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Improved visco-elastic analysis of laminated composite plates using an enhanced first-order deformation theory

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ABSTRACT

In this study, an efficient yet accurate method using an enhanced first-order shear deformation theory(EFSDT) is presented for the visco-elastic analysis of laminated composite plates. The main objective is to systematically modify the strain energy of first-order shear deformation theory(FSDT) based on a classical Reissner-Mindlin's plate theory. To this end, the in-plane warping functions based on the efficient higher-order plate theory(EHOPT) are synthesized into the FSDT to improve the performance. The relationships between the FSDT and EHOPT are systematically established via a strain energy transformation. The convolution theorem of Laplace transformation is employed to circumvent the overwhelming complexity of dealing with visco-elastic materials. The numerical results are compared to those available in literature.

1. INTRODUCTION

Recently, lightweight and high stiffness materials are demanded in various engineering applications. So, advanced structures made of laminated composite plates have been widely used in automobile, aerospace and many other branches of engineering industries due to their high stiffness to weight ratio.

With increasing utilization of laminated composite plates, numerous analysis models have been developed to accurately predict their static and dynamic responses. Starting with well-known conventional theories (CLPT; classical laminated plate theory, FSDT; first order shear deformation theory), many other refined higher order shear deformation theories developed in last three decades. However, most of them can not satisfy transverse shear stress conditions at surfaces and interfaces.

On the other hand, various zig-zag composite plate theories were also developed to improve their accuracy and efficiency. Among many proposed zig-zag theories, the efficient higher order plate theory (EHOPT) is known to be the best 5 D.O.F model because it can satisfy the transverse shear free conditions at top and bottom surfaces

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as well as the shear continuity conditions at interfaces (Cho 1993). However, EHOPT requires C^1 shape functions (slope continuity condition along the boundary of the element) for the finite element implementation, which result in heavy computational efforts.

The enhanced first order shear deformation theory (EFSDT), which only requires C^0-continuity in their finite element implementation, was developed in order to circumvent numerical issue of the EHOPT (Kim 2006). They systematically establish the relationships between the displacement fields of the EHOPT and FSDT via the strain energy transformation. In compliance with the relationships between them, one can come up with the FSDT-like theory. And their accuracy can be further improved by utilizing the recovery procedure.

Meanwhile, all of the mentioned theories have been analyzed linear elastic behavior of the composite structures. However, composite material is composed of elastic fibers and visco-elastic matrix which lead to visco-elastic behavior such as creep strain, stress relaxation and time-dependent failure. Thus, visco-elastic effects of the laminated composite plates should be considered for the reliable analysis. Several visco-elastic analysis for the dynamic response of the composite plates were performed in last three decades, and some researchers utilized the concept of the Laplace transformation for the visco-elastic analysis to solve the problem of time integral computational cost (Nguyen Sy, 2012).

In this paper, as a new way to address the aforementioned issues, EFSDT is applied to the visco-elastic problem and tested numerically. By employing Laplace transformation, time integrations of the Boltzmann superposition integral can be simplified as compared with elastic counterpart. In addition, the relationship between the two theories (EHOPT and FSDT) is systematically derived in the Laplace domain, so that one can come up with the theory incorporating the simplicity of FSDT as well as the accuracy of the EHOPT for the visco-elastic analysis. The numerical results obtained herein are compared to those available data in literature to demonstrate the accuracy and efficiency of the present theory.

2. MATHEMATICAL FORMULATION

In this paper, we consider laminated composite plates of thickness h, which are made of monoclinic material. Geometry and coordinates of the laminated composite plates is shown in Fig. 1. The reference plane of the laminated composite plates is referred as x_α, and the through-the-thickness position is denoted by x_3.

2.1 Constitutive equation for the visco-elastic material

Constitutive equation for the visco-elastic material is given by the form of Boltzmann superposition integral

$$\sigma_{ij}(t) = \int_0^t Q_{ijkl}(t-\tau)\dot{\varepsilon}_{kl}(\tau)d\tau, \quad \varepsilon_{ij}(t) = \int_0^t J_{ijkl}(t-\tau)\dot{\sigma}_{kl}(\tau)d\tau, \tag{1}$$

where t is time, and $Q_{ijkl}(t)$, $J_{ijkl}(t)$ represent the relaxation and compliance modulus.

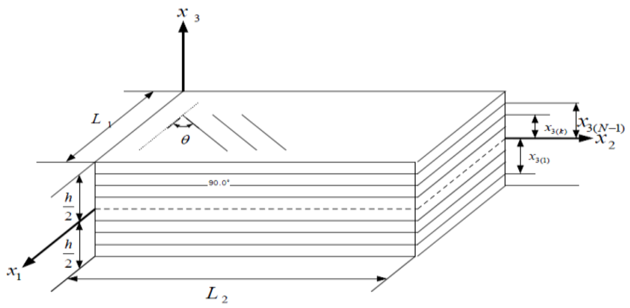

Fig. 1 Geometry and coordinates of the laminated composite plates

In this paper, Maxwell and Kelvin models are considered as the visco-elastic model. The relaxation modulus for the Maxwell model ($Q_{ijkl}^{M}(t)$) and the compliance modulus for the Kelvin model ($J_{ijkl}^{K}(t)$) can be expressed as follows:

$$Q_{ijkl}^{M}(t) = Q_{ijkl} e^{-a_M t}, \quad J_{ijkl}^{K}(t) = J_{ijkl}(1 - e^{-a_K t}). \tag{2}$$

By applying the convolution theorem of the Laplace transform, the constitutive equation for the visco-elastic material can be simplified as follows:

$$\sigma_{ij}^{*}(s) = sQ_{ijkl}^{*}(s)\varepsilon_{kl}^{*}(s), \quad \varepsilon_{ij}^{*}(s) = sJ_{ijkl}^{*}(s)\sigma_{kl}^{*}(s), \tag{3}$$

where superscript $(\ )^*$ represents the parameters in the Laplace domain.
Eq. (3) in Laplace domain is similar to those of the linear elastic constitutive equation which is based on Hook's law, so that computational efficiency can be improved as compared with the elastic counterpart.

2.1 Enhanced first order shear deformation theory for the visco-elastic material

In this section, an enhanced first order shear deformation theory for the visco-elastic analysis is derived by applying the strain energy transformation. The time-dependent displacement field of EHOPT can be expressed as

$$\begin{aligned} u_\alpha(x_i;t) &= u_\alpha^o(x_\beta;t) - u_{3,\alpha}^o(x_\beta;t)x_3 + \Phi_{\alpha\gamma}(x_3)\phi_\gamma(x_\beta;t), \\ u_3(x_i;t) &= u_3^o(x_\beta;t). \end{aligned} \tag{4}$$

And those of FSDT can be written as follows:

$$\begin{aligned} \bar{u}_\alpha(x_i;t) &= \bar{u}_\alpha^o(x_\beta;t) + \theta_\alpha(x_\beta;t)x_3, \\ \bar{u}_3(x_i;t) &= \bar{u}_3^o(x_\beta;t), \end{aligned} \tag{5}$$

where the superscript $(\ )^{o}$ denotes the variable at the reference plane, and $\Phi_{\alpha\gamma}(x_3)\phi_{\gamma}(x_{\beta};t)$ represents the through-the-thickness warping functions.

By applying the linearity of the Laplace transform, the displacement fields in Laplace domain can be given by

$$\begin{aligned} & u_{\alpha}^{*}(x_i;s) = u_{\alpha}^{o*}(x_{\beta};s) - u_{3,\alpha}^{o*}(x_{\beta};s)x_3 + \Phi_{\alpha\gamma}(x_3)\phi_{\gamma}^{*}(x_{\beta};s), \\ & u_3^{*}(x_i;s) = u_3^{o*}(x_{\beta};s). \end{aligned} \tag{6}$$

And

$$\begin{aligned} & \bar{u}_{\alpha}^{*}(x_i;s) = \bar{u}_{\alpha}^{o*}(x_{\beta};s) + \theta_{\alpha}^{*}(x_{\beta};s)x_3, \\ & \bar{u}_3^{*}(x_i;s) = \bar{u}_3^{o*}(x_{\beta};s). \end{aligned} \tag{7}$$

In order to derive the relationships between the two theories, the least-square approximation in the average sense is applied to the displacement fields of Eqs. (6) and (7). This least-square sense yields the following relationships

$$\bar{u}_{\alpha}^{o*} = u_{\alpha}^{o*} + C_{\alpha\gamma}\phi_{\gamma}^{*}, \quad \theta_{\alpha}^{*} + \bar{u}_3^{o*} = \bar{\gamma}_{3\alpha}^{*} = \Gamma_{\alpha\gamma}\phi_{\gamma}^{*}, \tag{8}$$

where

$$C_{\alpha\gamma} = \frac{1}{h}\left\langle \Phi_{\alpha\gamma} \right\rangle, \quad \Gamma_{\alpha\gamma} = \frac{12}{h^3}\left\langle x_3\Phi_{\alpha\gamma} \right\rangle, \quad \left\langle \cdot \right\rangle = \int_{-h/2}^{h/2} \cdot dx_3. \tag{9}$$

The visco-elastic strain energy expression for the EHOPT can be written in the compact form of

$$U_{EHOPT} = \mathop{\mathbf{I}}_{\eta}\left[\left\langle Q_{\alpha\beta\gamma\mu}^{*}(\eta)\varepsilon_{\gamma\mu}^{*}(\eta)\varepsilon_{\alpha\beta}^{*}(s-\eta) + Q_{3\alpha3\beta}^{*}(\eta)\gamma_{3\beta}^{*}(\eta)\gamma_{3\alpha}^{*}(s-\eta)\right\rangle\right], \tag{10}$$

where

$$\mathop{\mathbf{I}}_{\eta}\left[fun(\eta)\right] = \left(\frac{1}{s}\right)\frac{1}{2\pi i}\lim_{T\to\infty}\int_{c-iT}^{c+iT}\left[\eta(s-\eta)\cdot fun(\eta)\right]d\eta. \tag{11}$$

By using the relationships of Eq. (8), visco-elastic strain energy of the EHOPT can be transformed into that of the FSDT-like theory as follows:

$$U_{EHOPT} = U_{FSDT} + U_{Error}, \tag{12}$$

where U_{Error} represents the truncated strain energy, $C_{\alpha\gamma}$ and $\Gamma_{\alpha\gamma}$ can be determined by minimizing U_{Error} as close to be zero as possible.

After solving the problem with a FSDT-like theory (U_{FSDT}), the displacement field of EHOPT can be recovered in Laplace domain by substituting the relationships of Eq. (8) into the displacement field of Eq. (6) as:

$$\begin{aligned} u_\alpha^*(x_i;s) &= \bar{u}_\alpha^{o*}(x_\beta;s) - \bar{u}_{3,\alpha}^{o*}(x_\beta;s)x_3 + \left\{\Phi_{\alpha\gamma}(x_3) - C_{\alpha\gamma}\right\}\Gamma_{\gamma\mu}^{-1}\bar{\gamma}_{3\mu}^*(x_\beta;s), \\ u_3^*(x_i;s) &= \bar{u}_3^{o*}(x_\beta;s). \end{aligned} \tag{13}$$

Finally, by applying the inverse Laplace transform to the Eq. (13), the recovered displacement field in real time domain can be obtained as follows:

$$\begin{aligned} u_\alpha(x_i;t) &= \bar{u}_\alpha^{o}(x_\beta;t) - \bar{u}_{3,\alpha}^{o}(x_\beta;t)x_3 + \left\{\Phi_{\alpha\gamma}(x_3) - C_{\alpha\gamma}\right\}\Gamma_{\gamma\mu}^{-1}\bar{\gamma}_{3\mu}(x_\beta;t), \\ u_3(x_i;t) &= \bar{u}_3^{o}(x_\beta;t). \end{aligned} \tag{14}$$

3. NUMERICAL RESULTS AND DISCUSSION

To demonstrate the accuracy and efficiency of the EFSDT, cross-ply laminated composite rectangular plates are analyzed as numerical example. The material properties of each ply are given as

$$E_L / E_T = 25, \quad G_{LT} / E_T = 0.5, \quad G_{TT} / E_T = 0.2, \quad \nu_{LT} = \nu_{TT} = 0.25, \tag{15}$$

where L represents a parallel direction, T denotes perpendicular direction to the fiber. The visco-elastic coefficient for the Maxwell model (a_M) and Kelvin model (a_K) are assumed as

$$a_M = 0.01, \quad a_K = 1.00. \tag{16}$$

And the mechanical loading is prescribed on the top surface of the plates as follows:

$$p(t) = p_o H(t_0 - t), \tag{17}$$

where $H(t_0 - t)$ is the Heaviside unit step function which presents visco-elastic creep ($t_0 > t$) and recovery process ($t_0 < t$).

The time-dependent normalized in-plane displacements based on the EHOPT and FSDT are plotted in Fig. 2. Symmetric cross-ply lay-up for the Maxwell model (Fig. 2-a) and anti-symmetric cross-ply lay-up for the Kelvin model (Fig. 2-b) are considered. One can see that the in-plane displacements for the Maxwell model are over-estimated with

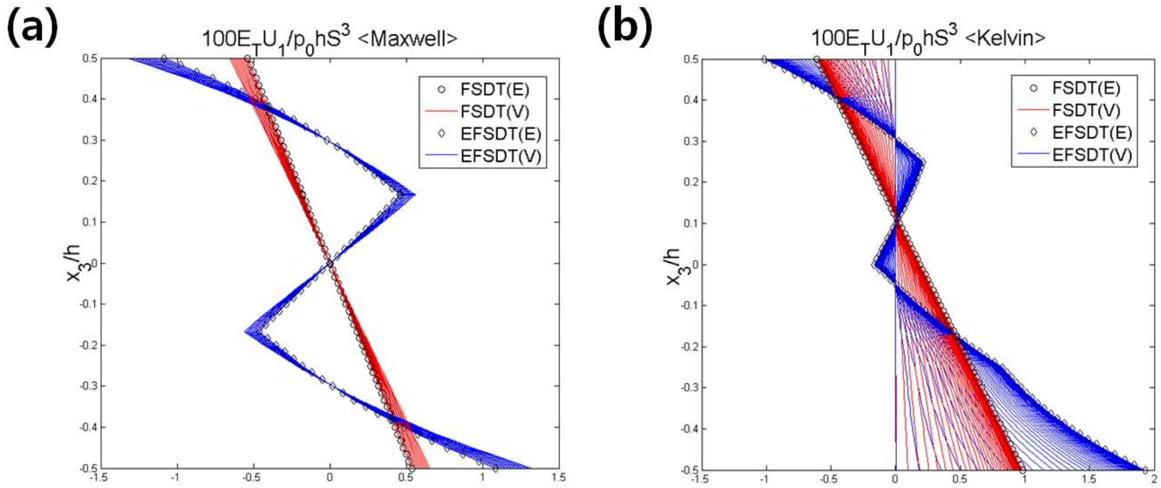

Fig. 2 In-plane displacements for the visco-elastic models

time as compared with those of elastic solution whereas the results of Kelvin model increase from zero value to the bound of the elastic solution. From the numerical results, it is noteworthy that the present theory provides reliable results as compared with those of FSDT by capturing severe zig-zag variation of in-plane displacements while it retains the same computational cost.

4. CONCLUSIONS

In this paper, an enhanced first-order shear deformation theory (EFSDT) in Laplace domain is proposed for the visco-elastic analysis of laminated composite plates. By applying Laplace transformation, complexity of the time integrations can be simplified as compared with elastic counterpart. The relationships between the FSDT and EHOPT are systematically derived by employing a strain energy transformation. And the accuracy can be further improved by utilizing the recovery procedure.

The numerical examples are demonstrated for the cross-ply laminated rectangular composite plates. Numerical results show that the present theory provides accurate results for the visco-elastic responses while it requires the same computational efforts as compared with FSDT.

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