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Statistical analysis of the concrete rectangular stress block parameters

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ABSTRACT

Performance of reinforced concrete members depends on many sectional and material variables that are statistically uncertain. Concrete properties and in particular equivalent rectangular stress block parameters are among those with high uncertainty. Thus, in any probabilistic evaluation of performance of reinforced concrete members, statistical models for these parameters are important. This paper describes fundamental statistical characteristics of the compressive stress distribution in the compressive zone of flexural members. Because most of the current design codes are using the equivalent rectangular stress block concept, the analysis in this study is based on this concept. A large database of experimental results on concrete equivalent rectangular stress block parameters is reviewed and discussed in this study. The database includes testing of plain concrete columns, reinforced concrete members such as eccentrically loaded columns and beams in pure flexure. With the aid of this large and updated experimental database, the uncertainty involved in the evaluation of the equivalent rectangular stress block parameters is investigated. Concrete stress block models in some of the current concrete design codes are reviewed and then, through probability-based model errors, these models are compared with the experimental data. Finally, using Monte Carlo Simulation, impact of uncertainty in the concrete stress block parameters on the ultimate flexural strength and curvature is studied. The results show that due to variations in material and sectional properties, a significantly higher variability exists in the ultimate curvature of reinforced concrete beam sections in comparison to strength and that the ultimate curvature is sensitive to more random variables comparing to the strength.

1. INTRODUCTION

The idea of using the ultimate strength of section in design dates back to the original concept of designing based on empirical failure loads. Design based on failure is often called the ultimate strength design. At the ultimate state, the concrete in the compression zone has a nonlinear stress distribution similar to its stress-strain

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relationship which is referred to as the actual stress distribution. The actual compressive stress distribution in the compressive zone of concrete flexural members is extremely difficult to measure and to adequately model. The first published ultimate load theory was conducted by Koenen (1886) who assumed a straight line distribution of concrete stress and a neutral axis at mid-depth (Mattock et al. 1961). Afterwards, many different stress distribution shapes in the compressive zone of reinforced concrete members have been proposed. Among those, the equivalent rectangular stress block was found to be the most practical and simplest model, with sufficient accuracy for design purposes.

Many researchers have been working on the stress-strain distribution of concrete since the beginning of the 20th century. In 1955, Hognestad and others (1955) presented test data demonstrating validity of the plasticity concepts involved in strength design. They developed a test set-up (shown in Fig. 1) to derive the concrete rectangular stress block parameters which later became widely accepted and used by many other researchers (Nedderman 1973; Kaar et al. 1978; Pastor 1986). Swartz et al. (1985) performed an experimental investigation of the flexural properties of higher strength concrete. Their results showed that the compressive stress block in beams at failure is curved and may be represented by a parabola. Ibrahim and Macgregor (1997) conducted an experimental investigation on rectangular stress block of high and ultra-high strength concrete. They concluded that the ACI rectangular stress block parameters overestimates the moment capacity of high and ultra-high strength concrete columns failing in compression. They also proposed some modifications to the rectangular stress block parameters. Mertol et al. (2008) studied the fundamental characteristics of the compressive stress distribution in the compression zone of flexural members with concrete compressive strengths up to 124 MPa. They recommended revisions for the provisions of ACI code for higher strength concrete.

According to the ultimate strength design theory, when the strain at an extreme edge of a concrete section reaches the concrete ultimate strain, the failure state is achieved. The procedure for ultimate strength design incorporates basic assumptions which are linear distribution of strain across the section, perfect bond between concrete and reinforcement, zero tensile capacity for concrete and derivation of concrete and reinforcement based on the material stress-strain relationship. Since 1956, the ultimate strength theory (based on the equivalent rectangular stress block) has become widely used in the ACI design code and other design codes worldwide. Current design codes use a probabilistic-based format for the strength limit state; however, in this format, concrete rectangular stress block parameters are generally treated as deterministic variables and the best-fit curves to the experimental data are used as being representative of the stress block parameters. A research conducted by Attard and Stewart (1998) is among the few studies in which a probabilistic model was proposed for the stress block parameters. Using this probabilistic model, they showed that for a ductile singly-reinforced rectangular section, the ultimate moment capacity is relatively insensitive to the stress block model. Estimates of the ductility level at the ultimate state and the column capacity in primary compression failure; however, are significantly affected by the choice of the stress block model.

This paper presents a statistical analysis on the concrete rectangular stress block parameters. With the aid of a very large and updated experimental database from over 200 tests on concrete specimens, the uncertainty involved in evaluating the equivalent rectangular stress block parameters is investigated. The effects of this uncertainty on the flexural strength and the ductility of singly-reinforced rectangular sections are then examined.

2. CONCRETE EQUAVALENT STRESS BLOCK PARAMETERS

The idea of using the equivalent rectangular stress distribution was first proposed by Emperger (1904) and then modified by Whitney (1937) for application to ultimate strength design and later experimentally verified by Hognestad et al. (1955) and Mattock et al. (1961). To obtain accurate as well as well-controlled data on flexure compression-loaded members, a test procedure for a series of experiments on C-shaped concrete specimens (see Fig. 1) subjected to axial load and bending moment was proposed by Hognestad et al. (1955) and later was used by several researchers. The position of neutral axis depth was kept fixed by continuously monitoring strains on one surface of the C-shaped specimen and adjusting the eccentricity of the applied force so that the strains on the neutral surface remain zero.

The actual stress distribution in the compression zone of concrete can be mathematically defined by three parameters k_1, k_2, and k_3 as shown in Figs. 1 and 2. These parameters are defined as follows: k_1 = the ratio of average compressive stress to the maximum compressive stress; k_2 = the ratio of the distance between the extreme fiber and the resultant of the compressive force to the neutral axis; and k_3 = the ratio of maximum compressive stress to the compressive strength of the concrete cylinder.

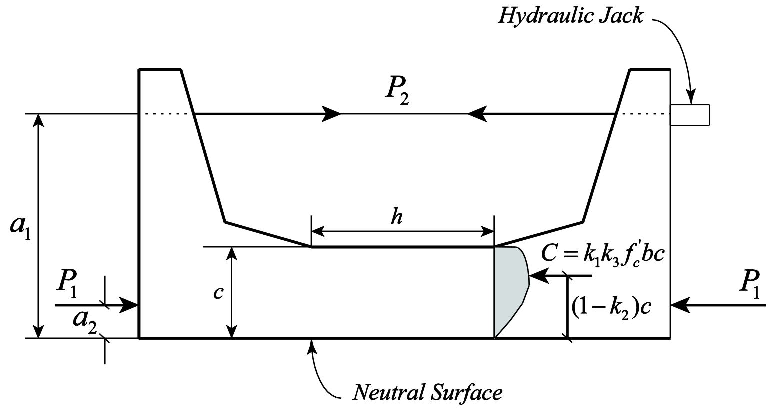


Fig. 1 Hognestad's test set-up for derivation of concrete stress block parameters

The three-parameter generalized stress block can be reduced to a two-parameter equivalent rectangular stress block, by keeping the resultant of the compressive force at the mid-depth of the assumed rectangular stress block. In order to determine flexural capacities, the magnitude k_1k_3 and position k_2 of the total compressive force are required. The rectangular stress block parameters, α_1 and β_1 are presented in Fig. 2

and can be defined as shown in Eq. (1).

$$\alpha_1 = \frac{k_1 k_3}{2k_2} \tag{1a}$$

$$\beta_1 = 2k_2 \tag{1b}$$

Some researchers used the experimental results from tests on reinforced concrete beams to derive the stress block parameters (Kahn et al. 1995; Mansur et al. 1997). Mansur et al. (1997) presented some simple equations for derivation of rectangular stress block from flexural tests on reinforced concrete beams. They concluded that the ACI equivalent stress block defined for normal concrete can be extended to high strength concrete.

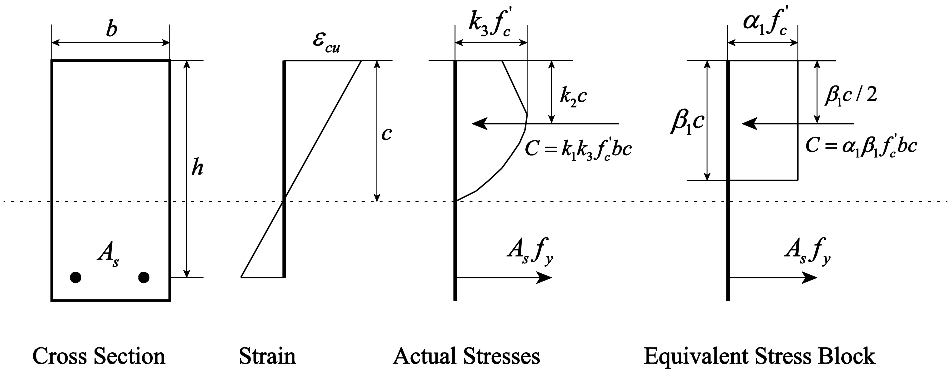


Fig. 2 Concrete stress distribution and the equivalent rectangular stress block

Ultimate concrete compressive strength is another important variable in the ultimate strength design. Although the ultimate flexural strength of reinforced concrete sections does not depend on this variable, it can noticeably affect the ultimate curvature of reinforced cross sections. Mattock et al. (1961) concluded that the value of 0.003 is a reasonably conservative value for ultimate strain of concrete. This value has been accepted by many design codes (NZS 3101 2006; ACI 318-08 2008; AS 3600 2009). Kahn et al. (1995) reported that the ultimate value of 0.003 is valid for concrete up to 102MPa and provided the best prediction of the ultimate moment. According to Mansur et al. study (1997), the maximum of 0.003 for concrete in compression may be extended to high strength concrete. Ibrahim and MacGregor (1996) results for ultimate concrete strain were considerably higher than the limiting value of 0.003. However, they concluded that based on the reported values in previous tests of C-shaped specimens, the value of 0.003 used by the ACI code, seems appropriate as a conservative lower bound of experimental data. One of the goals of this study is to derive a probabilistic model for the ultimate strain of concrete. This model is built based on extensive experimental data available from the current literature.

3. TEST DATA

In order to derive statistical models for the concrete rectangular stress block, the literature was surveyed extensively for experimental data on the concrete stress block parameters. When numerical values of strength were not tabulated in the original publications, approximate values were read from the published stress-strain curves. The data collected include test results from normal strength and high strength concrete, covering a wide range of concrete strengths. Table 1 shows the references and the number of collected data for each concrete stress block parameter.

Table 1 Summary of rectangular stress block parameters experimental data

Reference	$f_c^{'}$ range	Number of data points				
		k_1	k_2	k_3	k_1 k_3	ε_{cu}
Hognestad et al. (1955)	10-54	-	23	-	23	23
Rusch (1955)	5-72	-	8	-	8	8
Mattock et al. (1961)	18-60	-	13	-	13	13
Sargin et al. (1969)	28-32	-	-	3	-	3
Nedderman (1973)	80-100	-	9	-	9	9
Kaar et al. (1978)	25-100	34	34	34	34	34
Kaar et al. (1978)	20-50	3	3	3	3	3
Swartz et al. (1985)	50-80	8	8	8	8	8
Pastor (1986)	18-80	10	10	10	10	10
Schade (1992)	100-110	12	12	12	12	12
Ibrahim et al. (1996)	100-130	14	14	14	14	14
Mansur et al. (1997)	55-105	11	11	11	11	11
Yi et al. (Yi et al. 2002)	50-60	-	18	18	18	-
Tan et al. (Tan et al. 2005)	45-100	-	25	-	25	25
Mertol et al. (2008)	75-110	21	21	21	21	21
Ho et al. (2011)	25-50	-	-	-	-	8
Khadiranaikar et al. (2012)	60-130	33	33	33	33	33
$\sum$	**5-130**	**146**	**242**	**167**	**253**	**235**

As is shown in Table 1, a wide range of concrete compressive strength is covered in the gathered database. Furthermore, the database contains different types of concrete including lightweight and high performance concrete materials. In order to have consist ency amongst the gathered data, some unusually large or small specimen sizes were n ot considered in this study. The alternative could have been allowing for the specimen s ize effect as reported by Yi et al. (2002), but that approach was not chosen here for the sake of consistency. Some of the stress block parameters shown in Table 1 are derived using Hognestad's C-shape plain concrete bracket, while others are derived using tests on reinforced concrete beams.

4. STATISTICAL ANALYSIS

In the standard procedure of statistically analyzing the experimental data, first a model representing the average of the dataset is derived. Then, the disparity of the test points from that average model is assessed. For simplicity, in this study the values specified by ACI 318-11 design code for the stress block parameters are used as the predictive model. Eq. (2) summarizes the stress block parameters of the ACI 318-11 design code. These parameters originally were proposed by Honsestad et al. (1955) and until now have been used in the ACI code without any changes. The code-specified model is then compared with the collected test data. Because ACI code uses a two-parameter stress block in the design procedure using Eq. (1), the three parameter of the stress block (k_1, k_2, k_3) are converted to α_1 and β_1. Moreover, the statistical properties of k_3 parameter (the ratio of maximum compressive stress to the compressive strength of the concrete cylinder) are separately investigated. For this parameter, there is no specific code value, and the average will be compared to 1.0.

$$\alpha_1 = 0.85 \tag{2a}$$

$$0.65 \le \beta_1 = 1.05 - 0.00725 f_c^{'} \le 0.85 \tag{2b}$$

$$\varepsilon_{cu} = 0.0030 \tag{2c}$$

Fig. 3 illustrates variation of k_3 parameter with the concrete cylindrical strength. The evaluated mean and coefficient of variation of k_3 parameter are 0.995 and 0.095, respectively. The statistical correlation between concrete strength and k_3 parameter is found to be -0.27 which indicates that these variables are negatively correlated. Fig. 3 also shows that the 5^{th} and 95^{th} percentile levels of k_3 parameter are 0.85 and 1.17, respectively. Some design codes like the Australian code (AS 3600 2009) use the value of 0.90 for the k_3 factor. This value is not far from the 5^{th} percentile value obtained in this study.

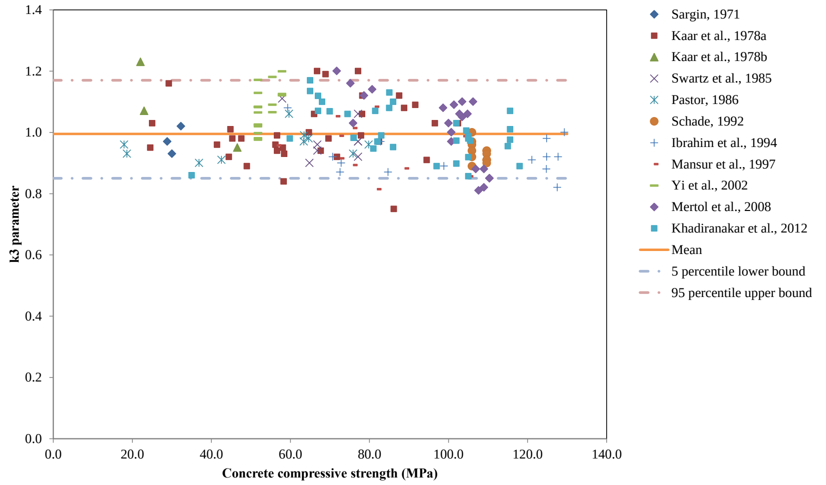
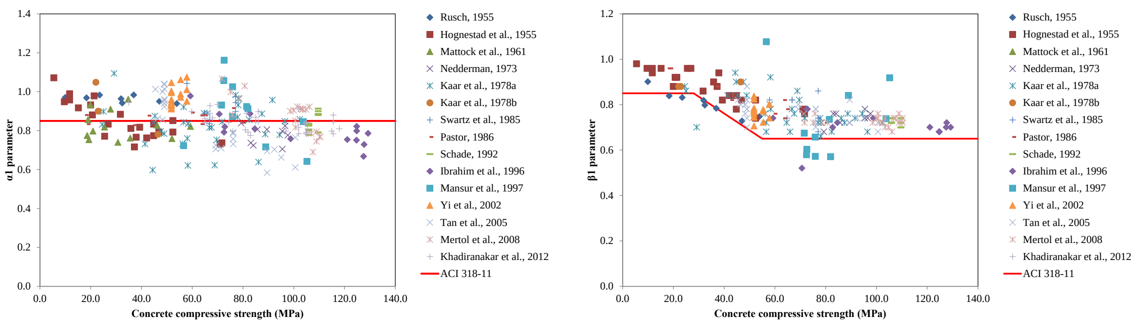


Fig. 3 Collected experimental results for k_3 parameter

Fig. 4 depicts variation of the two-parameter stress block parameters with respect to the concrete compressive strength. As is seen, there is considerable disparity amongst test data for the stress block parameters. Furthermore, the concrete compressive strength and the α_1 and β_1 parameters are negatively correlated. ACI code formula for β_1 is rightly reflecting this negative correlation; however, the code-specified value for α_1 is constant and does not show any negative correlation with the concrete compressive strength. Moreover, scatters shown in Fig. 4 demonstrate that the code-specified values for the concrete stress block parameters are not the best-fit values to the experimental data. In order to reduce the error in predicting the stress block parameters specifically for the high strength concrete, some researchers have suggested new equations that revise the code rectangular stress block (Ibrahim and MacGregor 1997; Mertol, Rizkalla et al. 2008). However, it should be mentioned that these modifications have been performed based on lower bound regression analysis and no probabilistic model fitting has been conducted.



(a) α_1 parameter (b) β_1 parameter

Fig. 4 Comparison of the ACI 318-11 stress block parameters with the experimental results

ACI code expression for β_1 parameter seems to be the lower bound of the experimental data. However, the value for α_1 parameters is close to the mean value and is not representing the lower bound value. It is worth mentioning that ideally the predictive model should capture the mean behavior of a phenomenon. The disparity of the actual data from the predictive model is called the model error associated with that model. Monti et al. (2009) have proposed a probabilistic approach for evaluating the predictive model. They have criticized the idea of using lower bound values for establishing any predictive model. According to their suggested proposal, a design model should be developed following the three following steps:

- Establishing a predictive model representing the average value of the considered phenomenon
- Finding the model error; the model error value is actually the ratio of the test to predicted value

- Present the model error by accounting for its probability distribution

In this study, it is assumed that the code-specified values for the stress block param eters could be used as predictive models for these parameters. Later on, in this study, t he model error associated with these models is statistically investigated.

The concrete crushing strain, ε_{cu}, at the extreme fiber that is obtained from the literat ure, expressed as a function of the concrete cylinder strength, is shown in Fig. 5. It can be observed that the results are scattered over a wide range. Some results are as low as 0.0020, whereas for the same concrete strength, some other researchers have obtai ned values as high as 0.005. The wide scatter of test data might be due to the difficulty in acquiring this information during a test. The measured values from the strain gauges only represent the strains before the concrete actually crushes. As the strain gauges ar e disturbed due to concrete distress, the real crushing strain might become much highe r than the previously recorded value.

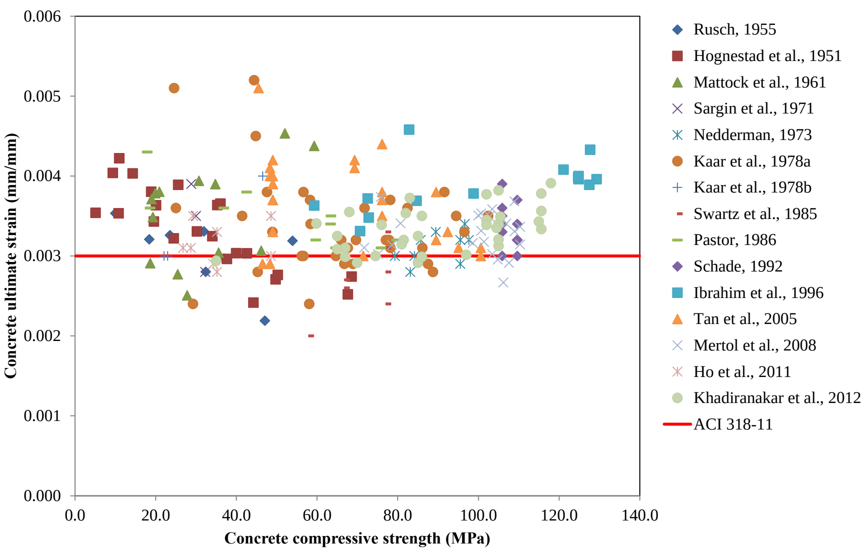


Fig. 5 Comparison of code specified value for ultimate concrete strength and experimental data

The model uncertainty is used to quantify the uncertainties associated with the assu mptions and simplifications used in the derivation of the theoretical model. The model u ncertainty associated with a particular mathematical model may be expressed in terms of the probabilistic distribution of the variable δ defined in Eq. (3).

$$\delta_M = \frac{Actual\ response}{\Pr edicted\ response} \tag{3}$$

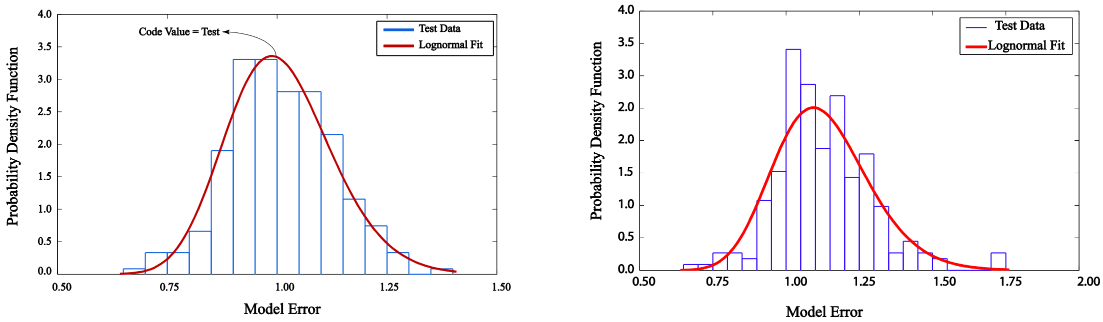
In this study, the procedure proposed by Monti et al. (2009) is used for the statistical analysis of the model error. However, as previously mentioned, instead of the average model, the code-specified model is used as the predictive model for each stress block p arameter. The mean and the coefficient of variation given in Table 2 are results of best-fit lognormal distribution for each stress block parameter. The correlation between the u ltimate concrete strength and each of the concrete stress block parameters are also sh own in Table 2. The mean and coefficient of variation values of model errors are very cl ose to the sample mean and coefficient of variation and this means that the lognormal distribution can well fit the experimental to code-specified ratios for the stress block par ameters. Result for the ultimate strain of concrete shows that the average strain of conc rete is about 1.13 × 0.003 = 0.0034. Some design codes such as the Canadian code (C SA A23.3-04 2004) use the ultimate strain of 0.0035 for concrete.

Table 2 Statistics obtained from stress block parameters model error

Parameter	α_1	β_1	ε_{cu}
Average	1.00	1.11	1.13
Coefficient of Variation	0.12	0.08	0.15
Correlation with $f_c^{'}$	-0.33	-0.65	-0.02

The results shown in Table 2 can be compared with those obtained by Attard and St ewart (1998). For example, they found the coefficient of variation for ultimate stain of co ncrete and k_2 (equivalent to β_1 parameter) parameters to be 0.19 and 0.03, respectivel y. It should be mentioned that they used different approach for statistical analysis of str ess block parameters and in their study they used fewer number of test data in compari son with the current research.

For further probabilistic investigation of stress block parameters, the best-fit lognorm al distributions for ultimate strain and α1 parameter are shown in Fig. 6. As the results i n Table 2 indicate, the scatter of ultimate strain data is more than that of α_1 parameter. According to the statistical results shown in Table 2, the bias factor of model error for ε_{cu} and β_1 variables is greater than 1.0. This shows that the ACI code-specified values f or these parameters are lower than the experimental results. However, for α_1, the code values are almost equal to the average of test data. Recently, researchers have sugges ted some recommendations for modifying code-specified values for α_1, especially for hi gh strength concrete (Mertol, Rizkalla et al. 2008). Unlike ACI, many other design code s reduce α_1 for high strength concrete (CSA A23.3-04 2004; EC2 2004). As previously discussed, establishing a lower bound expression for random variables like the stress b lock parameters is not an appropriate way to establish a predictive model, especially w hen these variables are used in a probabilistic procedure. The predictive model should represent the mean value of the experimental data, and the required safety measure sh ould be implemented separately in a broader probabilistic-based design procedure.



(a) α_1 parameter (b) ε_{cu} parameter

Fig. 6 Probability density function for α_1 and ε_{cu} parameters

5. EFFECT OF STRESS BLOCK UNCERTIANTY ON DUCTILITY AND STRENGTH

As in any structural analysis, the main variables are random in nature; the reinforced concrete section behavioral responses (the ultimate strength and curvature in this stud y) are probabilistic. In this paper, the ultimate strength (in terms of flexural capacity) an d ultimate curvature are derived from the section analysis. For a singly-reinforced concr ete beam section, the ultimate strength and curvature can be obtained using basic prin ciples of ultimate strength theory. Eq. (4) shows the resulted expressions for calculating the ultimate strength and curvature ductility of a singly-reinforced beam section. Statisti cal properties of these responses are investigated in this section.

$$M_u = A_s f_y \left(d - \frac{A_s f_y}{2\alpha_1 f_c^{'} b} \right) \tag{4a}$$

$$\varphi_u = \frac{\varepsilon_{cu}}{c} = \alpha_1 \beta_1 \frac{f_c^{'}}{f_y} \frac{b}{A_s} \varepsilon_{cu} \tag{4b}$$

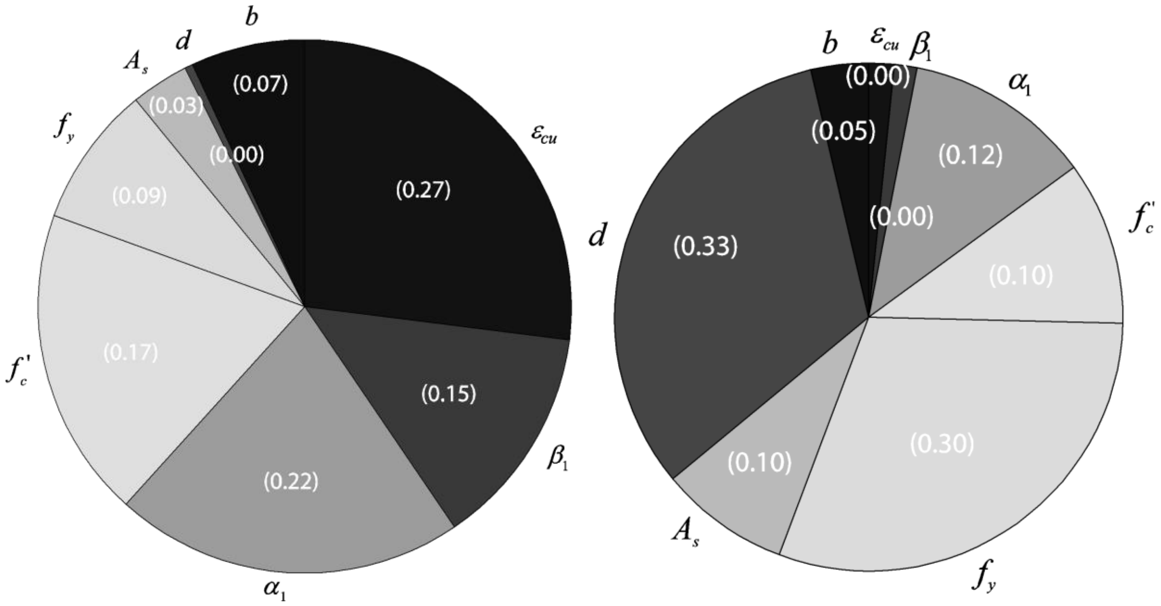
All dimensions used in Eq. 4 are shown in Fig. 2. Variables f_y and $f_c^{'}$ represent the steel yield stress and concrete ultimate strength, respectively. Eq. (4) shows that the ultimate curvature depends on all stress block parameters, while the ultimate strength only depends on one stress block parameter. Using sensitivity analysis, importance of each of the independent random variables in probabilistic behavior of ultimate strength and ductility is investigated. The statistical properties of basic random variables such as dimensions and material properties are taken from the available literature. Table 3 summarizes the probability models for all random variables affecting the ultimate strength and ductility. These properties are taken from Nowak et al. study (2003). Statistical properties of the stress block parameters resulted from the present study are shown in Table 2. The variable ρ in Table 3 represents the tensile rebar percentage. In this study, the rebar percentage is assumed to be half of the balance percentage.

Table 3 Statistical properties of the basic random variables

Variable	Nominal	Bias	COV
b	300 mm	1.01	0.04
d	1.5b	0.99	0.04
A_s	ρbd	1.00	0.015
f_y	420MPa	1.145	0.05
$f_c^{'}$	34MPa	1.10	0.10
α_1	ACI code	1.00	0.12
β_1	ACI code	1.11	0.08
ε_{cu}	ACI code	1.13	0.15

The distribution of ultimate strength and curvature can be simulated by means of Monte Carlo Simulation. Subsequently, the statistical property of system response can be derived based on the simulated results. In order to find the most important random variables affecting the ultimate strength and curvature, a probabilistic sensitivity analysis based on Pearson correlation coefficient is conducted. Two parameters have an influence on probabilistic sensitivities; the slope of the gradient and the width of the scatter of the random input variables. Furthermore, because the probabilistic sensitivities are based on Monte Carlo Simulation, any interaction among the input random variables will be correctly reflected in the probabilistic sensitivities.

There are 8 main random variables that affect the ultimate strength and tensile rebar stain. The pie chart in Fig. 7 presents the main random variables as well as normalized probabilistic sensitivity factors associated with each of these random variables. Results of the sensitivity analysis shown in Fig. 7 clearly indicate that the concrete stress block parameters ($\alpha_1, \beta_1, \varepsilon_{cu}$) have the biggest impact on the ultimate tensile rebar strain, while the ultimate strength is only sensitive to the α_1 parameter.



(a) Ultimate curvature (b) Ultimate strength

Fig. 7 Sensitivity of the ultimate strength and ductility to main random variables

In a code-based design procedure, the ultimate strain of concrete does not have any effect on the ultimate flexural strength. However, the maximum rebar area allowed by the design codes depends on this variable. By limiting the tensile rebar area, design codes try to provide minimum ductility in flexural design of reinforced cross sections (Kassoul et al. 2010). As Figure 7a clearly shows, the three stress block parameters ($\alpha_1, \beta_1, \varepsilon_{cu}$) have a great impact on the uncertainty of the ultimate curvature, while their effect on ultimate strength is very limited in comparison with contribution of other random variable. Therefore, in any reliability-based design procedure including ductility-based limit states, special attention should be paid to probabilistic models of stress block parameters. Currently, design code provisions for sectional ductility are not completely probabilistic and in any future reliability-based investigation of ductility limit states, statistical properties of stress block parameters can play an important role.

6. CONCLUSION

The statistics of the equivalent rectangular stress block parameters is studied in this research. Furthermore, effect of uncertainty in stress block parameters on ultimate stre ngth and curvature is investigated. Based on the results, the noticeable features of this research are summarized as follows:

- The rectangular stress block parameters obtained from a deterministic analysis (lower bound value based on regression analysis) differ from parameters derived from a probability-based procedure. Probabilistic models representing the model error associated with the current ACI 318 design code are presented in this study. These probabilistic models may be used for any future reliability analysis or code-calibration procedure.

- The statistical analysis of the test data for k_3 parameter (the ratio of maximum compressive stress to the compressive strength of the concrete cylinder) shows that the mean and coefficient of variation of this parameter are 0.995 and 0.095 respectively. The compressive strength of concrete and k_3 parameter are negatively correlated. This means that the higher is the concrete strength the lower would be the k_3 parameter.

- Statistical analysis of the stress block parameters shows that ACI design code provides a reasonable lower bound values for the concrete ultimate strain and β_1 parameter. The average of test data to code-specified values for these parameters is 1.11 and 1.13, respectively. On the other hand, the results show that for the α_1 parameter, the code-specified value almost represents the average of the test data. According to the collected data, there is no correlation between the concrete strength and the concrete ultimate strain. However, α_1 and β_1 parameters are negatively correlated to the concrete strength with the β_1 parameter showing higher correlation.
- Probabilistic sensitivity of the ultimate curvature (as an indicator of ductility) and the ultimate flexural strength of reinforced cross sections to the main independent random variables (sectional dimensions, material properties and the stress block parameters) are different. Ductility depends on all of the equivalent rectangular concrete stress block parameters, whereas strength is only dependent on one of the stress block parameters. This makes the ductility response more uncertain in comparison to the strength response.

- Deriving appropriate statistical models for the main random variables is an inevitable step in any probabilistic based code calibration. Unlike strength based limit states, for the section ductility, statistical properties of all concrete stress block parameters play an important role in any reliability analysis. Current literature does not adequately address these kinds of probabilistic models and special attention should be paid to this area.

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