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Seismic Assessment of Rocking Bridge Bents Using an Equivalent Rocking Block

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ABSTRACT

In light of the resurgence of rocking as a means of seismic isolation for modern structures/bridges, there is an increasing need to predict the response of complex rocking structures. To date, most analytical investigations into the rocking behavior assume a simple model involving a single rigid rocking block on a rigid half-space. This paper proposes the prediction of the rocking response of a complex (multiple block) rocking structure by considering the response of a single rigid rocking block. In particular, it describes a methodology to derive a dynamical equivalence between a planar rocking bridge bent and the archetypal rocking block. Through the proposed concept of the (dynamically) equivalent rocking block, this approach makes the vast existing research on the rocking block useful for the treatment of more realistic rocking structures. To investigate the efficient of the proposed approach, the study assesses the seismic response of a bridge bent designed to rock during earthquake excitation. The results verify the ability of the proposed approach to capture the global rocking behavior and confirm the ample seismic stability of the symmetric planar rocking bridge bent.

Keywords: rocking bridges, equivalent rocking block, structural dynamics, earthquake engineering

1. INTRODUCTION

The idea of allowing rocking behaviour as a means of seismic isolation is not new. Early studies and applications on rocking date back to the 60's and the 70's (Housner 1963, Kelly and Tsztoo 1977). Recently, the concept of re-centering structures, which use some degree of rocking motion to isolate a structure from the stresses induced by earthquakes, is re-gaining momentum as a modern seismic design alternative. In this context, numerous studies on the rigid rocking block have generated a wealth of knowledge about rocking behaviour. However, there is still a need to predict the expected response and assess the seismic performance of modern and more complex rocking systems, such as rocking bridge bents (frames).

To meet this need, this paper proposes the prediction of the rocking response of a multiple block rocking structure by considering the response of a single rigid rocking block. In particular, it describes a methodology to derive, an exact or approximate, equivalence between a complicated rocking structure and the single rocking block. Through the concept of the (dynamically) equivalent rocking block, this approach makes the vast existing research on the rocking block useful for the treatment of more realistic rocking structures. In this paper the focus is on a rocking bridge bent (frame) configuration found in prefabricated bridge technology, though the proposed methodology is applicable to a wide range of structures (DeJong and Dimitrakopoulos 2013).

2. REVIEW OF THE ARCHETYPAL ROCKING BLOCK RESPONSE

This section presents briefly the equations of motion for a rocking block. Consider a rigid block subjected to a horizontal ground motion with acceleration time history $\ddot{u}_g(t)$ (Fig. 1). Assuming pure rocking behavior, the block will uplift and rotate about the point O (or O'), once the ground acceleration exceeds a minimum magnitude:

$$\lambda = \frac{\ddot{u}_{g,\min}}{g} \tag{1}$$

where α is the slenderness, g is the gravitational acceleration and λ is the dimensionless uplift parameter. The equations of motion during pure rocking are (Housner 1963, among others):

$$\ddot{\theta} = p^2\left(-\sin\left[\alpha\,\mathrm{sgn}(\theta)-\theta\right]-\frac{\ddot{u}_g}{g}\cos\left[\alpha\,\mathrm{sgn}(\theta)-\theta\right]\right) \tag{2}$$

where sgn() is the sign function and p is the rocking frequency parameter; for a rectangular block (Fig. 1), $p=\sqrt{3g/(4R)}$.
The pertinent linearized equations of motion are:

$$\ddot{\theta} = p^2\left(\alpha\,\mathrm{sgn}(\theta)-\theta-\frac{\ddot{u}_g}{g}\right) \tag{3}$$

Under the assumption of pure rocking, when the block returns to its initial position (θ = 0), impact takes place, the pivot point changes, and the rotation switches sign. A simple way to treat impact is with a coefficient of restitution η, which yields the post- ($\dot{\theta}^+$) impact angular velocity as a ratio of the pre- ($\dot{\theta}^-$) impact velocity:

$$\dot{\theta}^+ = \eta\dot{\theta}^- \tag{4}$$

As in reality the coefficient of restitution is application and material specific, this study will treat it as an independent parameter (Dimitrakopoulos and DeJong 2012).

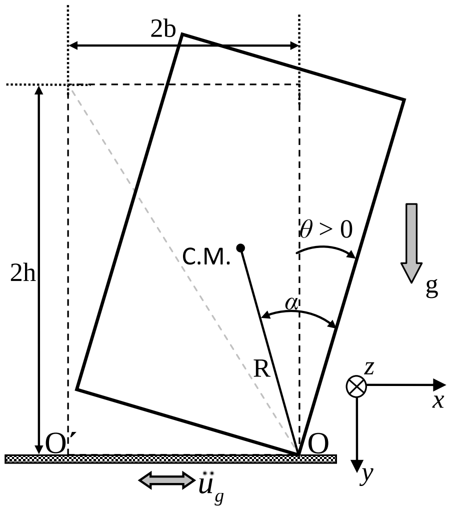

Fig. 1 Rocking block geometry

3. DYNAMICALLY EQUIVALENT ROCKING STRUCTURES

The literature on the rocking response of the rigid block is prolific and has generated a wealth of knowledge about rocking behavior (Dimitrakopoulos and DeJong 2012). The evaluation of more complex rocking structures though, is comparatively scarce. One reason is that it requires the derivation and solution of more complicated equations of motion.

Firstly, the present section presents a methodology for the definition of a single block, dynamically equivalent with a multi-block rocking structure. For instance, an asymmetric rocking frame/bent (Fig. 2, assuming different heights H_1 and H). Then it focuses on the simpler case of the symmetric rocking bent (Fig. 2, assuming heights H_1 and H are the same) and simplifies the previously obtained general equations.

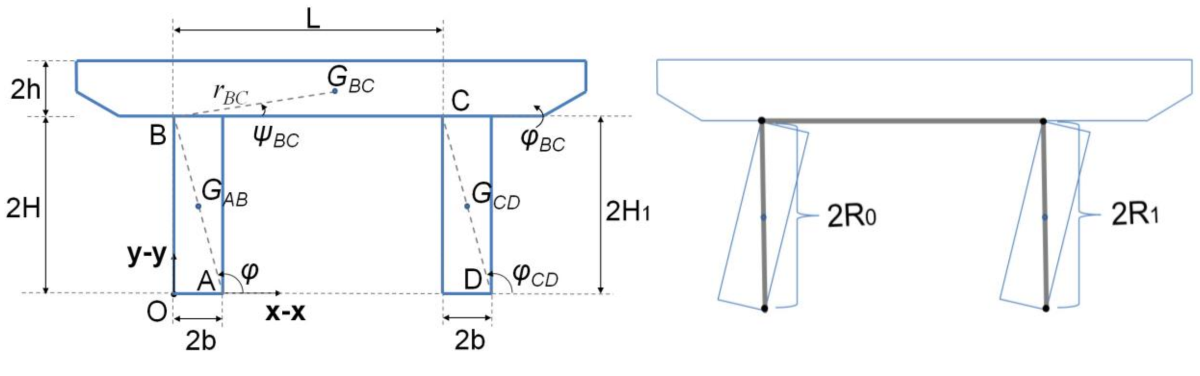

Fig. 2 The examined free-standing rocking bridge bent: geometry (left) and kinematic mechanism (right)

The equation of motion of such multiple block rocking mechanisms (e.g. Fig. 2) can be derived using Lagrange's equation:

$$\frac{\partial}{\partial t}\left(\frac{\partial T}{\partial \dot{\phi}}\right)-\frac{\partial T}{\partial \phi}+\frac{\partial V}{\partial \phi}=Q \tag{5}$$

where T is the kinetic energy, V is the potential energy, Q is the generalized force and ϕ is the generalized coordinate which describes the rocking motion. For multiple-block rocking mechanisms (e.g. the bridge bent of Fig. 2), Eq. (5) yields an equation of the following form:

$$I_{nl}(\phi)\ddot{\phi}+J_{nl}(\phi)\dot{\phi}^2-G_{nl}(\phi)g=-B_{nl}(\phi)\ddot{u}_g \tag{6}$$

where I_{nl}, J_{nl}, G_{nl} and B_{nl} are nonlinear functions of the generalized coordinate. Equation (6) is not equivalent with the rocking block equation of motion (2). Still, an approximate local equivalence is feasible (DeJong and Dimitrakopoulos 2013) around the unstable equilibrium position. In particular, for small amplitude vibrations about an equilibrium point, Lagrange's equation (5) assumes the linearized form (Meirovitch 1986):

$$\left.\frac{\partial^2 T}{\partial \dot{\phi}^2}\right|_{\phi-\phi_{cr}\approx 0}\ddot{\phi} \;+\; \left.\frac{\partial^2 V}{\partial \phi^2}\right|_{\phi-\phi_{cr}\approx 0}\left(\phi-\phi_{cr}\right)=Q\big|_{\phi-\phi_{cr}\approx 0} \tag{7}$$

where the point of unstable equilibrium (ϕ_{cr}) is determined from:

$$\left.\frac{\partial V}{\partial \phi}\right|_{\phi-\phi_{cr}\approx 0}=0 \tag{8}$$

After some algebra, Eq. (7) yields:

$$I_{eq}\ddot{\phi}+G_{eq}\left(\phi-\phi_{cr}\right)g=-B_{eq}\ddot{u}_g \tag{9}$$

where G_{eq}, B_{eq}, and I_{eq} are constants that are specific to the kinematics of the unstable equilibrium configuration defined as:

$$I_{eq}=\left.\frac{\partial^2 T}{\partial \dot{\phi}^2}\right|_{\phi-\phi_{cr}\approx 0} \qquad G_{eq}=\frac{1}{g}\left.\frac{\partial^2 V}{\partial \phi^2}\right|_{\phi-\phi_{cr}\approx 0} \qquad B_{eq}\ddot{u}_g=-Q\big|_{\phi-\phi_{cr}\approx 0} \tag{10}$$

Equation (9) differs from the pertinent equation of the rocking block (3) in that the excitation term is scaled differently than the stiffness term ($B_{eq} \neq G_{eq}$). For this reason the authors introduced (DeJong and Dimitrakopoulos 2013) the following transformation of variables:

$$\theta(t)=\frac{\phi(t)}{\kappa} \tag{11}$$

where θ represents the rotation of the equivalent single block, while ϕ represents the rocking rotation of the multi-block mechanism, and the scaling parameter κ is given by:

$$\kappa = \kappa_1 = \frac{B_{eq}}{G_{eq}} \tag{12}$$

Or, for effectively non-slender structures (DeJong and Dimitrakopoulos 2013) by:

$$\kappa = \kappa_2 = \frac{\phi_{cr}}{\lambda} \tag{13}$$

With the help of the transformation (11), the equation of motion (9) assumes the form:

$$\ddot{\theta} = p_{eq}^2 \left(\theta_{cr} - \theta - \frac{\ddot{u}_g}{g} \right) \tag{14}$$

which is now directly equivalent with Eqn. (3), and for which:

$$p_{eq} = \sqrt{gG_{eq} / I_{eq}} \quad ; \quad \theta_{cr} = \phi_{cr} / a_{sc} \tag{15}$$

The methodology summarized above hinges on a local approximation of the nonlinear equations of motion around the (unstable) equilibrium position and offers a dynamically equivalent rocking block for more complicate multi-block rocking structures. In the following, we revisit with the help of the proposed methodology the rocking response of a symmetric rocking frame/bent. More complicated examples (e.g. a two-block mechanism and the three-block mechanisms including a non-symmetric rocking frame) can be found in DeJong and Dimitrakopoulos (2013).

(1) Kinematic analysis

The kinematics of a three-block rocking mechanism (Fig. 2) is far from trivial. Still, the instantaneous configuration of the three-block mechanism can be captured with a single generalized coordinate, which herein is selected as the angle φ of segment AB with respect to the positive *x*-axis. The rocking amplitude is measured as the rotation with respect to the initial position $\phi = \varphi_0 - \varphi$ (Fig. 2). The angular velocities are then derived from the pertinent rotations by differentiating with respect to time (DeJong and Dimitrakopoulos 2013):

$$\dot{\varphi}_{BC}(\varphi, \dot{\varphi}) = f_{BC}(\varphi) \cdot \dot{\varphi}, \quad \dot{\varphi}_{CD}(\varphi, \dot{\varphi}) = f_{CD}(\varphi) \cdot \dot{\varphi} \tag{16}$$

(2) Equation of motion

The potential energy of the three-block mechanism can be expressed as:

$$V = g\left[\left(m_{AB} + 2m_{BC}\right)R_0 \sin\varphi + m_{BC} r_{BC} \sin\left(\varphi_{BC} + \psi_{BC}\right) + m_{CD}\left(2H - 2H_1 + R_1 \sin\varphi_{CD}\right)\right] \tag{17}$$

where m_{AB}, m_{BC} and m_{CD} are the masses of blocks AB, BC and CD, respectively. The kinetic energy can be expressed as:

$$T = \frac{1}{2} I_{AB}\dot{\varphi}^2 + \frac{1}{2} I_{BC}\left(f_{BC}(\varphi)\cdot\dot{\varphi}\right)^2 + \frac{1}{2} I_{CD}\left(f_{CD}(\varphi)\cdot\dot{\varphi}\right)^2 + \frac{1}{2} m_{BC}\left[\left(2R_0\right)^2 + 4R_0 r_{BC}\cos\left(\varphi - \varphi_{BC} - \psi_{BC}\right) f_{BC}(\varphi)\right]\dot{\varphi}^2 \tag{18}$$

where I_{AB} is the mass moment of inertia of AB with respect to the pivot point A, and I_{BC} and I_{CD} are the equivalent quantities for members BC and CD.

The equation of motion is derived from Lagrange's equation (5) and can be written in the form of Eq. (6) with:

$$\begin{aligned}
I_{nl}(\varphi) &= \begin{Bmatrix} I_{AB} + I_{BC}\left(f_{BC}(\varphi)\right)^2 + I_{CD}\left(f_{CD}(\varphi)\right)^2 + \\ m_{BC} 4R_0\left[R_0 + r_{BC}\cos\left(\varphi - \varphi_{BC} - \psi_{BC}\right) f_{BC}(\varphi)\right] \end{Bmatrix} \\
J_{nl}(\varphi) &= -\begin{Bmatrix} I_{BC} f_{BC}(\varphi) f'_{BC}(\varphi) + I_{CD} f_{CD}(\varphi) f'_{CD}(\varphi) + \\ 2m_{BC} R_0 r_{BC}\left[\cos\left(\varphi - \varphi_{BC} - \psi_{BC}\right) f'_{BC}(\varphi) - \sin\left(\varphi - \varphi_{BC} - \psi_{BC}\right)\left(1 - f_{BC}(\varphi)\right) f_{BC}(\varphi)\right] \end{Bmatrix} \\
G_{nl}(\varphi) &= -\left\{\left(m_{AB} + 2m_{BC}\right)R_0\cos\varphi + m_{BC} r_{BC}\cos\left(\varphi_{BC} + \psi_{BC}\right) f_{BC}(\varphi) + m_{CD} R_1 \cos\varphi_{CD} f_{CD}(\varphi)\right\} \\
B_{nl}(\varphi) &= \begin{Bmatrix} m_{AB} R_0 \sin\varphi + m_{CD} R_1 \sin\left[\varphi_{CD}(\varphi)\right] f_{CD}(\varphi) \\ + m_{BC}\left[2R_0 \sin\varphi + r_{BC}\sin\left[\varphi_{BC}(\varphi) + \psi_{BC}\right] f_{BC}(\varphi)\right] \end{Bmatrix}
\end{aligned} \tag{19}$$

(3) Symmetric Frame

When the rocking frame is symmetric (φ_{BC} =0, $\varphi_{CD} = \theta$, $I_0 = I_{AB} = I_{CD}$, $R = R_0 = R_1$, and $I_0 = I_{AB}$ and $m = m_{AB} = m_{CD}$) the equation of motion (6) simplifies to:

$$\left(I_0 + 2m_{BC}R^2\right)\ddot{\varphi} = -\left(m + m_{BC}\right)R\left(g\cos\varphi - \ddot{u}_g \sin\varphi\right) \tag{20}$$

And the uplift parameter becomes:

$$\lambda = \frac{\ddot{u}_{g,\min}}{g} = \tan\alpha \tag{21}$$

Equation (20) verifies the direct equivalence between the symmetric rocking frame and a rocking block, as previously identified (Makris and Vassiliou 2013).

(4) Linearized equation of motion

With the help of Eq. (10) the equation of motion can be linearized about the point of (unstable) equilibrium (Eq. (7)) and assume the form of Eq. (9), in which:

$$
\begin{aligned}
I_{eq} &= \begin{Bmatrix} I_{AB} + I_{BC} f_{BC}\left(\varphi_{cr}\right)^2 + I_{CD} f_{CD}\left(\varphi_{cr}\right)^2 + \\ m_{BC}\, 4R_0 \left[R_0 + r_{BC}\cos\left(\varphi_{cr} - \varphi_{BC}\left(\varphi_{cr}\right) - \psi_{BC}\right) f_{BC}\left(\varphi_{cr}\right)\right] \end{Bmatrix} \\
G_{eq} &= -\begin{Bmatrix} m_{BC}\, r_{BC} \left[\cos\left(\varphi_{BC}\left(\varphi_{cr}\right) + \psi_{BC}\right) f'_{BC}\left(\varphi_{cr}\right) - \sin\left(\varphi_{BC}\left(\varphi_{cr}\right) + \psi_{BC}\right)\left(f_{BC}\left(\varphi_{cr}\right)\right)^2\right] \\ + m_{CD} R_1 \left[\cos\left(\varphi_{CD}\left(\varphi_{cr}\right)\right) f'_{CD}\left(\varphi_{cr}\right) - \sin\left(\varphi_{CD}\left(\varphi_{cr}\right)\right)\left(f_{CD}\left(\varphi_{cr}\right)\right)^2\right] - \left(m_{AB} + 2m_{BC}\right) R_0 \sin\varphi_{cr} \end{Bmatrix} \\
B_{eq} &= \begin{Bmatrix} m_{AB} R_0 \sin\varphi_{cr} + m_{CD} R_1 \sin\left(\varphi_{CD}\left(\varphi_{cr}\right)\right) f_{CD}\left(\varphi_{cr}\right) \\ + m_{BC}\left[2R_0 \sin\varphi_{cr} + r_{BC}\sin\left(\varphi_{BC}\left(\varphi_{cr}\right) + \psi_{BC}\right) f_{BC}\left(\varphi_{cr}\right)\right] \end{Bmatrix}
\end{aligned}
\qquad (22)
$$

where the critical rotation is: $\varphi_{cr} = \pi / 2 \Rightarrow \phi_{cr} = \alpha$.

The errors caused by the proposed linearization approach are quantified and discussed in (DeJong and Dimitrakopoulos 2013) with respect to different types of rocking structures. In summary, the error introduced is acceptable for seismic assessment where the uncertainty involved in predicting expected ground motions is large.

4. ANALYSIS AND ASSESSMENT OF THE SEISMIC RESPONSE

This section examines the response of a free standing symmetric rocking frame (Fig. 2). The bent consists of two columns with 1.2 m diameter, 6.1 m height each (i.e. the column aspect ratio is 5.0 Fig. 2) and the pier/deck mass ratio is taken as 10. It is assumed that the beam-column and the column-foundation connections are simple (unilateral) contacts. Hence, this structural configuration is a pure rocking version of the bridge bent recently proposed as an effective structural system for rapidly constructible prefabricated bridges (Hieber *et al*. 2005) (Fig. 2). In (Hieber *et al*. 2005) the bridge bent is not a free-standing rocking structure, but the rocking mechanism is enhanced with the addition of post-tensioned (PT) tendons for increased self-centering.

Regardless, to assess the seismic behaviour of the free standing rocking frame/bent, we consider a well-known set of historic ground motions, which are scaled so as to yield a probability of exceedance of 2% in 50 years (SAC 1997). Fig. 3 offers sample time-history results. Despite the fact that the ground motions considered are scaled to the maximum credible earthquake level, and that the bridge bent is assumed to be free-standing, the structure survives all ground motions of Table 1. It is reminded that when the structure survives the motion (i.e. does not overturn), it eventually self-centers and hence, there is no permanent rotation and/or expected damage for all strong earthquake excitations considered (Table 1).

Further, Fig. 3 compares the nonlinear solutions of Eq. (6) with the proposed equivalent linearized block solutions of Eq.(9). The linear and nonlinear predictions compare well (Fig. 4). The prediction is better when the block nears the point of instability, as the dynamic properties of the equivalent linearized block are defined with respect to the point of instability (the unstable configuration). These conclusions verify that the proposed linear formulation is more than adequate when considering the uncertainty of earthquake loading magnitude and the simplifications made to derive the SDOF analytical model.

Number	Record	Magni-tude	Scale Factor	DT (s)	Dura-tion (s)	PGA (cm/sec^2)
SE22	1992 Mendocino	7.1	0.98	0.02	59.98	476.22
SE23	1992 Erzincan	6.7	1.27	0.005	20.775	593.60
SE24	1992 Erzincan	6.7	1.27	0.005	20.775	529.06
SE25	1949 Olympia	6.5	4.35	0.02	79.98	878.23
SE27	1965 Seattle	7.1	10.04	0.02	81.82	1722.40
SE28	1965 Seattle	7.1	10.04	0.02	81.82	1364.70
SE30	1985 Valpariso	8	2.9	0.025	99.975	1543.50
SE31	1985 Valpariso	8	3.96	0.025	99.975	1246.20
SE32	1985 Valpariso	8	3.96	0.025	99.975	884.43
SE35	1978 Miyagi-oki	7.4	1.78	0.02	79.98	595.07
SE36	1978 Miyagi-oki	7.4	1.78	0.02	79.98	768.62

Table 1 Ground motions (probability of exceedance of 2% in 50 years)

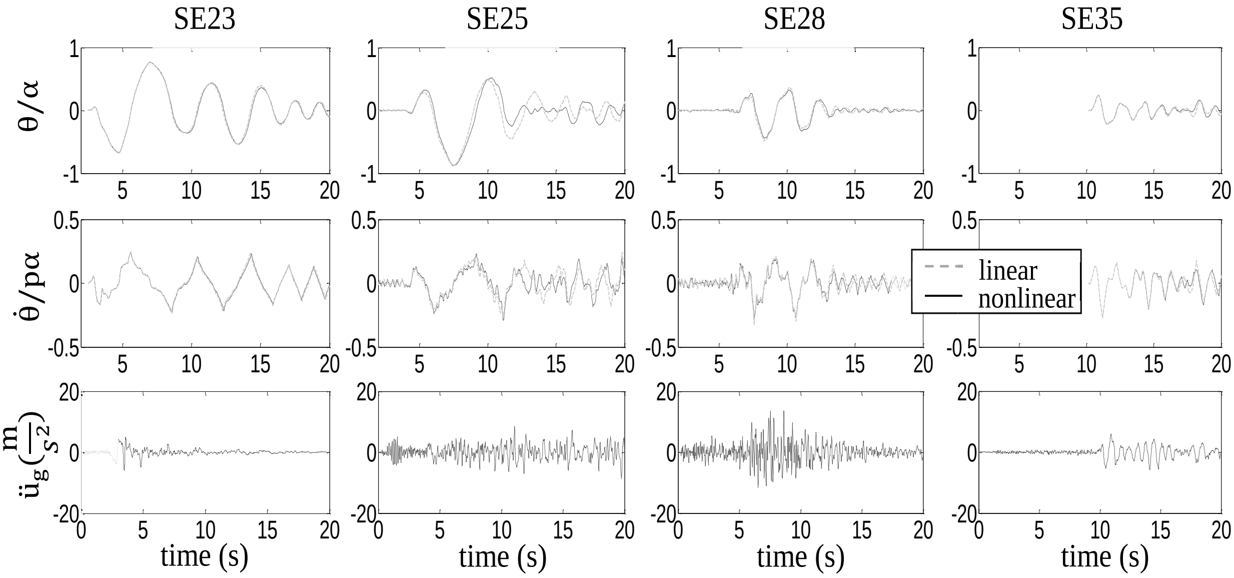

Fig. 3 Seismic response of a symmetric rocking frame for different earthquake records (bottom). Top: dimensionless rocking rotation and middle: dimensionless angular velocity

Hence, the equivalent rocking block approach could allow for a rapid first order approximation of the expected global rocking response (as in Fig. 3 and Fig. 4). Further, the equivalent rocking block approach could be combined with the abundant knowledge regarding the rocking block response. For instance, the available closed form solutions derived for pulse-type records and/or the use of dimensionless variables (e.g. Dimitrakopoulos and DeJong 2012) which specify the fundamental rocking parameters.

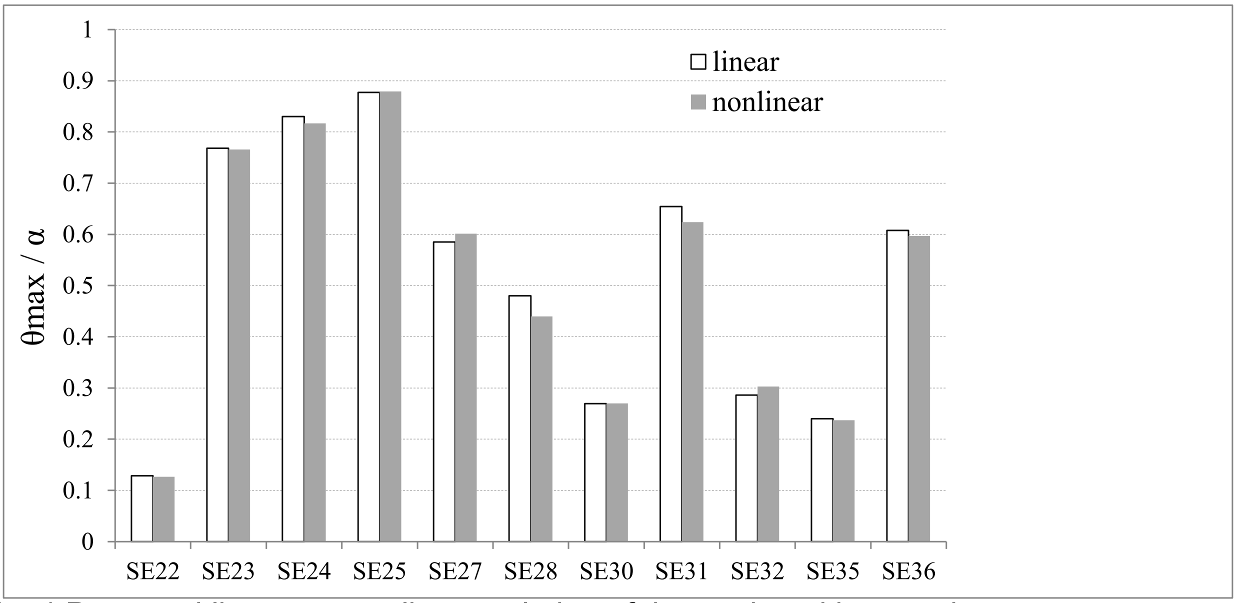

Fig. 4 Proposed linear vs. nonlinear solution of the peak rocking rotation

5. CONCLUSIONS

This study considers equivalence between a multiple block rocking structure (the rocking bridge bent) and the archetypal rocking block. The proposed methodology hinges on a local linearization of the nonlinear equation of motion of the complex structure, for small (rocking) rotations close to the instability configuration, and it offers an approximate dynamical equivalence with a single rocking block. The study shows that the proposed approach could effectively allow for a rapid first order approximation of the expected global rocking response and offers the pertinent equations of motion. As an example, the seismic behavior of a planar rocking bridge bent is examined, using both the ("exact") nonlinear formulation and the proposed linear approximation. The results of the two (nonlinear and linear) predictions are in excellent agreement. The results also confirm the ample seismic stability of the symmetric rocking frame, even though the earthquake excitations examined correspond to the "maximum credible earthquake" level.

The proposed concept of the (dynamically) equivalent rocking block approach could make the vast existing research on the rocking block useful for the treatment of more realistic rocking structures.

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