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Asymptotic analysis of laminated composite plate including interlayer slips

*Jaehun Lee[1], Jun-Sik Kim[2] and Maenghyo Cho[3]

[1], [3] *Department of Mechanical and Aerospace Engineering, Seoul National University, Seoul 151-742, Korea*
[2] *Department of Intelligent Mechanical Engineering, Kumoh National Institute of Technology, Gumi 730-701, Korea*
[1] *spin9900@snu.ac.kr*

ABSTRACT

An asymptotic analysis is carried out for laminated composite plate including interlayer slips. Based on the formal asymptotic expansion of field variables and the scaling of the thickness coordinates, the recursive governing plate equations are systematically derived. Obtained solutions via the asymptotic expansion guarantee the exact solutions converging to the elasticity solutions. Taking the interlayer slip effect into account using through-the-thickness finite element method, calculated warping modes show the displacement jump at each interlayers. For numerical results, various layups including soft core sandwich plate are considered to demonstrate the validity of the present asymptotic approach.

1. INTRODUCTION

Composite materials have been widely used in various engineering fields due to their excellent properties; high strength-to-weight, stiffness-to-weight ratio and lightweight characteristics. Especially, composite laminates consist of reinforcing fiber and matrix are used as primary loading structures. Due to the anisotropy and inhomogeneity of the composite laminates, it is hard to obtain accurate static and dynamic behaviors of the multilayered structures. Thus, numerous studies have been carried out to analyze and design the laminated composite structures. Comprehensive reviews can be found in the surveys. (Reddy 1994, Carrera 2003)

Among the various approach, asymptotic method equipped with rigorous mathematical foundation is a proper option. Since the method can be mixed with the finite element analysis which is a very powerful tool for engineering approach, one can carry out an efficient and accurate analysis; the asymptotic analysis of composite laminates with finite element formulation can be found in Yu (2002) and Kim (2008, 2009), the former based on variational asymptotic method, and the latter used formal asymptotic method.

[1] Graduate Student
[2], [3] Professor

Difficult subject in analyzing composite laminates is a failure analysis. Due to the stress concentration between the layers, delamination is generated and propagates along the imperfect region of the multilayered structures. Thus, one needs to predict accurate behaviors of composite laminates including interfacial imperfections. Based on the higher-order plate theory, Cheng (1996) applied spring-layer model to represent the weakened interface effect. Also, recently, Kim (2011) developed enhanced first-order shear deformation model based on the spring-layer model. However, there is no research using an asymptotic approach to analyze the composite laminates with weakened interfaces. In this study, we developed an asymptotic model for the analysis of composite laminates including weakened interfaces. Different from the higher-order theories which assume the in-place displacement as a basis function, the asymptotic plate model has no assumption except the plate geometry: thinness of the structure. Thus, the present asymptotic approach can provide the mathematically rigorous analysis model for the prediction of the mechanical behaviors of multilayered structures with weakened interfaces.

2. ASYMPTOTIC FORMULATION

A laminated composite plate with weakened interfaces is shown in Fig. 1. Firstly, a small parameter is defined to be ‘h/a’, in which ‘h’ and ‘a’ represent the thickness and characteristic length of the plate, respectively. The coordinates are scaled to apply asymptotic expansion method as follows:

$$y_1 = x_1, \qquad y_2 = x_2, \qquad y_3 = x_3 / \epsilon \tag{1}$$

The scaled governing equations of linear elasticity are expressed by substituting Eq. (1) into the original field equations.

$$\begin{aligned}
&\sigma_{ij,j} + \tilde{b}_i = 0 & & \frac{1}{\epsilon}\mathrm{L}_3^t\sigma + \mathrm{L}_\alpha^t\sigma_{,\alpha} + \tilde{\mathbf{b}} = \mathbf{0} \\
&\sigma_{ij} = C_{ijkl}\varepsilon_{kl} & \rightarrow \quad & \sigma = \frac{1}{\epsilon}\mathbf{C}\mathrm{L}_3\mathbf{u}_{,3} + \mathbf{C}\mathrm{L}_\alpha\mathbf{u}_{,\alpha} \\
&\varepsilon_{kl} = \frac{1}{2}(u_{k,l} + u_{l,k}) & & \varepsilon = \frac{1}{\epsilon}\mathrm{L}_3\mathbf{u}_{,3} + \mathrm{L}_\alpha\mathbf{u}_{,\alpha}
\end{aligned} \tag{2}$$

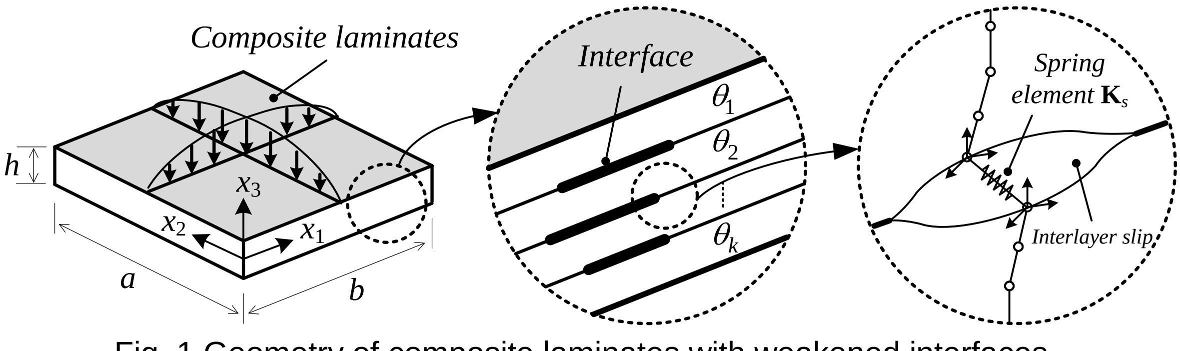

Fig. 1 Geometry of composite laminates with weakened interfaces

Displacement field, stress and strain are asymptotically expanded using small parameter. By substituting the expanded variables as shown in Eq. (3) into the field equation in Eq. (2), one can derive governing equations of the asymptotic expansion method. Details can be seen in the work of Kim (2009).

$$
\begin{aligned}
\mathbf{u}(\vec{x}) &= \mathbf{u}^{(0)}(y_\alpha) + \sum_{n=1}^{\infty} \epsilon^n \mathbf{u}^{(n)}(\vec{y}), \\
\varepsilon(\vec{x}) &= \varepsilon^{(0)}(\vec{y}) + \sum_{n=1}^{\infty} \epsilon^n \varepsilon^{(n)}(\vec{y}), \\
\sigma(\vec{x}) &= \sigma^{(0)}(\vec{y}) + \sum_{n=1}^{\infty} \epsilon^n \sigma^{(n)}(\vec{y})
\end{aligned}
\tag{3}
$$

For the microscopic analysis which is a calculation of through-the-thickness deformation modes, displacement field are decomposed into two parts: fundamental and warping solutions. The fundamental solution represents the global behavior of the plate, and the warping solution is a local deformation of the layered structure. Starting from the virtual work principle, the microscopic governing equations can be derived as follows:

$$
\begin{cases}
\int_h \delta(\mathbb{L}_3 \mathbf{u}_{w,3}^{(2)})^t \mathbf{C}(\Phi \mathbf{e}^{(1)} + \mathbb{L}_3 \mathbf{u}_{w,3}^{(2)}) dy_3 = 0 \\
\mathbf{u}_w(\vec{y}) = \mathbf{N}_u(y_3)\bar{\mathbf{u}}_w(y_\alpha)
\end{cases}
\rightarrow \ \mathbf{K}\bar{\mathbf{u}}_w^{(2)} + \mathbf{F}_{3E}\mathbf{e}^{(1)} = 0
\tag{4}
$$

where, the details of Eq. (4) are presented in the work of Kim (2009). In order to consider the interlayer slip effect, a spring element is introduced, and the microscopic finite element equations is changed as

$$
\mathbf{K}_s \bar{\mathbf{u}}_s^{(2)} = \mathbf{f}_s \ \ \rightarrow \ \ \mathbf{K}^* \bar{\mathbf{u}}_w^{(2)*} + \mathbf{F}_{3E}^* \mathbf{e}^{(1)} = 0
\tag{5}
$$

By solving Eq. (5), one can obtain the warping modes of the plate. The macroscopic governing equations are the same to those of the asymptotic plate model of the perfectly bonded plate. In this paper, deriving the macroscopic equations is omitted, and detailed procedures can be seen in the work of Kim (2009).

3. NUMERICAL RESULTS

To verify the proposed asymptotic plate model with weakened interfaces, we considered an anti-symmetric ([0/90/0/90]), relatively thick (S=4) and rectangular (b=a) composite laminate. The material properties are presented by

$$
E_L = 25E_T, \ \ G_{LT} = 2.5G_{TT}, \ \ \nu_{LT} = \nu_{TT} = 0.25
\tag{6}
$$

Double sinusoidal loadings are applied at the top surface of the composite plate, and

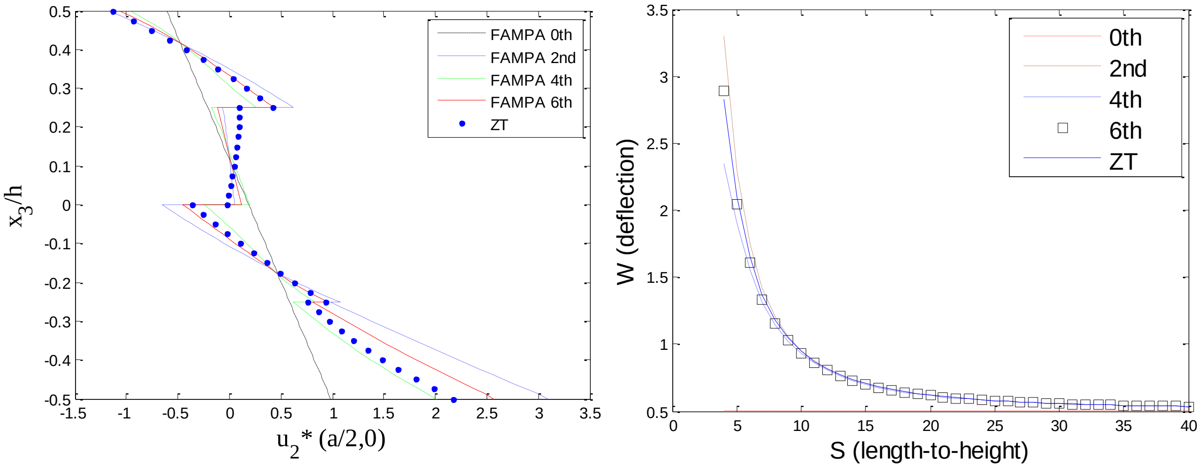

Fig. 2 In-plane displacement and center deflection of [0/90/0/90] laminate.

the corresponding boundary condition is simply-supported. As shown in Fig. 2, the in-plane displacement has a jump at the interface of the plate. FAMPA denotes the present formal asymptotic plate model, and ‘ZT’ represents higher-order zigzag theory. The asymptotic results shows a converging behavior as increasing the order of the solution. FAMPA 0th represent the classical Kirchhoff-Love plate model which cannot express the interlayer slip effect. The second graph in Fig. 2 shows the center deflection of the laminate. The present asymptotic solution correlates well with that of zigzag theory. The 6th solution of asymptotic method is in between 2nd and 4th solutions. Thus, one can expect that the present asymptotic results will be converge to the exact elasticity solution of weakened interface model as the case of perfectly bonded composite laminates (Kim 2009).

4. CONCLUSIONS

In this study, static behavior of composite laminates including weakened interfaces was investigated using formal asymptotic method. In the microscopic through-the-thickness analysis, the spring-layer model was easily applied to the one-dimensional finite element model, resulting the jump behavior of the warping distribution. The plate equation derived from the three-dimensional virtual work principle was exactly the same to that of the original perfectly bonded one. The present results were compared to those of higher-order zigzag model, and show the accurate prediction of the weakened interface behaviors. The major contribution of the present work is that the proposed method can be the benchmark solution to the multilayered structures like composite laminates and sandwich plate with weakened interfaces. Since there is no exact elasticity solution for the weakly bonded multilayered structures, the present asymptotic approach which guarantees the three-dimensional exact solution is an efficient and proper way to compare various kinds of analysis models.

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