The 2013 World Congress on
Advances in Structural Engineering and Mechanics (ASEM13)
Jeju, Korea, September 8-12, 2013

A nonlinear computational asymptotic formulation for slender anisotropic structures

*Jun-Sik Kim[1)]

[1)] *Department of Intelligent Mechanical Engineering, Kumoh National Institute of Technology, Gumi, Gyeongbuk 730-701, Korea*
[1)] *junsik.kim@kumoh.ac.kr*

ABSTRACT

Zeroth order asymptotic cross-sectional modes for nonlinear strains are calculated for composite beams. The modes have an impact on the macroscopic beam formulation of geometrically moderate nonlinear slender structures. The approach starts with three-dimensional elasticity by taking the geometric slenderness of beam structures. This allows us to systematically split the original 3D problem into microscopic cross-sectional analysis and macroscopic beam analysis. To this end, the cross-sectional coordinates are scaled by employing the small parameter ε which is also applied to the Green tensor. Thus one can obtain an asymptotically correct geometrically nonlinear beam formulation.

1. INTRODUCTION

An asymptotic method is a very power tool for the analysis of elastic bodies, which is especially useful when we do not know exact solutions (Kim et. al, 2008). Most asymptotic approaches have been made for linear elastic bodies including composite materials and smart materials. However applications to nonlinear anisotropic elastic bodies are rare in literature, because the asymptotic expansion for nonlinear problems could be very difficult.

In this paper, a computational asymptotic method is employed to overcome such a difficulty so that one can derive a consistent nonlinear composite beam model. Although many nonlinear composite beam models have been reported in literature, we do not know whether they are mathematically consistent or not. To this end, we start from the Green strain tensor and three-dimensional elasticity, and then, take the advantage of being geometrically slender. After complicate but straightforward manipulation, one can obtain the zeroth-order asymptotic sectional modes that will guide us to an asymptotically correct macroscopic beam formulation.

[1)] Professor

2. FORMULATION

To apply the asymptotic expansion method, one needs to scale the coordinate first.

$$x_1 = y_1,\ x_2 = \frac{y_2}{\varepsilon},\ x_3 = \frac{y_3}{\varepsilon}, \qquad \varepsilon \equiv \frac{h}{l_c} \tag{1}$$

where h is the maximum dimension of the cross-section of beams, and l_c is the characteristics length of the beam. In addition, the Green strain tensor is pre-scaled as follows:

$$e_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i} + \varepsilon u_{k,i} u_{k,j}\right) \tag{2}$$

which is reasonable because the nonlinear terms are relatively small. The virtual work principle is given as:

$$\int_V \sigma_{ij} \delta e_{ij} dV = 0 \tag{3}$$

Substituting Eqs. (1) and (2) into Eq. (3) yields the recursive form of the virtual work that can be separated into two problems such as microscopic problems (cross-sectional analysis) and macroscopic problems (one-dimensional beam formulation).

The first non-trivial warping functions for linear strains are shown in Fig. 1, where the four cross-sectional modes (tension, two moments, and torsion) are observed. These warping functions are smeared into the macroscopic beam stiffness that would be asymptotically correct up to the first order (classical beam model).

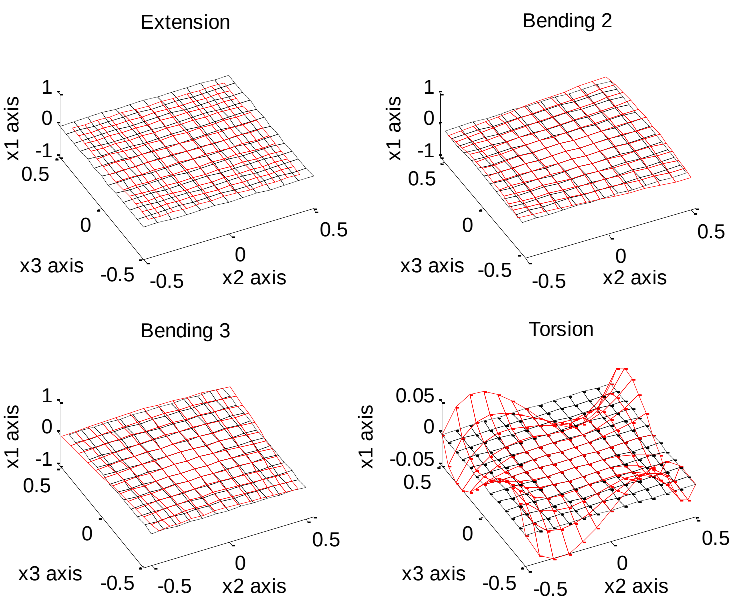

Fig. 1 Cross-sectional deformation modes of linear strains for the beam with solid rectangular cross-section

Unlike the warping functions for linear strains, those for nonlinear strains have six functions. The six warping functions corresponding to the six nonlinear strain components are illustrated in Fig. 2, where one can clearly see that the shear deformation presents due to the nonlinear deformation. This is of interest because the first order asymptotic expansion is expected to be a classical approximation (i.e., no shear deformation). Unlike the shear deformation appeared at the linear strain, the nonlinear shear warping is straight. This looks reasonable since the beam is supposed to be slender (the first order approximation).

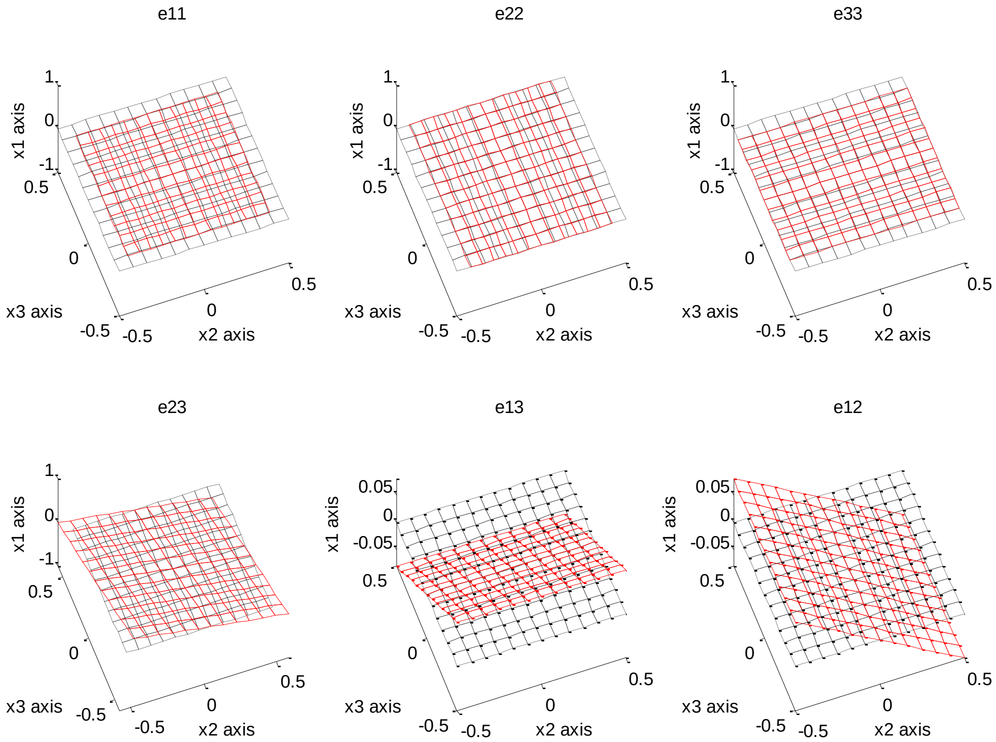

Fig. 2 Cross-sectional deformation modes of nonlinear strains for the beam with solid rectangular cross-section

REFERENCES

Kim, J.-S., Cho, M. and Smith, E.C. (2008) “An asymptotic analysis of composite beams with kinematically corrected end effects,” *Int. J. Solids Struct*., 45, 1954-1977.

Kim, J-S, Hill, J.R. and Wang, K. W. (2011) “An asymptotic approach for the analysis of piezoelectric fiber composite beams,” Smart Mater. Struct., 20(2), 025023:1-15.