

Displacement Demand of Rocking Systems in Conditions of Moderate Ground Shaking

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ABSTRACT

Simple models for estimating earthquake induced peak displacement demand of structures and their risks of overturning are presented in this paper. The presented models circumvent the need of deciding which ground motion predictive expression to adopt and what accelerograms to use, and the need of lengthy and cumbersome non-linear dynamic analyses. The displacement based methodology so presented is distinguished from conventional codified methods of aseismic design of structures which are based on representing seismic actions as equivalent quasi-static lateral forces. There are structural systems which can be displaced to a certain generous limit in an earthquake without sustaining damage and be able to fully recover all the displacements after the event. Realistic prediction of the displacement demand of such (rocking) system is therefore crucial to achieving satisfactory design outcomes.

1. INTRODUCTION

This paper is concerned with the behavior of structures undergoing large displacement and the actions of overturning when subject to seismic actions. Shaker table experiments were first conducted on non-deformable free-standing rectangular objects with symmetrical distribution of mass. Experimental results were augmented by computer simulations using algorithms that had been verified to develop fragility curves for overturning. Observations from the experiments, and simulations, provided insight into factors controlling the risks of overturning in spite of the idealized conditions of the test setup. Importantly, expressions have been derived to correlate the maximum estimated tilt of the object with elastic response spectral parameters that are easy to calculate (Section 2). The challenge of estimating the value of the peak displacement demand for low-moderate seismicity regions where data is scarce has been overcome by the introduction of generic predictive models (Section 3). Meanwhile, shortcomings

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of codified response spectrum models and uniform hazard spectra have been explained. It is recommended that critical design earthquake scenarios and their associated peak displacement demand will need to be identified. The assessment methodology that has been introduced may be applied to free-standing furniture and storage items, mechanical/electrical equipment, blockwalls and precast concrete columns that are in support of buildings featuring a soft-storey (Section 4). Last and not least, there are potentials of adapting the methodology for the design of innovative unbonded post-tensioned precast concrete frames and retaining wall structures.

2. OVERTURNING RISKS

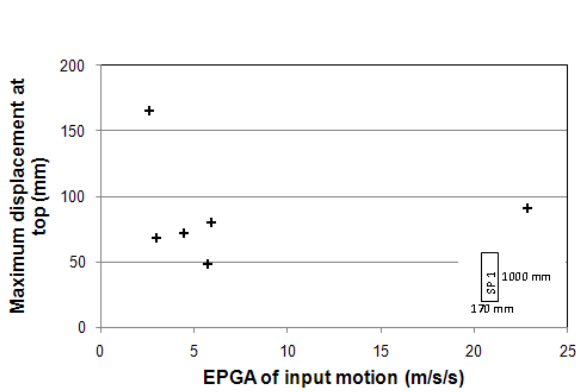
As a non-deformable free-standing object responds to base excitations the effective natural period of vibration is not constant but varies with the amplitude of displacement. Consequently, the (usual) response spectral acceleration, or displacement, value which corresponds to a pre-defined natural period of vibration cannot be identified. Thus, broad based ground motion parameters such as peak ground acceleration (PGA) or peak ground velocity (PGV) will need to be employed to correlate with the risks of overturning. In recent shaker table experimentations of free-standing objects undertaken by the authors (Kafle *et al.*, 2012) the amount of maximum tilt of the object as indicated by its maximum displacement at the top was correlated against values of base motion parameters namely maximum response spectral acceleration (RSA_{max}) velocity (RSV_{max}) and displacement (RSD_{max}).

The RSA_{max} parameter which characterises the response potentials of short period systems has been found to have poor correlations with the displacement behavior of the object and its risks of overturning despite the “short period” behaviour in the initial at-rest conditions. In contrast, the RSD_{max} parameter (which is related directly to the peak displacement of the ground) has been found to have very strong correlations with the risks of overturning. These trends are well illustrated in Figures 1a & 1b in which measured results from shaker table experiments are presented, and in Figures 2a & 2b in which results simulated from a validated algorithm developed initially in Lam *et al.* (2003) are presented.

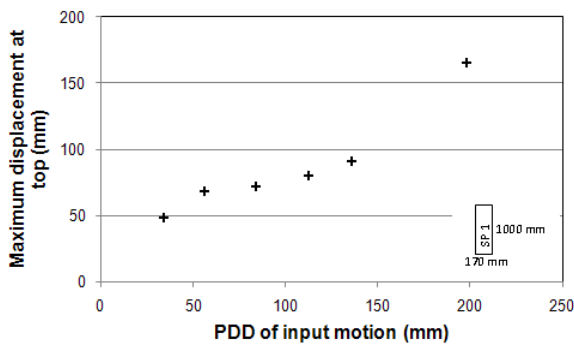
Clearly, the risks of overturning of the tested objects on the shaker table were controlled by the displacement amplitude of the table (and not the value of its peak acceleration). Objects of varying height and aspect ratio were tested repetitively by both the shaker table, and by simulations, to reveal the underlying trends. The important and somewhat counter intuitive finding was that the risks of overturning did not increase monotonically with increasing height, nor aspect ratio, of the object when their thickness in the direction of ground shaking was kept constant. The risks of overturning was also seen to decrease gradually with increasing size of the object when the aspect ratio was kept constant. These trends are well illustrated by the fragility curves of Figures 3 and 4 (Kafle *et al.*, 2012).

An expression showing the threshold peak displacement demand (RSD_{max}) value of the ground shaking to impose a 5% chance of overturning was derived following an

extensive parametric study carried out by the authors (equation 1). This expression is consistent with recommendations from an earlier study by Al Abadi *et al.* (2006) in which a major parametric study involving extensive analytical simulations was undertaken to identify the risks category of overturning as function of the object thickness and RSD_{max} value of the base excitations (equations 2a – 2c).

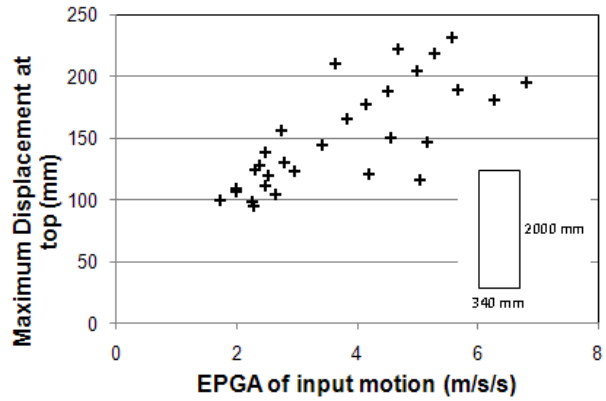


(a)

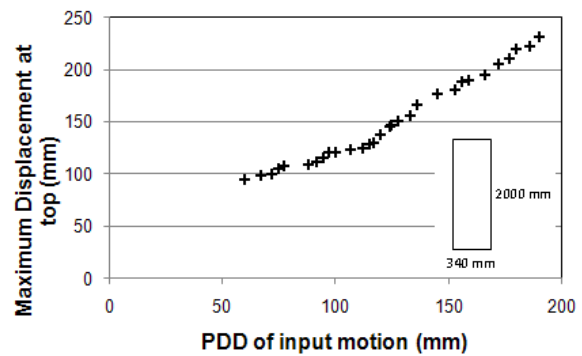


(b)

Figure 1 Results from shaker table tests



(a)



(b)

Figure 2 Results from validated simulations

$$RSD_{max} = 35.8 \times e^{0.004t} \quad \text{for 5\% probability of overturning} \quad (1)$$

where t is the thickness of object in direction of ground shaking.

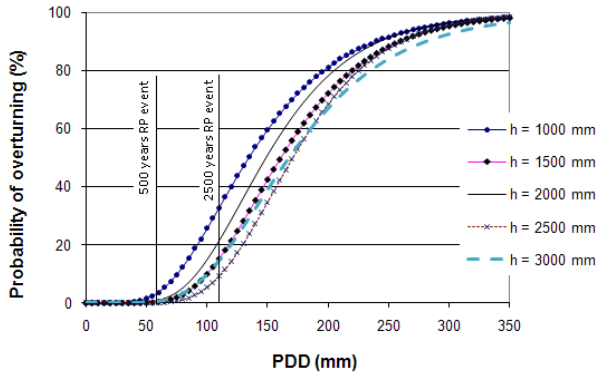
$$RSD_{max} \geq \frac{2}{3}t \quad \text{[high risks]} \quad (2a)$$

$$\frac{2}{3}t > RSD_{max} > \frac{1}{3}t \quad \text{[moderate risks]} \quad (2b)$$

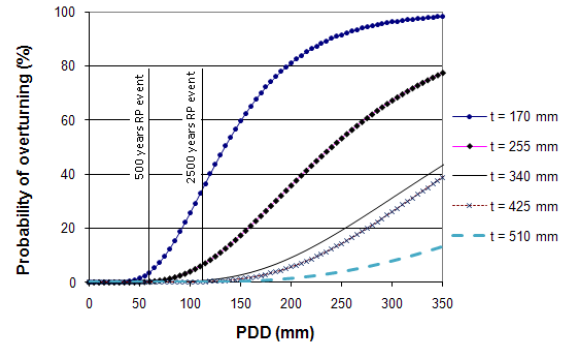
$$RSD_{max} \leq \frac{1}{3}t \quad \text{[low risks]} \quad (2c)$$

The value of the peak displacement demand (RSD_{max}) is clearly the controlling parameter. The findings as described have profound potentials in the design and assessment of structures for countering overturning but would require further

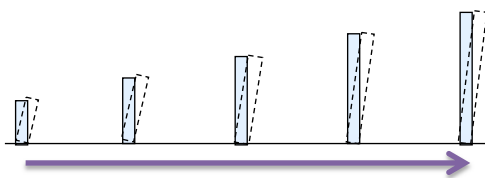
verifications, and modifications, to take into account conditions which are different to that of a free-standing object which has symmetrical distribution of mass.



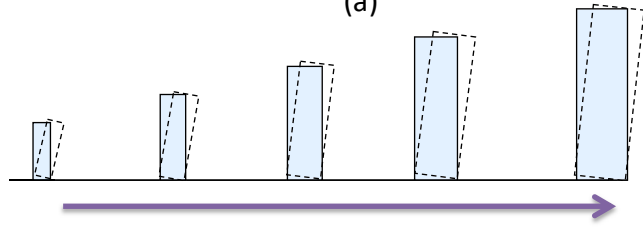
(a)



(a)



Similar Overturning risks
(b)



Decreasing risks of Overturning
(b)

Figure 3 Trend of overturning risks (height) Figure 4 Trend of overturning risks (size)

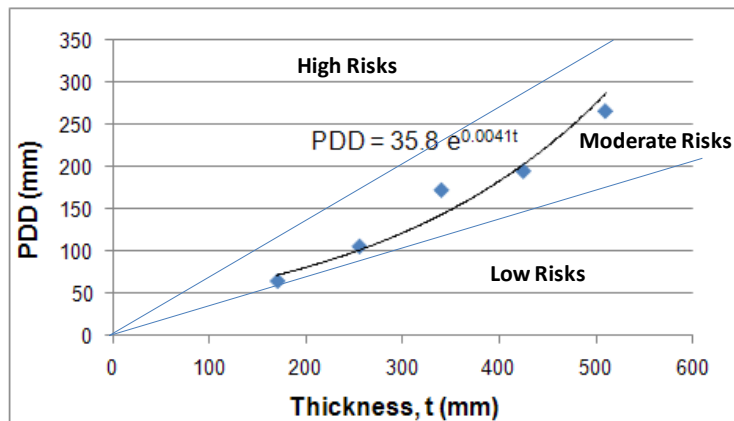


Figure 5 PDD value and risks of overturning

3. PEAK DISPLACEMENT DEMAND PREDICTIONS

Measurements of the peak displacement demand, or peak floor displacement, can be done easily on the shaker table in the laboratory in order that the risks of overturning of an object can be estimated reliably using equation 1, or equations 2a – 2c. However,

the displacement behavior of the ground in natural conditions is very difficult to measure and to estimate, given that ground accelerations are recorded in earthquakes, not ground displacement. In theory, values of the peak displacement demand (RSD_{max}) can be determined using equation (3a) once the response spectral acceleration (RSA) value in the long period range is known. Alternatively, equation (3b) can be used to convert a response spectral velocity (RSV) value into a response spectral displacement (RSD) value.

$$RSD = RSA \times \left(\frac{T}{2\pi} \right)^2 \quad (3a)$$

$$RSD = RSV \times \left(\frac{T}{2\pi} \right) \quad (3b)$$

It should be noted that, however, the value of RSD as derived from equation (3a), or equation (3b), and a conventional codified uniform hazard spectrum model (usually of the flat-hyperbolic form in the acceleration format) would increase indefinitely with increasing natural period of vibration instead of arriving at a definitive peak value. In this situation the value of RSD_{max} may be calculated in accordance with what has been stipulated at a natural period of 5s given that most civil engineering structures experiencing rocking behavior are expected to respond within this range of natural period. The determination of peak ground displacement, or response spectral displacement with natural periods exceeding 2 seconds, has always been a challenge. With vast improvements to the quality of the recordings and post-processing of data, many ground motion prediction models are now able to provide predictions of spectral ordinates of up to 5 seconds. In the rest of this paper, the value of the peak displacement demand (RSD_{max}) is defined as the maximum response spectral displacement value up to the period limit of 5 seconds.

Values of RSD_{max} so inferred in this manner from a codified response spectrum model can be overly conservative depending on the underlying assumptions in the development of the spectrum. Response spectrum models that are currently stipulated by the Eurocode (EC8), Australian Standard (AS1170.4, 2007) and the New Zealand Standard (NZS 1170.5, 2004) feature two distinct corner periods (T_1 and T_2) which divide the response spectrum into three regions: namely the *acceleration*, *velocity* and *displacement* controlled regions. The value of parameters RSA_{max} , RSV_{max} and RSD_{max} are then unambiguously defined in a response spectrum model of this form which was first proposed by Newmark-Hall (1982) and can be presented in the respective (acceleration, velocity and displacement) formats of Figures 6a – 6c.

Given that overturning is a highly non-linear phenomenon the risks of occurrence can be grossly overstated by the use of a uniform hazard spectrum (UHS) to infer the value of the peak displacement demand. To overturn a structure, or an object, the response spectral acceleration value based on initial at-rest condition (usually of a short natural period) would need to be sufficiently high to initiate rocking motion in the first place. Once rocking motion has commenced the probability of overturning is then controlled by the (long period) peak displacement demand properties of the ground shaking.

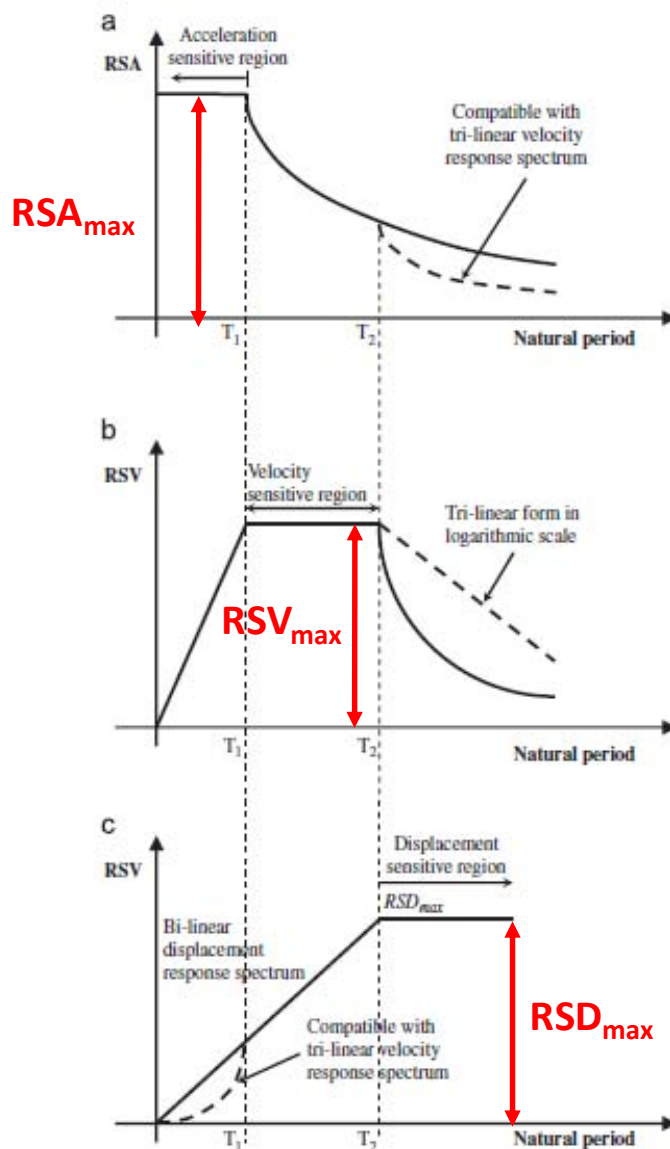


Figure 6 Response spectrum presented in different formats

Thus, overturning can only be resulted if the ground shaking in one event is filled with both low and high frequency contents. In other words, the structure will never overturn if the ground shaking is only filled with high frequency, or low frequency, contents, and not both. Because of this combined requirement a uniform hazard spectrum may give misleading information given that a multitude of earthquake scenarios is encapsulated in one spectrum. For areas which are under the threat of both local earthquakes and large magnitude distant earthquakes it has never been the intention that a structure will need to be designed to cater for both types of earthquakes striking it at the same time. However, such (most unlikely) loading combination is implicit in a uniform hazard spectrum model. Consequently, overturning risks are often evaluated for conditions which do not really exist. In summary, existing codified seismic hazard maps and response spectrum models may not in itself be sufficient to provide realistic predictions of potential overturning actions.

The design earthquake scenario based approach is introduced next. Earthquake scenarios can be determined by (i) *de-aggregation* or (ii) *calibration*. *De-aggregation* is numerically intensive and is only justified in areas where probabilistic seismic hazard analysis (PSHA) has been undertaken on the basis of a reliable, and accurate, model of earthquake sources and attenuation properties of the surrounding region. Few PSHA models in regions of low and moderate seismicity would qualify. An alternative, simpler, approach is by *calibration* wherein earthquake scenarios are selected in order that response spectra associated with individual scenarios are matched with different parts of a codified response spectrum. Ground motion parameters such as PGA or PGV values may also be used to identify compatible design earthquake scenarios. For example, earthquake scenarios of M6 at R = 30 km, or that of M6.5 at R = 45 km, have been identified to generate median PGV values of 60 mm/s (or 0.08g) for the generic rock conditions of south-eastern Australia (Lam *et al.*, 2005) whereas M6 at R = 20 km, or M6.5 at R = 30 km, for PGV values of 100 mm/s (or 0.14g). These two sets of earthquake scenarios can be used to represent seismic hazard of capital cities on the eastern seaboard of Australia for return period of 500 years and 2500 years respectively. Response spectra simulated for these calibrated earthquake scenarios are effectively enveloped by the respective codified response spectrum model.

In the design earthquake scenario based approach ground motion predictive expressions (GMPEs), which is traditionally known as “attenuation models”, can be employed for obtaining accurate estimates of the value of RSA_{max} , RSV_{max} or RSD_{max} for pre-defined earthquake scenarios. Accelerograms recorded, or synthesized, for pre-determined conditions can also be grouped in accordance with specific magnitude-distance (M-R) and site class combinations.

Many competing models of GMPEs have been developed. There can be challenges on the part of the engineer in selecting the “correct” blend given the inter-model differences. The Next Generation Attenuation (NGA) model is aimed at bringing together expert recommendations into a unified framework. Five well known GMPE blends: (i) Boore & Atkinson (ii) Campbell & Bozorgnia (iii) Chiou & Youngs (providing an update of the model by Sadigh) (iv) Idriss and (v) Abrahamson & Silva constituting the NGA were developed during the period 2007-2010 using the strong motion database of the Pacific Earthquake Engineering Research Centre (PEER) at the University of California at Berkeley. In spite of the same dataset being used Inter-model inconsistencies amongst the NGA model are still significant and this presents challenges to engineers. Furthermore, most of the data were collected from Western North America, Taiwan, Turkey, Iran and Italy and there are little representations from low-moderate seismicity regions.

In a comparative study undertaken by the authors the value of the RSA_{max} and RSV_{max} parameters predicted from a diversity of GMPEs sourced from western and eastern North America and Europe were correlated with distance when the earthquake magnitude was kept constant (Figures 7a & 7b). It is shown in both figures that the median predictions from one model can be well over 100% higher than another model

for identical magnitude-distance combinations. In areas of low-moderate seismicity with a paucity of strong motion data it can be difficult to decide which model to select in order to best represent regional and local conditions. Importantly, predictions of RSD_{max} from a diversity of models are much better constrained as shown in Figure 8. In view of the good consistencies amongst so many GMPEs sourced around the globe it is feasible to have a generic model for estimating peak displacement demand for worldwide applications.

Table 1 is an excerpt of recommendations published by the authors in Lumantarna *et al.* (2012). The empirical model as summarized in Table 1 is compared in Figure 9 with two alternative models recommended by the authors in earlier publications: (i) stochastic simulations of the seismological model of Eastern North America (Lam *et al.*, 2000b) (ii) theoretical fault rupture model of Lam & Chandler (2005). These two models are defined by equations (4a) and (4b) respectively.

$$RSD_{max}(mm) = 12 \times (0.2 + 0.8(M - 5)^{2.3}) \times \gamma_D \times G \times \beta_D \quad (4a)$$

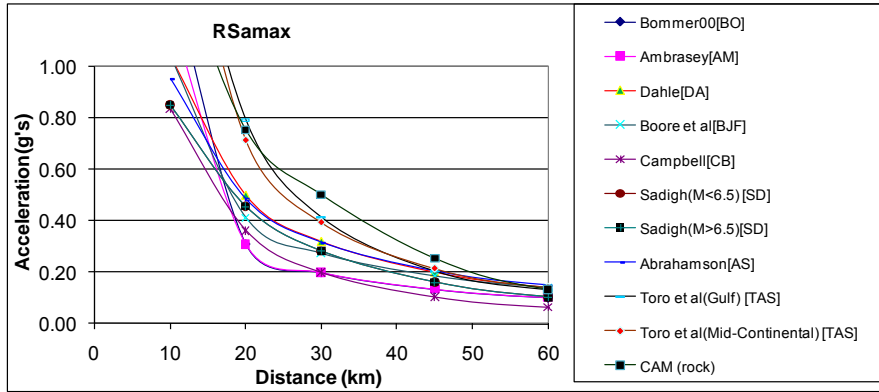
$$RSD_{max}(mm) = 10^{M-5} \times \gamma_D \times G \times \beta_D \quad (4b)$$

where $\gamma_D = 1.5$ for Generic Rock; $G = \frac{30}{R}$ and $\beta_D = \left(\frac{30}{R}\right)^{0.003R}$ as summarized in Lam *et al.* (2010)

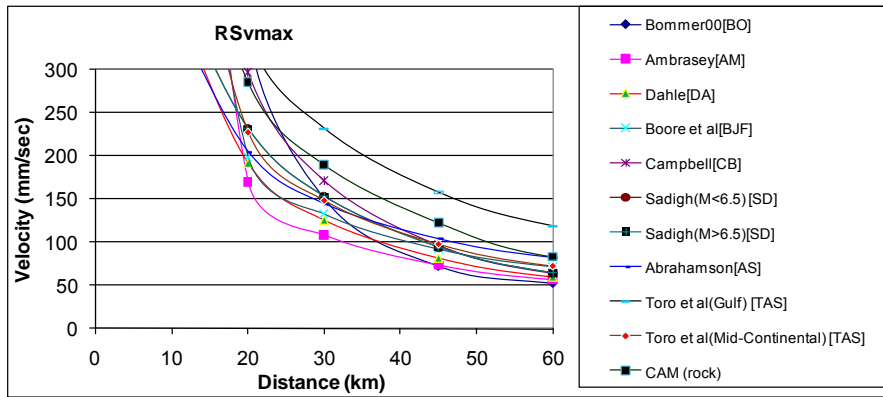
The expressions of Figure 9 in conjunction with equations (1) & (2a) – (2c) enable the risks of overturning to be predicted for any rigid object which is founded on a rock site and at a site-source distance of 10 – 50 km. Consistent predictions from these expressions which were developed in different timeframe using different approaches re-assure the generality of the predictive relationships.

Figure 10 is the schematic diagram showing how a response spectrum on rock sites can be modified to account for the effects of soil amplifications (Lam *et al.*, 2001). Site amplification is a major topic in its own right. A model for predicting displacement amplification on flexible soil sites is presented in Tsang *et al.* (2006).

In situations where non-linear time-history analyses are required accelerograms will need to be retrieved and collated for analyses. The PEER database which contains some 1600 accelerograms from 58 mainshocks of shallow earthquakes is amongst the most well known database of earthquake accelerograms and is open for public access. However, the size of the database is much reduced if motions recorded on intermediate/soft soil sites or from over 50 km distance have been excluded. Sixty-four recordings from rock or stiff soil sites can be found in the magnitude bin of M6 (M5.75 - M6.25) but only from eight events; and eighty records in the magnitude bin of M6.5 (M6.25 – M6.75) but only from seven events. The number of events that can be incorporated into a study is even less if analyses are restricted to certain distance range of interests. Thus, the amount of records that have been archived to date is not adequate to address variabilities between events. Readers are recommended to resort to the use of artificial accelerograms to augment recorded data (Lam *et al.*, 2000a).



(a) RSA_{max}



(b) RSV_{max}

Figure 7 RSA_{max} and RSV_{max} values from Ground Motion Models for M6.5 event

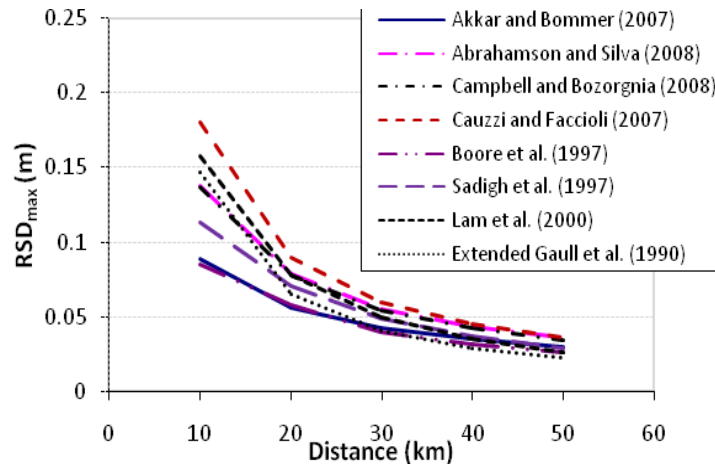


Figure 8 RSD_{max} values from Ground Motion Models for M6.5 event
(The listing of references cited in the figure legend can be found in Lumantarna et al., 2012)

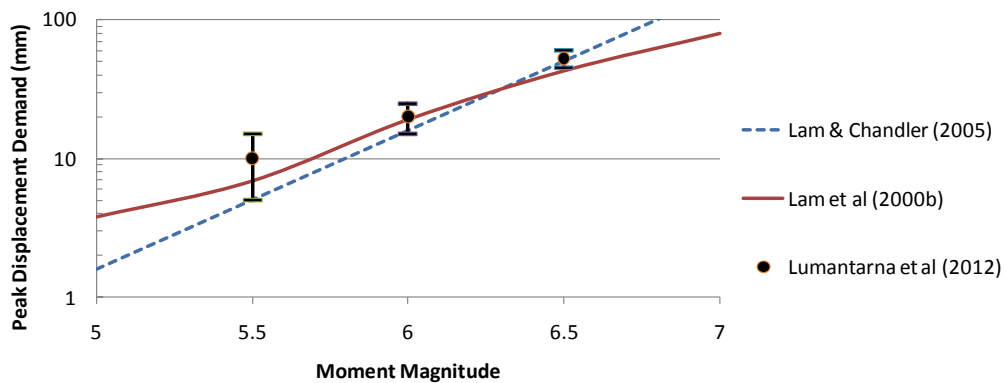


Figure 9 Peak Displacement Demand on Generic Rock Sites

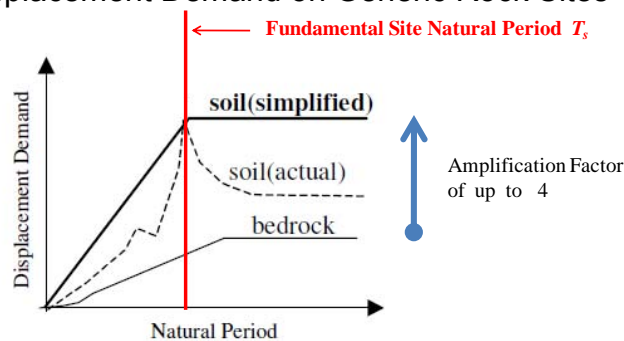


Figure 10 Site Effects in Displacement Spectrum Format

Table 1
Median model predictions of peak displacement demand (mm).

	<i>R</i> = 10 km	<i>R</i> = 20 km	<i>R</i> = 30 km	<i>R</i> = 40 km	<i>R</i> = 50 km
M5.5	23 (15–30)	15 (10–20)	10 (5–15)	8	5
M6	68 (45–90)	34 (25–40)	20 (15–25)	15	10
M6.5	135 (90–180)	75 (55–90)	55 (45–60)	38 (30–45)	33 (25–40)

Notes: (a) Mid range values are shown. (b) Upper and lower bound values are shown in brackets where there are significant inter-model discrepancies. (c) Values shown in *italics* are much less well constrained and are associated with scenarios of very low probability of occurrences in a region of low-moderate seismicity.

4. STRUCTURAL ENGINEERING APPLICATIONS

In summary, the overturning assessment procedure comprises the following steps:

1. Identify the design earthquake scenarios expressed in terms of M-R combinations and site class.
2. Check that the earthquake scenario would generate a level of acceleration which is sufficient to initiate rocking of the object using conventional force-based calculation techniques.
3. Determine the peak displacement demand value for each of the identified earthquake scenarios using Table 1, Equations (4a) & (4b) and/or Figure 9.
4. Determine the effective thickness of the object in the direction of ground shaking.

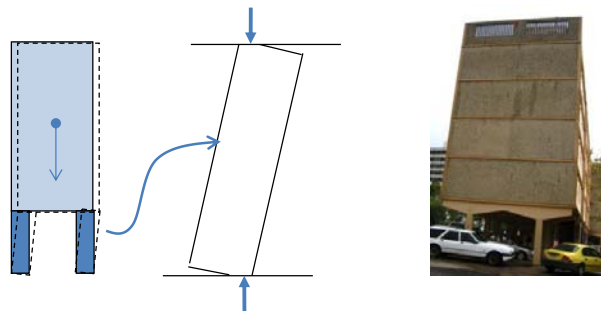
5. Identify the risks of overturning using equations (1) & (2a-2c) and/or Figure 5.

The procedure as outlined above can be applied for the assessment of free-standing objects, furniture items, mechanical/electrical equipment as well as unreinforced block walls and brickwalls (Figure 11a). In cases where there are holding down devices to secure the item in place the procedure may still be applied to cater for the extreme scenario where the holding down devices fail to resist the overturning actions. Thus, the methodology presented in this paper provides estimates of the last line of defence against overturning.

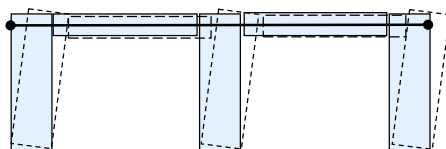
The same procedure has been adapted for assessing the risks of collapse of precast concrete columns which provide support of buildings featuring a soft storey (Figure 11b). Given that the column is carrying most of the load from above and not its own weight the boundary conditions are different to that of a free-standing object. Thus, equation (3c) will need to be modified to take into account some 50% increase in displacement capacity. It can be shown that the column is deemed to be subject to a low risks of overturning if the peak displacement demand value is equal to, or exceeded by, half the effective thickness of the column in the direction of ground shaking. The effective thickness of the precast concrete column is the relative offset of the centroid of the reaction forces developed at both ends of the column and is a fraction of its physical thickness (Wibowo *et al.*, 2011).



(a) Free-standing furniture, equipment, block/brick walls



(b) Precast concrete columns



(c) Unbonded post-tensioned precast concrete frame



(d) Retaining/abutment walls

Figure 11 Applications of the overturning assessment methodology

The displacement demand model introduced in this paper can also be used to assess damage inflicted by pounding of the base of the structure, or equipment item, against the ground (Al Abadi *et al.*, *in press*).

The assessment procedure can potentially be adapted for the displacement capacity assessment of innovative systems featuring precast concrete frames with connections that are jointed by unbonded post-tensioned cables for the mitigation of seismic hazards (Figure 11c). There are also potentials to adapt the methodology for assessing retaining walls and bridge abutments; further research is required to study the influence of the interaction of the backfill with the wall experiencing overturning (Figure 11d).

5. CONCLUSIONS

The displacement behavior of non-deformable free-standing objects experiencing rocking actions, and their risks of overturning, have been found to correlate well with the value of the peak displacement demand (RSD_{max}) of the applied base excitations. Expressions have been derived to determine the level of risks of overturning as function of RSD_{max} and thickness of the object in the direction of earthquake ground shaking.

Codified response spectrum models and uniform hazard spectra might not in itself provide realistic predictions of the value of RSD_{max} . It is proposed that compatible earthquake scenarios be identified by *de-aggregation*, or by *calibration*, based on the following criteria: (i) response spectral accelerations generated by the scenario are to match with certain parts of the targeted response spectrum (ii) the level of accelerations should be sufficient to initiate rocking of the object.

Once the critical earthquake scenarios have been identified the value of RSD_{max} on generic rock sites can be found using generic relationships that are summarized in Figure 9. The effects of displacement amplification on flexible soil sites could then be incorporated to provide predictions of the site specific value of RSD_{max} .

The assessment procedure introduced in this paper has been applied for the assessment of free-standing objects, furniture items, mechanical/electrical equipment as well as unreinforced block walls and brick walls. The same procedure has also been adapted for assessing the risks of collapse of precast concrete columns supporting buildings featuring a soft-storey.

The procedure can potentially be used for the displacement capacity assessment of innovative systems featuring precast concrete frames with connections that are jointed by unbonded post-tensioned cables, and for assessing retaining walls and bridge abutments.

REFERENCES

- Al-Abadi, H., Gad, E.F., Lam, N.T.K. & Petrolito, J. (in press) "A Simple Model for Estimating Shocks in Unrestrained Building Contents in an Earthquake" *Journal of Earthquake Engineering*. Manuscript no.UEQE-2012-1487. Accepted 28 December 2012.
- Al Abadi, H., Lam, N.T.K. & Gad, E. (2006). "A Simple Displacement Based Model for Predicting Seismically Induced Overturning". *Journal of Earthquake Engineering*. 10(6): 775-814.
- AS 1170.4(2007), Structural Design Actions – Part 4 Earthquake Actions, Standards Australia, Sydney, 2007.
- EC 8 (BS EN 1998-1:2004): Eurocode 8 part 1: Design of structures for earthquake resistance: General rules, seismic actions and rules for buildings.
- Kafle, B., Lam, N.T.K., Gad, E.F. & Wilson, J.L. (2011). "Displacement Controlled Rocking Behaviour of Rigid Objects", *Journal of Earthquake Engineering and Structural Dynamics*, 40: 1653-1669.
- Lam, N.T.K., Wilson, J.L. & Tsang, H.H. (2010), "Modelling Earthquake Ground Motions By Stochastic Methods", *Stochastic Control*, SCIYO Publisher, Chapter 23 : 475 - 492.
- Lam, N.T.K., Chandler, A.M. (2005). Peak Displacement Demand in Stable Continental Regions, *Earthquake Engineering and Structural Dynamics*. John Wiley & Sons Ltd, 34: 1047-1072.
- Lam, N.T.K., Wilson, J.L. & Srikanth, V. (2005). Accelerograms for dynamic analysis under the New Australian Standard for Earthquake Actions, *Electronic Journal of Structural Engineering*. 5 : 10-35.
- Lam, N.T.K., Griffith, M.C., Wilson, J.L. & Doherty, K. (2003) Time History Analysis of URM walls in out-of-plane flexure, *Journal of Engineering Structures*. Elsevier Science Publisher. 25(6):743-754.
- Lam, N.T.K., Wilson, J.L., Chandler, A.M. (2001) Seismic Displacement Response Spectrum Estimated from the Frame Analogy Soil Amplification Model, *Journal of Engineering Structures*, 23, 1437-1452.
- Lam, N.T.K., Wilson, J.L. and Hutchinson, G.L., 2000a: "Generation of Synthetic Earthquake Accelerograms Using Seismological Modelling : A Review ", *Journal of Earthquake Engineering* , 4(3), 321-354.
- Lam, N.T.K., Wilson, J.L., Chandler, A.M. and Hutchinson, G.L., 2000b : "Response Spectral Relationships for Rock Sites Derived from The Component Attenuation Model ", *Earthquake Engineering and Structural Dynamics*, 29(10), 1457-1490.
- Lumantarna, E., Lam, N.T.K., Wilson, J.L. and Griffith, M.C. (2010). "Inelastic Displacement Demand of Strength Degraded Structures", *Journal of Earthquake Engineering*. 14: 487-511.
- Lumantarna, E., Lam, N.T.K. & Wilson, J.L. (in press) "Displacement Controlled Behaviour of Asymmetrical Single-Storey Building Models" *Journal of Earthquake Engineering*. Manuscript no.UEQE-2012-1471. Accepted 21 Feb 2013.

- Lumantarna, E., Wilson, J.L. & Lam, N.T.K. (2012) "Bi-linear displacement response spectrum model for engineering applications in low and moderate seismicity regions" *Soil Dynamics and Earthquake Engineering*, 43: 85-96.
- Newmark, N.M., Hall, W.J. (1982). *Earthquake spectra and design*. EERI Monograph, Earthquake Engineering Research Institute, California.
- NGA (Next Generation Attenuation Models), Pacific Earthquake Engineering Research Centre http://peer.berkeley.edu/ngawest/nga_models.html
- NZS 1170.5:2004 Structural design actions - Earthquake actions - New Zealand
- PEER (2000), Pacific Earthquake Engineering Research Centre, Regents of The University of California, < <http://peer.berkeley.edu/smcat/index.html>>.
- Tsang, H.H., Chandler, A.M. & Lam, N.T.K.(2006) "Estimating non-linear site response by single period approximation". *Journal of Earthquake Engineering and Structural Dynamics*. 35(9): 1053-1076.
- Wibowo, A., Wilson, J.L., Gad, E.F., Lam, N.T.K. and Collier, P.A. (2011), "Drift capacity of a precast soft-storey building in Melbourne", *Australian Journal of Structural Engineering*, 11(3): 177-193.