ). This condition can be expressed as:

$$k_{S1} = k_{S4} = ak_{S2} = ak_{S3} \quad (6)$$

where  $0 < a < 1$  (Lamberti and Martinelli, 1998).

### Damping parameter

The Rayleigh damping is used to simulate the energy dissipation in the caisson system

as follows (Wilson, 2004):

$$[C] = \alpha[M] + \beta[K] \quad (7)$$

in which  $\alpha$  and  $\beta$  are the mass and stiffness damping coefficients. Due to orthogonality conditions of mass and stiffness matrices, this equation can be rewritten as:

$$\xi_n = \alpha/2\omega_n + \beta\omega_n/2 \quad (8)$$

where  $\xi_n$  is the critical-damping ratio; and  $\omega_n$  is the natural frequency.

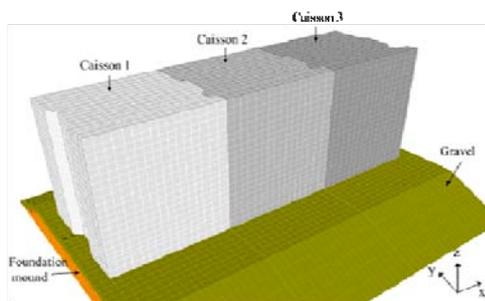
If the damping ratios (e.g.,  $\xi_i$  and  $\xi_j$ ) corresponding to two specific frequencies (e.g.,  $\omega_i$  and  $\omega_j$ ) are known, the two Rayleigh damping factors (i.e.,  $\alpha$  and  $\beta$ ) can be evaluated.

### 3. NUMERICAL VALIDATION OF SIMPLIFIED MODEL OF HARBOR CAISSON

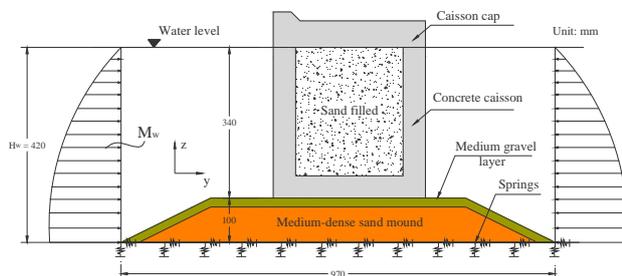
#### 3.1 3-D FE model of Harbor Caissons

A 3-D FE model of harbor caissons is simulated using SAP2000 software as shown in Fig. 2(a). An elastic modulus of 24 GPa (28-day strength of 28 MPa) is designed for concrete caissons. Soil parameters are selected according to Look (2007). The material properties are listed in Table 1.

The elastic characteristic of the sea bed (dense sand) is described by an area spring system (see Fig. 2(b)). The spring constant of the sea bed is selected as 96 MN/m/m<sup>2</sup> (Bowles, 1996). The interlocking condition is simulated by y-directional 1-D links at the shear-keys (see Fig. 2(c)). The stiffness of links is assumed to be 25 MN/m/m<sup>2</sup>. For all modes in the 3-D FE model, 5% of the damping ratio is assumed (Gao et al., 1988). The added mass of sea water ( $M_w$ ) is calculated by Westergaard's hydrodynamic water pressure equation (Westergaard, 1933). It should be noted that Eq. (3) is simplified form Westergaard's equation.



(a) 3-D FE model



(b) Boundary condition

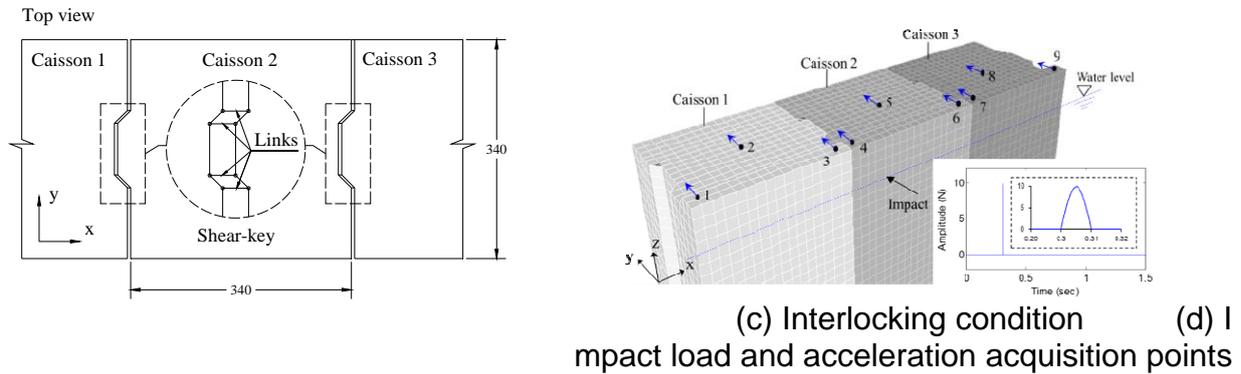


Fig. 2 3-D FE model of three interlocked caissons

Table 1. Material properties of the 3-D FE model

	Medium gravel	Medium-dense sand	Concrete
Mass density (kg/m <sup>3</sup> )	2100	2000	2400
Elastic modulus (MPa)	50	30	24000
Poisson's ratio	0.3	0.325	0.2

An impact force in y-direction is applied perpendicularly to the front wall of Caisson 2. The y-directional acceleration signals are measured at nine points (i.e., 1-9) on the top of the caisson caps as shown in Fig. 2(d). The sampling frequency is set as 1 kHz. Fig. 3 shows acceleration signals in y-direction of points 2, 5 and 8. It is observed that the vibration of Caisson 2 is propagated into Caisson 1 and Caisson 3. However, the vibration amplitudes of the unexcited caisson are only about a half of that of the excited one. This implies that a certain amount of energy is apparently subtracted from the excited caisson by wave propagation along the caisson system (Lamberti and Martinelli, 1998).

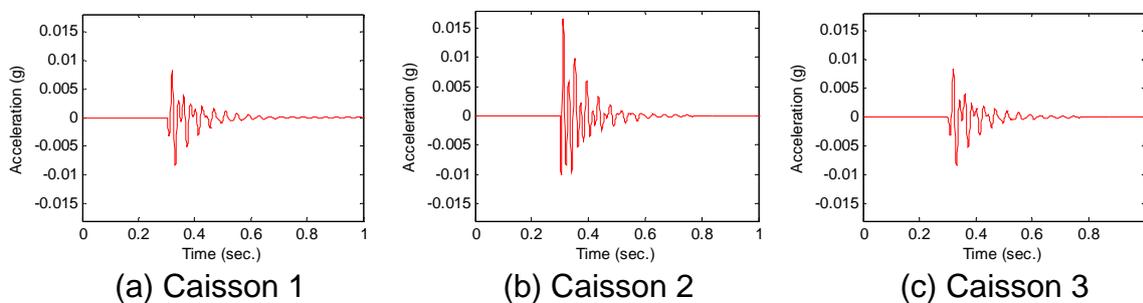


Fig. 3 Y-directional acceleration signals of 3-D FE model

### 3.2 Simplified Model of 3-D FE simulation

The simplified model of the 3-D FE model is established using the proposed theoretical model. The total horizontal masses for three caissons are 149.52 kg. The stiffness parameters are determined by matching vibration responses of the simplified model and the 3-D FE model using try-and-error method. The modulus of subgrade reaction of the foundation mound is selected as  $25 \times 10^6$  N/m<sup>3</sup> which is equivalent with that of medium dense sand (Bowles, 1996). By using Eq. (5), the spring constants of the foundation mound are calculated as  $2.89 \times 10^6$  N/m for three caissons. By assuming  $k_{S1} = k_{S4} = 0.5k_{S2} = 0.5k_{S3}$  (Martinelli and Lamberti, 1998), the stiffness of the middle and last shear-keys are obtained as  $3.179 \times 10^6$  N/m and  $1.59 \times 10^6$  N/m, respectively. For calculating the damping term, the first two natural frequencies (see Table 2) and the critical damping ratio (5%) of the 3-D FE model are utilized. The calculated mass and stiffness damping coefficients are, respectively, 10.387 and 0.000218. To solve the equations of motion, the Runge–Kutta scheme is utilized (Press et al., 1988).

#### Vibration Response in Time Domain

It is noted that in the 3-D FE model, acceleration signals on the top of caissons are measured, whereas acceleration signals of the simplified model are computed at the mass centroids of the caissons. Therefore, the following procedure is performed to estimate the acceleration signals of the mass centroids of the caissons in the 3-D FE model. Firstly, acceleration signals of additional locations on the front walls (points 10, 11 and 12) are measured as described in Fig. 4. By comparing the acceleration signals of the upper points (3, 6 and 9) and the lower points (10, 11 and 12), the inclinations of the caissons can be obtained. Secondly, the mass centroid of each caisson is computed considering the added mass of sea water and added mass of soil by (see Fig. 4). Thirdly, for each caisson unit, the acceleration signal of the mass centroid (C1, C2 or C3) is linearly-estimated based on its inclination ( $\alpha_1, \alpha_2$  or  $\alpha_3$ ) and the measured signal at the top center location (point 2, point 5 or point 8). The estimated y-directional acceleration signals at the caissons' centroids in the 3-D FE model are used to compare with those of the simplified model in Fig. 5. It can be seen in the figure that the signals of both models are well-matched.

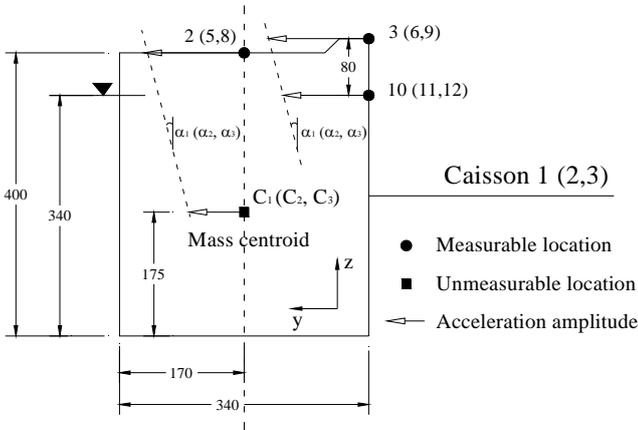


Fig. 4 Linear relationships of acceleration signals

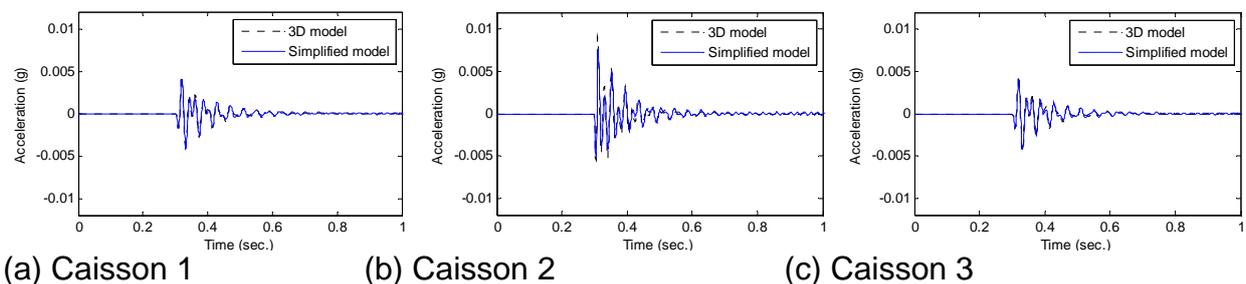


Fig. 5. Y-directional acceleration signals at caissons' centroids in 3-D FE model and its simplified model

### Vibration Response in Frequency Domain

The PSDs of y-directional acceleration signals of the caissons' centroids are computed for the both models (i.e., simplified model and 3-D FE model), as shown in Fig. 6. In the figure, the magnitudes and frequencies of the first two peaks obtained from the two models are well-matched.

The extracted mode shapes and corresponding natural frequencies are shown in Fig. 7 and given in Table 2, respectively. It can be seen that the modal parameters of the simplified model are almost consistent with those of the 3-D FE model.

The modal analysis of the 3-D FE model is carried out in SAP2000 software. It is observed that three caissons mostly move together in the same phase for the first mode, but in the opposite phase for the second mode. These results are well comparable with those sketched in Fig. 7.

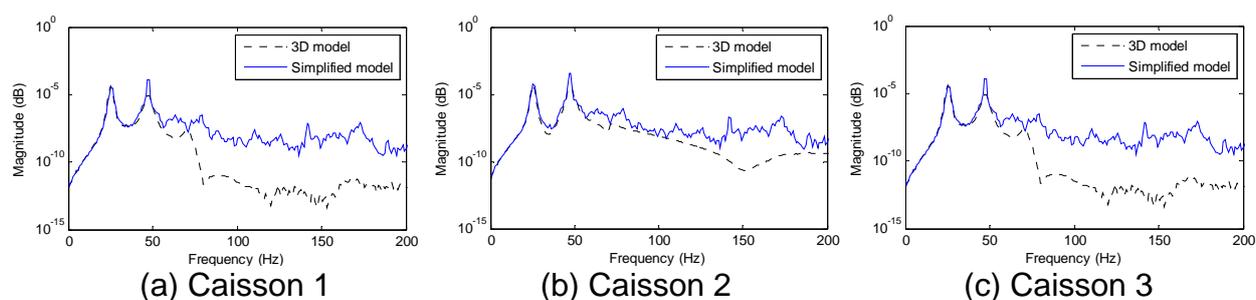


Fig. 6. Y-directional power spectral density of 3-D FE model and its simplified model

Table 2. Natural frequencies of 3-D FE model and its simplified model

Mode	Natural frequency (Hz)		
	FE model	Simplified model	Difference
Mode 1	25.33	25.88	2.13%
Mode 2	47.59	47.36	-0.49%

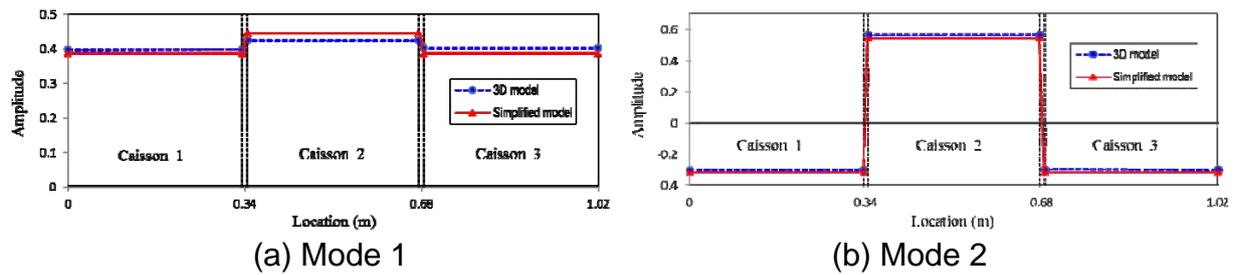


Fig. 7. Y-directional mode shapes of 3-D FE model and its simplified model

## 4. EXPERIMENTAL VALIDATION OF SIMPLIFIED MODEL OF HARBOR CAISSON

### 4.1 Experimental Model of Harbor Caisson

Harbor caisson structure was experimentally simulated on two dimensional wave tank. To simulate installed condition of the real caisson, a lab-scale caisson and its foundation was set up on the foundation mound with tetrapods as shown in Fig. 8(a). Foundation mound was constructed and covered by armor gravel. Water level was set to 34 cm height from bottom of the caisson as shown in Fig. 8(b). As the interlocking condition, two concrete blocks which have shear key were installed on both sides of the caisson and fitted to wall of the water tank.

Vibration responses were measured by acceleration acquisition system which consists of accelerometer, signal conditioner, terminal block, DAQ card, and laptop. Accelerometers used for the test are PCB393B04 model which has  $\pm 5g$  of measureable range and  $1V/g$  of sensitivity. The accelerometers were installed on the cap concrete to measure the vibration response of the caisson as shown in Fig. 8(b). Y-directional hammer impact on the caisson wall was employed to excite the caisson.

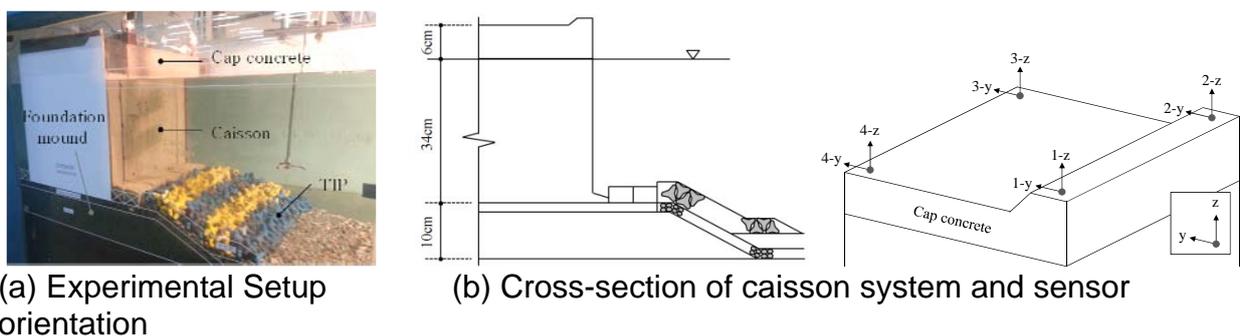


Fig. 8 Experimental setup for a lab-scale caisson in the 2D wave tank

### 4.2 Simplified Model of Experimental Harbor Caisson

The simplified model of the experimental model is established using the proposed theoretical model. The stiffness parameters are determined by matching vibration

responses of the simplified model and the experimental model using try-and-error method. The modulus of subgrade reaction of the foundation mound is selected as  $25 \times 10^6 \text{ N/m}^3$  which is equivalent with that of medium dense. By using Eq. (5), the spring constants of the foundation mound are calculated as  $1.33 \times 10^6 \text{ N/m}$  for caisson 2 and  $2.19 \times 10^5 \text{ N/m}$  for caisson 1 and 3. In the same manner of that of 3D FE model, the stiffness of the middle and last shear-keys are obtained as  $2.62 \times 10^5 \text{ N/m}$  and  $9.04 \times 10^5 \text{ N/m}$ , respectively.

For calculating the damping term, the first two natural frequencies (see Table 3) and damping ratio of two modes (i.e. 8.34% for mode 1 and 3.45% for mode 2) are utilized. The calculated mass and stiffness damping coefficients are, respectively, 2.841 and  $3.17 \times 10^{-5}$ .

Vibration Response in Time Domain

Acceleration signals on the top of caissons are measured, whereas acceleration signals of the simplified model are computed at the mass centroids of the caissons in the same manner of that of 3D FE model. Fig. 9(a) shows acceleration signal of experimental model and its simplified model. Acceleration response of experimental model includes noise and unexpected excited source. However, the signals of both models were well-matched, comparatively.

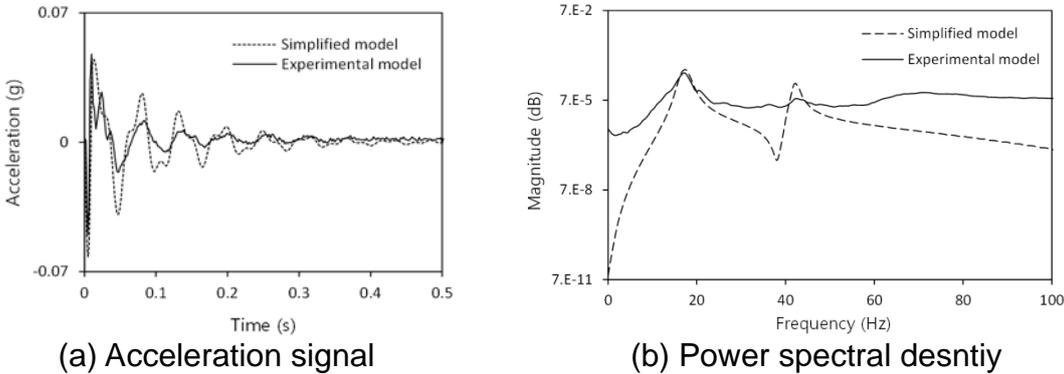


Fig. 9 Y-directional vibration response at caissons’ centroids in the experimental model and its simplified model

Table 3. Natural frequencies of experimental model and its simplified model

Mode	Natural frequency (Hz)		
	Experimental model	Simplified model	Difference
Mode 1	17.09	17.58	2.87%
Mode 2	41.99	41.99	0%

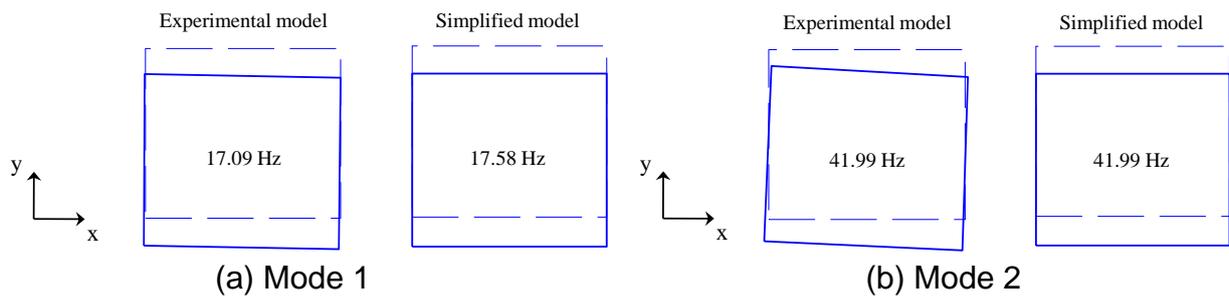


Fig. 10 Mode shapes of the experimental model and its simplified model

### Vibration Response in Frequency Domain

The PSDs of y-directional acceleration signals are computed for the experimental model and the simplified model, as shown in Fig. 9(b). In the figure, the magnitudes and frequencies of the first two peaks obtained from the two models are well-matched. The extracted mode shapes and corresponding natural frequencies are shown in Fig. 10 and given in Table 3, respectively. It can be seen that the modal parameters of the simplified model are almost consistent with those of the experimental model.

## 5. CONCLUSION

In this study, a simplified model of caisson system for stability monitoring was presented. First, a simplified model of the caisson system was designed on the basis of the characteristics of existing harbor caisson structures. A mass-spring-dashpot (MSD) model allowing only the sway motion was formulated. In the model, each unit is connected to adjacent ones via springs and dashpots to represent the condition of interlocking mechanisms. Next, the MSD model was evaluated for a 3-D FE model of the caissons and a lab-scale caisson to examine the accuracy of the simplified model's vibration responses.

The proposed simplified model of caisson system estimated the horizontal vibration, successfully. The vibration features (i.e., power spectral density, natural frequency and mode shape) of 3-D FE model and experimental model of caisson system were well consistent in its simplified models. Hence, the simplified model was reliable for the dynamic analysis of the caisson system.

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## REFERENCES

- Bowles, J. E., "Foundation Analysis and Design (5th Edition)", *McGraw-Hill*, 1996
- Gao, M., Dai, G. Y., and Yang, J. H., "Dynamic Studies on Caisson-type Breakwaters", *Proceedings of 21st Conference on Coastal Engineering*, Torremolinos, Spain, 1988, pp. 2469-2478.
- Goda, Y., "Dynamic Response of Upright Breakwater to Impulsive Force of Breaking Waves", *Coastal Engineering*, Vol. 22, 1994, pp. 135-158.
- Lamberti, A., and Martinelli, L., "Prototype Measurements of the Dynamic Response of Caisson Breakwaters," *Proc. 26th ICCE*, Copenhagen, Denmark, 1998.
- Lee, S. Y., Nguyen, K. D., Huynh, T. C., Kim, J. T., Yi, J. H. and Han, S. H., "Vibration-Based Damage Monitoring of Harbor Caisson Structure with Damaged Foundation-Structure Interface", *Smart Structures and Systems*, Vol. 10(6), 2012, pp. 517-547.
- Look, B., "Handbook of Geotechnical Investigation and Design Tables", *Taylor & Francis*, 2007.
- Oumeraci, H., and Kortenhaus, A., "Analysis of the Dynamic Response of Caisson Breakwaters", *Coastal Engineering*, Vol. 22, 1994, pp. 159-183.
- Press, W.H., Flannery, B. P., Teukolsky, S. A. and Vetterling, W. T., "Numerical Recipes – the Art of Scientific Computing", *Cambridge University Press*, Cambridge, 1988.
- Richart, F. E., Hall Jr., J. R., Woods, R. D., "Vibration of Soils and Foundations", *Prentice Hall Inc.*, 1970.
- Smirnov, G.1  
N., and Moroz, L.R., "Oscillations of Gravity Protective Structures of a Vertical Wall Type", *IAHR, Proc. 20th Congress*, Vol. 7, 1983, pp. 216-219.
- Vink, H.A.Th., "Wave Impacts on Vertical Breakwaters", *Master's thesis*, Faculty of Civil Engineering, Delft University of Technology, The Netherlands, 1997.
- Westergaard, H.M., "Water Pressures on Dams during Earthquakes", *T. Am. Soc.*, Vol. 98(2), 1933, pp. 418-432.
- Wilson, E. L., *Static and Dynamic Analysis of Structures (4<sup>th</sup> edition)*, *Berkeley, CA: Computers and Structures, Inc.*, 2004.
- Yoon, H. S., Lee, S. Y., Kim, J. T., and Yi, J. H., "Field Implementation of Wireless Vibration Sensing System for Monitoring of Harbor Caisson Breakwaters," *International Journal of Distributed Sensor Networks*, Vol. 2012, 2012, pp. 1-9.