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Approximated Gradient Based Seismic Control of 3D Irregular Buildings Using Multiple Tuned-Mass-Dampers

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ABSTRACT

Irregular structures are often found to be in danger under seismic action, more such than their symmetric counterparts. By using external passive damping devices, such as Tuned-Mass-Dampers (TMD), existing structures could be controlled and brought to behave within limits of desired performance, thus performance based design (PBD) could be efficiently achieved. The authors have previously presented a practical PBD analysis/redesign (A/R) procedure for the allocation and sizing of multiple TMDs in 3D irregular structures undergoing seismic loadings. This paper modifies this procedure, making it more computationally efficient, as well as more cost efficient. It is shown that by using the methodology presented herein, a desired performance level is successfully targeted by adding near-optimal amounts of mass at various locations and tuning the TMDs to dampen several of the structure's frequencies. This is done using analysis tools only, and since the formulations are general, and apply to all types of structures, the methodology presented is recommended for practical use.

1. INTRODUCTION

Past experience shows that irregular structures are, in general, more seismically vulnerable than regular ones, and experience more damage due to earthquakes. Reducing the amount of damage the structure experiences following a ground motion is of much importance. By using external passive damping devices, existing structures could be controlled and brought to behave within limits of desired performance, thus performance based design (PBD) could be efficiently achieved. Tuned-Mass-Dampers (TMD) have been shown to be able to eliminate most of the steady state motion of a linear single degree of freedom system under a harmonic loading of a given frequency, if properly tuned (e.g. Den-Hartog 1940; Warburton 1982; Soong and Dargush 1997). Using such TMDs for the seismic control of multi degree of freedom buildings with multiple modes contributing to their response is still limited. This is due to their tuning nature, which may seem to be in contrast with the multi-modal response of structures,

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in general, and irregular structures in particular, under such loads. The use of multiple TMDs for the multi-modal seismic control of buildings may overcome this obstacle (Clark, 1998). Thus, adopting MTMDs as means of structural control, might be beneficial, as they have some very desirable characteristics such as their simple behavior, which is easy to model, their relative low cost and their efficiency in multi-hazard control (wind and earthquake mitigation).

TMDs and MTMTDs (the latter referring to a case where the dampening of more than one mode in each direction is targeted) have been previously used in 3D structures (Almazan et al. 2012; Jangid and Datta 1997; Li and Qu 2006; Petti and Iuliis 2009; Lin et al. 2010; Lin et al. 2011; Singh et al. 2002; Ahlawat and Ramaswamy 2003; Desu et al. 2006 Desu et al. 2007; Lin et al. 1999). Nonetheless, there had been no simple, computationally efficient methodology for PBD seismic retrofitting by means of MTMDS. The authors have recently presented a simple and practical PBD analysis/redesign procedure for the allocation and sizing of multiple TMDs in 3D irregular structures (Lavan and Daniel 2013). This paper modifies the procedure previously proposed by the authors (Lavan and Daniel 2013), making it more computationally efficient as well as more cost efficient. The modified procedure is more computationally efficient as there is no need to evaluate mean-square response at various frequencies, as required by the previous analysis/redesign procedure suggested. In addition, it is more cost efficient as the solution attained is closer to the actual optimal solution of the optimization problem considered. Here, the first stage of redesign includes redesign of the sum of masses of all TMDs at a specific location, based on the RMS response at that location. Thereafter, in the second stage of redesign, the masses of TMDs at the same location, tuned to various frequencies, are updated based on approximated gradients. Equal gradients were chosen to distribute the mass amongst TMDs at the same location so as to simulate a KKT solution (see for example, Bazaraa and Shetty 1979) to the optimization problem, promising the solution will be near optimum.

Using the proposed methodology, a desired performance level is successfully targeted by adding near-optimal amounts of mass at various locations and tuning them to dampen several of the structure's frequencies. In addition, the proposed methodology is general, and therefore suitable for use in all types of structures, regardless of the extent of their irregularity, their shape or type.

2. PROBLEM FORMULATION

2.1 Performance measures

High acceleration levels can cause severe damage to nonstructural systems, sensitive equipment within the structure, as well as cause discomfort to humans occupying the buildings. Under these circumstances, reduction of acceleration is very important. In addition, reducing them can reduce the base-shear and overturning moments (Soong and Dargush 1997; Chen and Wu 2001). Also, as will be seen herein, reduction of accelerations often leads to considerable reduction of inter-story drifts, and therefore structural damage, as well.

As for the cost of control, in the case of TMDs, this is determined based on the amount of added mass (through direct cost of material, void floor space for the control system, added gravitational forces to the existing structural system). As more mass is needed, the solution becomes more expensive, and therefore less cost-effective.

2.2 Problem formulation

The problem is formulated so as to try minimize control forces (masses of all dampers) while limiting RMS accelerations (in frequency domain) at peripheral locations to limits set by the performance criteria. Accelerations are limited at all peripheral locations of all floors, as they are the largest accelerations expected within the floor limits. The problem is formulated as:

$$
\begin{aligned}
&\min J = \sum_{l}^{\text{all locations}} \sum_{f}^{\text{all frequencies}} \left(\mathbf{m}_{TMD}\right)_{l,f} \\
&s.t. \\
&\frac{RMS\left(\left(\ddot{\mathbf{x}}_{\mathrm{p}}^{\mathrm{t}}\right)_l\right)}{a_{\mathrm{all}}^{\mathrm{RMS}}} \leq 1.0 \qquad \forall l = 1,2,\ldots,N_{locations}
\end{aligned}
\tag{1}
$$

where $(\mathbf{m}_{\mathrm{TMD}})_{l,f}$ is the mass of the TMD located at peripheral location l tuned to frequency f, $a_{\mathrm{all}}^{\mathrm{RMS}}$ is the allowable RMS total acceleration, $RMS\left(\left(\ddot{\mathbf{x}}_{\mathrm{p}}^{\mathrm{t}}\right)_l\right)$ is the root mean square of the total acceleration at location l (the l^{th} term of $RMS\left(\ddot{\mathbf{x}}_{\mathrm{p}}^{\mathrm{t}}\right)$), and $N_{locations}$ is the number of locations constrained ($=4N_{floors}$ where N_{floors} is the number of floors).

3. PROPOSED SOLUTION SCHEME

The aforementioned optimization problems could be solved using formal optimization tools, as has been proposed for the seismic design using other types of energy dissipation devices (e.g. Takewaki 2000a; Takewaki 2000b; Takewaki et al. 1999; Lavan and Levy 2005; Lavan and Levy 2006a; Lavan and Levy 2006b; Lavan and Levy 2010). However, those require knowledge and tools that are less familiar to practicing engineers. The analysis/redesign solution scheme proposed herein is aimed at finding the locations, masses and tuning frequencies of MTMDs that satisfy the constraints while reducing the total mass of the MTMDs, thus achieving the goals of the performance based design. This procedure is expected to lead to a cost efficient solution, that is close to the formal optimal one.

3.1 Full Resources Utilization Design

Design methods that are based on fully stressed characteristics go back to the classical design of trusses under static loads, whereby the weight is minimized for a given allowable stress. For that problem, it had been widely accepted that the optimal design yields a: *statically determinate fully stressed design, with members out of the design having strains smaller than the allowable*. (Cilley, 1900). This has been proven in several occasions, using various approaches. Later, Levy and Lavan (2006) considered the minimization of total added viscous damping in frame structures subjected to ground accelerations while constraining inter-story responses. Their optimal solutions indicated that: "*At the optimum, damping is assigned to stories for which the local performance index has reached the allowable value. Stories with no assigned damping attain a local performance index which is lower or equal to the allowable*." That is, the optimal solutions having "fully stressed" characteristics.

The authors have also proposed a full resources utilization design (FRUD) criteria for an efficient design of MTMDs in 3D irregular structures (Lavan and Daniel 2013). The FRUD criteria were stated as follows: *TMDs are assigned only to peripheral locations for which the RMS acceleration has reached the allowable value under the assumed PSD of input acceleration. In addition, at each location where mass dampers are placed, TMDs of a given frequency are assigned only to frequencies for which the output spectral density is maximal.* When comparing the attained designs to formal optimal ones it was seen that while the first part of the statement holds in the optimal design, the second part, while leading to very efficient designs (close to optimal), does not lead to the true optimum.

In this paper, the second part of the statement is modified to lead to designs that are closer to optimum while reducing the computational effort required (as different frequency domain tools are used). It is postulated that an efficient optimal, or close to optimal, selection of locations and sizes of MTMDs in structures, under a stochastic ground acceleration input, possesses the following characteristics: *TMDs are assigned to peripheral locations for which the RMS acceleration has reached the allowable value under the assumed PSD of input acceleration. In addition, at each location to which TMDs are added, TMDs of a given frequency are assigned only to frequencies for which the gradient of the RMS response at that location with response to the TMD that mode is set to dampen is most negative.* The second part of this statement is based on optimality criteria methods that are aimed at satisfying the KKT conditions for optimum solutions (see e.g. Bazaraa and Shetty 1979). An assumption on the KKT conditions is made- that only one constraint is active at the optimum (i.e. response at a single location). While this assumption may not always hold, it seems to lead to results that are close to the optimum, even when more constraints are active. Such optimality criteria have been used, for the formal optimal design of other types of energy dissipation devices, by Takewaki (1997).

3.2 Analysis/redesign algorithm

Solutions to problems, which possess FRUD characteristics, are efficiently achieved iteratively using a two step algorithm in each iteration cycle. In the first step an analysis is performed for a given preliminary design, whereas in the second step the design is changed using a recurrence relationship that targets full utilization of the resources. The recurrence relation can be generally written as:

$$x_l^{(n+1)} = x_l^{(n)} \cdot \left(\frac{pi_l^{(n)}}{pi_{allowable}} \right)^P \tag{2}$$

where x_l is the value of the design variable associated with the location l, pi_l is the performance measure of interest for the location l, $pi_{allowable}$ is the allowable value for the performance measure, n - the iteration number and P - a convergence parameter. The advantages of the analysis/redesign algorithm include its simplicity, the need to use analysis tools only, and the fairly small computational effort that lies in the small number of analyses required for convergence. Such analysis/redesign procedure will be utilized here to attain full resources utilization designs where the mass, frequency and locations of MTMDs within framed structures is to be determined.

4. DESIGN METHODOLOGY

Step 1: An allowable RMS acceleration is chosen. The mass, damping and stiffness matrices of the structure are assembled according to the relevant dynamic DOFs. Solution of the eigenvalue problem determines the structure's natural frequencies and mode shapes. A power spectral density (PSD), $S(\omega)$, for the input acceleration is chosen (e.g. white-noise, which gives a constant PSD; Clough-Penzien; filtered Kanai-Tajimi PSD (Clough and Penzien 1995 etc.). RMS accelerations under the chosen input PSD are computed for each of the structure's DOFs (e.g. using Lyapunov's equation, see e.g. Kwakernaak and Sivan 1972), and then transformed to peripheral coordinates.

Step 2: If for any peripheral coordinate, l, the RMS acceleration obtained is larger than the allowable RMS acceleration, MTMDs are added to suppress the acceleration produced. Each TMD of mass $(\mathbf{m}_{\mathrm{TMD}})_{l,f}$ is assigned with a DOF for its displacement relative to the ground. At each location, N_{mode} TMDs are potentially added, to suppress N_{mode} original frequencies of the structure.

The response of each mode could be evaluated based on a SDOF equivalent system. For the sake of simplicity, in this work Den-Hartog (1940) / Warburton (1982) properties were chosen. Nonetheless, more advanced criteria could easily be used with the proposed methodology. In the case of optimal Den-Hartog properties the following initial properties are taken for the dampers:

1. For each peripheral coordinate, the initial mass of all TMDs located at that coordinate is taken as certain predetermined percentage of the structure's mass (say 1%). It is divided equally between the dampers situated at the same location:

$$(\mathbf{m}_{\mathrm{TMD}})_{l,f} = (0.01 \cdot M_{structure})/N_{mode} \qquad (3)$$

where l represents the damper's location, f represents the mode dampened and $M_{structure}$ is the structure's total mass. The mass ratio $(\boldsymbol{\mu}_{\mathrm{TMD}})_f$ of all TMDs tuned to frequency f is calculated as the ratio between the effective TMD mass of all TMDs tuned to frequency f and the f^{th} modal mass of the structure. This mass ratio is defined as:

$$(\boldsymbol{\mu}_{\mathrm{TMD}})_f = \left(\phi_f^{\mathrm{T}} \cdot \mathbf{T}^{\mathrm{T}} \cdot \mathfrak{D}\left((\mathbf{m}_{\mathrm{TMD}})_f\right) \cdot \mathbf{T} \cdot \phi_f\right) / \left(\phi_f^{\mathrm{T}} \cdot \mathbf{M}_{\mathbf{original}} \cdot \phi_f\right) \qquad (4)$$

where ϕ_f is the f^{th} mode-shape of the bare structure, $[\mathbf{M}_{\mathbf{original}}]$ is the bare frame's mass matrix, $\mathbf{T}$ is the transformation matrix to peripheral locations, and $\mathfrak{D}((\mathbf{m}_{\text{TMD}})_f)$ is a diagonal matrix with the terms $(\mathbf{m}_{\text{TMD}})_{1:Nlocations,f}$ sitting on the diagonal.

2. Each TMD's stiffness is determined according to the frequency of the mode which is dampened by the TMD. The frequency is tuned to:

$$(\boldsymbol{\omega}_{\text{TMD}})_f = (\boldsymbol{\omega}_{\text{n}})_f \big/ \left(1 + (\boldsymbol{\mu}_{\text{TMD}})_f\right) \tag{5}$$

where $(\boldsymbol{\omega}_{\text{n}})_f$ is the frequency f to be dampened. The compatible stiffness is:

$$(\mathbf{k}_{\text{TMD}})_{l,f} = (\mathbf{m}_{\text{TMD}})_{l,f} \cdot \left((\boldsymbol{\omega}_{\text{TMD}})_f\right)^2 \tag{6}$$

3. Each TMD's damping ratio is determined according to:

$$(\boldsymbol{\xi}_{\text{TMD}})_f = \sqrt{\left(3 \cdot (\boldsymbol{\mu}_{\text{TMD}})_f\right) \Big/ \left(8 \cdot \left(1 + (\boldsymbol{\mu}_{\text{TMD}})_f\right)^3\right)} \tag{7}$$

and the matching damping coefficient:

$$(\mathbf{c}_{\text{TMD}})_{l,f} = 2 \cdot (\mathbf{m}_{\text{TMD}})_{l,f} \cdot (\boldsymbol{\xi}_{\text{TMD}})_f \cdot (\boldsymbol{\omega}_{\text{n}})_f \tag{8}$$

Step 3: The mass, damping and stiffness matrices of the damped frame are updated. Peripheral RMS accelerations are then reevaluated.

Step 4: TMD's masses are re-determined using two stages; the total mass of all dampers located at a given location is first determined. This is followed by the distribution of that mass between all TMDs at that location, having various tuning frequencies. Following the change in mass, the stiffness and modal damping ratio of each TMD are also updated while keeping the Den-Hartog principles intact, using Eqs. (5) - (8). The two-stage analysis/redesign procedure is carried out iteratively until convergence, in the following way:

Stage 1: The first stage of redesign includes evaluation of the total mass of TMDs at each location, promising the existence of the first part of the postulate. This is formulated using:

$$\left(\mathbf{m}_{\text{TMD, total}}^{(n+1)}\right)_l = \sum_{f=1}^{\substack{\text{all}\\ \text{frequencies}}} \left(\mathbf{m}_{\text{TMD}}^{(n+1)}\right)_{l,f} = \sum_{f=1}^{\substack{\text{all}\\ \text{frequencies}}} \left(\mathbf{m}_{\text{TMD}}^{(n)}\right)_{l,f} \cdot \left(\frac{RMS\left(\left(\ddot{\mathbf{x}}_{\text{p}}^{\text{t}\,(n)}\right)_l\right)}{a_{\text{all}}^{\text{RMS}}} \right)^P \tag{9}$$

where $(\cdot)^{(n)}$ is the value at iteration n, $\left(\mathbf{m}_{\text{TMD, total}}^{(n+1)}\right)_l$ is the total mass of all dampers at location l, and P is a constant which influences the convergence and convergence rate. A large P will result in a faster but less stable convergence of the above equation.

Based on the authors' experience, a P in the range of 0.1-2.0 should be satisfying in terms of stability, convergence and fair amount of iterations.

Stage 2: In the second stage of redesign, the total mass obtained at each location is distributed between N_{mode} dampers (dampening modes $\left(\boldsymbol{\omega}_{\mathrm{n}}\right)_f$) at that same location l, promising the existence of the second part of the postulate, using the following:

$$\left(\mathbf{m}_{\mathrm{TMD}}^{(n+1)}\right)_{l,f} = \left(\mathbf{m}_{\mathrm{TMD}}^{(n)}\right)_{l,f} \left(\frac{ -\dfrac{\partial\left(RMS\left(\left(\ddot{\mathbf{x}}_{\mathrm{p}}^{\mathrm{t}\,(n)}\right)_l\right)\right)}{\partial\left(\mathbf{m}_{\mathrm{TMD}}\right)_{l,f}} }{ \max\limits_f \left(-\dfrac{\partial\left(RMS\left(\left(\ddot{\mathbf{x}}_{\mathrm{p}}^{\mathrm{t}\,(n)}\right)_l\right)\right)}{\partial\left(\mathbf{m}_{\mathrm{TMD}}\right)_{l,f}} \right) } \right)^P \cdot NF \tag{10}$$

and:

$$NF = \frac{\left(\mathbf{m}_{\mathrm{TMD,\,total}}^{(n+1)}\right)_l}{\sum\limits_{f=1}^{\substack{\text{all}\\ \text{frequencies}}} \left(\mathbf{m}_{\mathrm{TMD}}^{(n)}\right)_{l,f} \left(\frac{ -\dfrac{\partial\left(RMS\left(\left(\ddot{\mathbf{x}}_{\mathrm{p}}^{\mathrm{t}\,(n)}\right)_l\right)\right)}{\partial\left(\mathbf{m}_{\mathrm{TMD}}\right)_{l,f}} }{ \max\limits_f \left(-\dfrac{\partial\left(RMS\left(\left(\ddot{\mathbf{x}}_{\mathrm{p}}^{\mathrm{t}\,(n)}\right)_l\right)\right)}{\partial\left(\mathbf{m}_{\mathrm{TMD}}\right)_{l,f}} \right) } \right)^P} \tag{11}$$

where the approximated gradient $\dfrac{\partial\left(RMS\left(\left(\ddot{\mathbf{x}}_{\mathrm{p}}^{\mathrm{t}\,(n)}\right)_l\right)\right)}{\partial\left(\mathbf{m}_{\mathrm{TMD}}\right)_{l,f}}$ is evaluated based on the following equation:

$$\frac{\partial\left(RMS\left(\left(\ddot{\mathbf{x}}_{\mathrm{p}}^{\mathrm{t}\,(n)}\right)_l\right)\right)}{\partial\left(\mathbf{m}_{\mathrm{TMD}}\right)_{l,f}} \approx \frac{\left(\mathbf{T}\cdot\phi_f\right)_l^2 \cdot \Gamma_f^2}{RMS\left(\left(\ddot{\mathbf{x}}_{\mathrm{p}}^{\mathrm{t}\,(n)}\right)_l\right)} \cdot \frac{\phi_f^{\mathbf{T}} \mathbf{B_d}^{\mathbf{T}} \dfrac{\partial \mathfrak{D}(\mathbf{m})}{\partial\left(\mathbf{m}_{\mathrm{TMD}}\right)_{l,f}} \mathbf{B_d}\phi_f}{\phi_f^{\mathrm{T}} \cdot \mathbf{M}_{\mathbf{original}} \cdot \phi_f} \cdot \frac{\partial\left(a_{MS}\left(\omega_f\right)_{\max}\right)}{\partial\left(\boldsymbol{\mu}_{\mathrm{TMD}}\right)_f} \tag{12}$$

where the matrix $\mathbf{B_d}$ is a transformation matrix, used to assign the TMDs within the structure, Γ_j is the participation factor of mode j, defined as $\Gamma_j = \dfrac{\phi_f^{\mathrm{T}} \cdot \mathbf{M}_{\mathbf{original}} \cdot \mathbf{e}}{\phi_f^{\mathrm{T}} \cdot \mathbf{M}_{\mathbf{original}} \cdot \phi_f}$ where **e** is the excitation direction vector with values of zero and one for DOFs perpendicular and parallel to the excitation direction, respectively , and the gradient $\dfrac{\partial\left(a_{MS}\left(\omega_f\right)_{\max}\right)}{\partial\left(\boldsymbol{\mu}_{\mathrm{TMD}}\right)_f}$ is derived based on an empirical formula obtained using curve-fitting, and is:

$$\frac{\partial\left(a_{MS}\left(\omega_f\right)_{\max}\right)}{\partial\left(\boldsymbol{\mu}_{\mathrm{TMD}}\right)_f} = -3.15099 \cdot \left(\left(\boldsymbol{\mu}_{\mathrm{TMD}}\right)_f + 0.003\right)^{-1.12041} \cdot \left(\boldsymbol{\omega}_{\mathrm{n}}\right)_f \cdot S\left(\left(\boldsymbol{\omega}_{\mathrm{n}}\right)_f\right) \tag{13}$$

where $S\left((\omega_n)_f\right)$ is the value of the input PSD $S(\omega)$ at the frequnecy $\omega = (\omega_n)_f$.

In deriving the approximated gradient, it was assumed that, approximately, the equations of motion of the damped structure are not coupled when transformed to the modal coordinates of the undamped structure, and that the TMDs tuned to dampen a certain mode do not affect the response of other modes. For this approximated gradient, the forces in TMDs due to the ground's movement are neglected, and it is assumed that forces in all TMDs are created only due to the structure's movement.

Step 5: Repeat steps 3-4 until convergence of the mass is reached.

5. EXAMPLE

The following 8-story asymmetric setback RC frame structure (Fig. 1) introduced by Tso and Yao (1994) is retrofitted using MTMDs for a deterministic ensemble of ground motions exciting the structure in the "y" direction). A uniform distributed mass of 0.75 ton/m^2 is taken. The column dimensions are 0.5m by 0.5m for frames 1 and 2 and 0.7m by 0.7m for frames 3 and 4. The beams are 0.4m wide and 0.6m tall. 5% Rayleigh damping for the first and second modes is used. A 45% reduction of the RMS total acceleration in the bare structure is desired. The response is analyzed under a Clough-Penzien filtered Kanai-Tajimi PSD with parameters fitted to the average FFT values of the SE10/50 ground-motion ensemble. The design variables are the locations and properties of the individual tuned mass dampers. The dampers are to potentially be located in the peripheral frames, where they are most effective, and as the excitation is in the "y" direction only, dampers will be assigned only to the peripheral frames 1 (lower 4 floors), 3 (upper 4 floors) and 4, to dampen frequencies of modes which involve "y" and "θ".

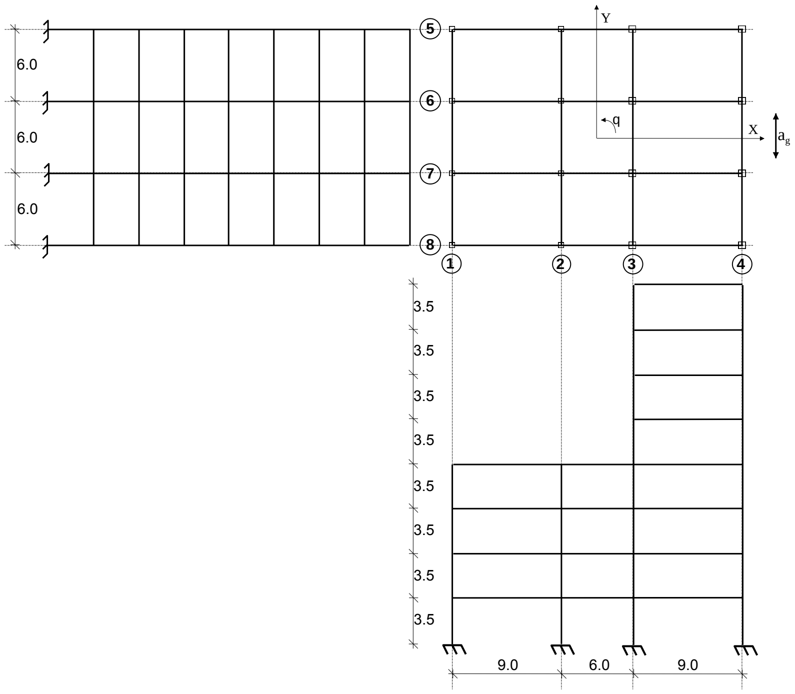

Fig. 1 Eight-story setback structure

Step 1: The mass, inherent damping and stiffness matrices of the frame in the dynamic DOFs were constructed. The natural frequencies, of the structure were determined. The first 10 modes are: 6.88s (x), 7.36s (y,θ), 10.37s (y,θ), 16.04s (x), 17.88s (y,θ), 22.61s (y,θ), 33.87s (x), 35.96s (y,θ), 43.84s (y,θ), 50.00s (x). RMS accelerations of the undamped structure are evaluated under the Clough-Penzien filtered Kanai-Tajimi PSD with parameters: $\omega_g = 13 \, {rad}/{\sec}$, $\xi_g = 0.98$, $S_0 = 1$, $\omega_f = 1.5 \, {rad}/{\sec}$, and $\xi_f = 0.9$. The allowable RMS acceleration for all peripheral accelerations was earlier adopted as 55% of the maximum peripheral RMS acceleration of the bare frame, giving: $a_{\text{all}}^{\text{RMS}} = 16.17$.

Step 2: 160 TMDs were added, as a first guess, with initial properties as given in Table 1. Those are comprised of 10 dampers each tuned to a different mode frequency (of modes related to "y" and "θ") at each of the 16 peripheral locations of frames 1, upper 4 floors of frame 3, and frame 4.

Step 3: The mass, stiffness and damping matrices were updated. With the newly-updated matrices and the same PSD input, new peripheral RMS accelerations were evaluated. Some of the peripheral accelerations in frames 1, 3 and 4 exceeded the allowable.

Table 1. Initial properties of TMDs

No. TMD	Mode to dampen	Initial mass (ton)	Initial natural frequency (rad/sec)	Initial damping ratio
1-16	2	287.1	7.18	9890.2
17-32	3	287.1	10.20	98977.
33-48	5	287.1	17.57	989.91
49-64	6	287.1	22.15	989..9
65-80	8	287.1	35.42	9897.0
81-96	9	287.1	42.48	989090
97-112	11	287.1	56.36	989.22
113-128	12	287.1	65.92	989..2
129-144	13	287.1	70.90	98971
145-160	15	287.1	92.57	989..0

Step 4: The problem has not converged, and thus the TMDs' properties were altered, using the recurrence relations of Eqs. (9) - (11) and *P*=1 as the convergence parameter, giving updated total masses at each DOF. The total mass of each peripheral coordinate was then distributed between the 10 dampers at the same location using Eqs. (10) and (11). Iterative analysis/redesign as described in Eqs. (9) - (11) while altering the mass of the damper is carried out until convergence to allowable levels. Upon convergence, the mass of added dampers are shown in Table 3. TMDs with non-zero properties were located at frame number 1 (at floor 4), number 3 (at floor 8) and number 4 (at floor 8), which are the top floors for each part of the setback frame. The final properties of each added TMD are shown in Table 2. All assigned TMDs add

up to 9.33% of the original structure's mass. For all practical reasons, TMDs with small masses can be neglected without effecting the response of the structure.

Table 2. Final properties of added TMDs

Frame	Floor	Mode to dampen	Final mass (ton)	Final stiffness (kN/m)	Final damping ratio
1	4	.	0.50	53.08	0.0375
3	8	2	75.76	3299.48	0.1763
3	8	3	5.06	540.53	0.0375
3	8	5	57.60	1174.66	0.2195
3	8	8	11.27	1423.66	0.0650
4	8	11	6.30	2008.83	0.0676
4	8	2	4.39	191.36	0.1763
4	8	6	5.35	2639.43	0.0796

Finally, an analysis of the retrofitted structure yields the peripheral RMS accelerations shown in Fig. 2. Also in Fig. 2 the total amount of mass at each floor is shown. As can be seen, only locations who had reached the maximum allowable RMS total acceleration were assigned with added absorbers, making the solution obtained a fully-stressed design.

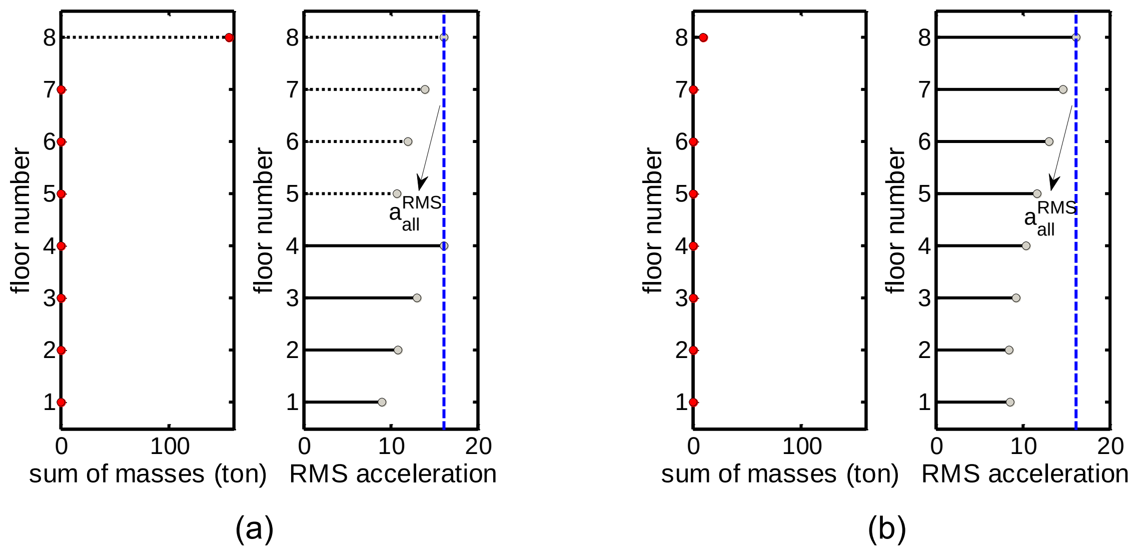

Fig. 2 Peripheral RMS accelerations of structure with final TMDs (continuous or dashed) and sum of added masses (dots) (a) frame 1 (floors 1-4) and 3 (floors 5-8) and (b) frame 4 .

Fig. 3 presents the convergence of the design variables (masses) and the performance measure (acceleration). As can be seen in Fig. 3, convergence is reached within about 250 iterations (although practically only 60 iterations are required).

The results attained were compared to results attained using formal optimization and the previous analysis/redesign-based methodology presented by the authors (Lavan and Daniel 2013), and are presented in Table 3. Note that the number of iterations/ function evaluations needed is given for comparison-of-convergence-sake, however, as different tools are used in each methodology (formal optimization tools with sensitivities in the first, frequency response analysis at all frequencies in the second, and approximated gradient without full frequency analysis in the third), computational effort cannot be compared directly.

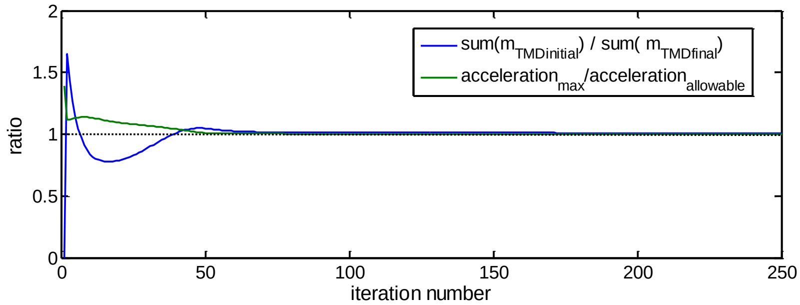

Fig. 3 Convergence of normalized sum of masses (objective function) and maximum normalized RMS acceleration (constraint).

Table 3. Results comparison

	% added mass	Number of iterations/ function evaluations*
Formal optimal solution	8.34%	~360
A/R frequency response	9.78%	~40
A/R approximated gradient	9.33%	~250

* Note that as different tools are used in each methodology computational effort cannot be compared directly.

6. CONCLUSIONS

A performance-based methodology for the retrofitting of 3D irregular structures was presented. This methodology makes use of an iterative two-step analysis/redesign procedure to limit RMS absolute acceleration levels at all peripheral locations to an allowable level. A previous methodology for solution of the same problem was previously presented by the authors. It used a similar analysis/redesign procedure that was based on an equal frequency responses of modes associated with TMDs dampening them for the second stage of the redesign step. This led to a solution that was based on analysis tools only, and was fast-converging, but the results were only close to optimal (compared to the formal optimal solution of the problem). Herein, a different variation on the analysis/redesign method was suggested, so as to still only use analysis tools, but based on a modified refined criteria for the second stage of the redesign step, that is based on optimality conditions. This approach, while still fast converging, led to results that were closer to the optimal results than the first analysis/redesign methodology used. In addition, thanks to the approximated expression of the gradient, and it's use in the second stage of the redesign step, full frequency analysis is not needed in this analysis/redesign variation, as opposed to the one originally suggested, making the proposed methodology even more computationally effective.

Results showed that MTMDs can be used for seismic design of structures, and that those can reduce accelerations to a desired level. TMDs tuned to several modes and located in different peripheral locations are utilized to obtain effectiveness. The results obtained were indeed close to the optimal solution of the problem, as was shown. The advantages of the design methodology presented herein include its simplicity, relaying on analysis tools only, it's fast convergence, it's generality and suitability to many problems, regardless of irregularities, all of which make it applicable for use in practical design

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