Damage Analysis of Delaminated Smart Composite Structure Using Frequency Analysis

Bin Huang[1)] and *Heung Soo Kim[2)]

[1), 2)] *Department of Mechanical, Robotics and Energy Engineering, Dongguk University,*
Seoul, 100-715, Korea
[2)] *heungsoo@dgu.edu*

ABSTRACT

This paper investigates an analytical modeling in frequency analysis of smart composite structure with delamination. The modeling is based on an electrical-mechanical coupled improved layerwise theory and finite element method. The general modal reduction method is applied to solve the second order differential equation. It is expected that delamination not only weakens the strength of structure, but also affects dynamic characteristics. The results well proved that there exists significant shift of natural frequencies in frequency analysis of piezoelectric sensor output due to delamination. It is found that the delamination locations also influenced the natural frequencies of smart composite structure. Thus, the proposed methodology can be a useful tool in damage detections of smart composite structure.

1. INTRODUCTION

The engineering use of composite laminates has been growing over the last few decades. Due to their high stiffness-to-weight ratio and high strength-to-weight ratio, they have great potential in replacing the conventional metal and nonmetal materials. Since recent decades, smart composite laminate structures have been developed and widespread used in aircrafts, vehicle industries. These smart composite laminates inherit the both advantages of two materials, composite and smart materials. However, application of smart composite laminate is very limited to their complicated failure mechanism. The failures, such as crack and delamination, pre-existed during manufacturing or generated by external loads severely influence the service life of smart composite laminates.

Until now, numerous researches have been conducted in the analysis of composite laminates with delamination. It has been found that the existence of delamination not only weakens the strength of structure, but also influences the dynamic characteristics. Thus, it is possible to detect the delamination in composite

---

[1)] Graduate Student
[2)] Professor

laminates by analyzing the dynamic response of the structure. The mathematical modeling of composite laminate is normally based on the assumed displacement fields with additional terms, such as Heaviside unit step function with multiper, which are capable of explaining the discontinuity of displacement fields at the delaminated interface. Cho (1997) and Kim developed a higher-order zigzag theory for buckling and post buckling problem in composite plates and shells with multiple delaminations. Chattopadhyay (2003) developed a piezoelectric-mechanical coupled higher-order theory to address the transient response of piezoelectric composite laminates with through thickness delamination. Due to electrical and mechanical coupling nature, the coupling mechanism becomes more complicated. Kim and Ghoshal (2006) developed an improved layerwise theory to model smart composite plates with discrete delamination for dynamic analysis in both linear and nonlinear aspects.

In the present study, an improved layerwise theory with FEM implementation is developed for the frequency analysis of smart composite laminates with delamination. To investigate the dynamic response, one actuator and three sensors are surface bonded and electric harmonic loading is exerted to the actuator. It is expected to obtain delamination information from the frequency domain of tip displacement and sensor outputs. Finally, a numerical example using symmetric cross-ply laminate with single delamination will be given. The actuator locations and delamination locations are also investigated which probably influence the frequency response. It is expected that the proposed model would be efficient in predicting the delamination influence to the dynamic nature of smart composite laminates.

2. MATHEMATICAL MODELING

The mathematical modeling of smart composite laminates with delamination at ply interface adopts an improved layerwise displacement field, as shown in Eq. (1), proposed by Kim (2003).

$$U_1^k(x,y,z,t) = u_1 + A_1^k(z)\phi_1 + B_1^k(z)\phi_2 + C_1^k w_{,x} + D_1^k w_{,y} + \overline{E}_1^j \overline{w}_{,x}^j + \overline{F}_1^j \overline{w}_{,y}^j + \sum_{j=1}^{N-1} \overline{u}_1^j H(z - z_j)$$

$$U_2^k(x,y,z,t) = u_2 + A_2^k(z)\phi_1 + B_2^k(z)\phi_2 + C_2^k w_{,x} + D_2^k w_{,y} + \overline{E}_2^j \overline{w}_{,x}^j + \overline{F}_2^j \overline{w}_{,y}^j + \sum_{j=1}^{N-1} \overline{u}_2^j H(z - z_j)$$

$$U_3^k(x,y,z,t) = w(x,y,t) + \sum_{j=1}^{N-1} \overline{w}^j(x,y,t) H(z - z_j) \tag{1}$$

where A_i^k , B_i^k , C_i^k , D_i^k , $\overline{E}_i^j$ and $\overline{F}_i^j$ are layerwise coefficients and expressed in terms of geometry and material properties. $\overline{u}_i^j$ and $\overline{w}^j$ represent possible jumps in displacement fields caused by delamination allowing slipping and separation between sub-laminates, and z_j denotes the delaminated interface.

For electrical-mechanical coupling, the linear constitutive relation and higher order electric potential field are considered for piezoelectric material. And then finite element method is implemented with four-node plate element. Thus, the equations of motion are derived using variational principle as follows.

$$\delta\pi_u = -\int_{t_0}^{t}\left[\int_V(\rho\ddot{u}_i\delta u_i + \sigma_{ij}\delta\varepsilon_{ij} + \gamma\dot{u}_i\delta u_i)dV + \int_S t_i\delta u_i dS\right]dt = 0$$
$$\delta\pi_\phi = -\int_{t_0}^{t}\left[\int_V D_i\delta\phi_{,i}dV + \int_S q_e\delta\phi dS\right]dt = 0 \qquad (3)$$

where $\delta\pi_u$ and $\delta\pi_\phi$ denote the energy functionals of mechanical and electrical fields, respectively.

By using the variational principle, the equations of motion are obtained and written in the matrix form as follows.

$$[M]\{\ddot{X}\} + [C]\{\dot{X}\} + [K]\{X\} = [F] \qquad (4)$$

To obtain the frequency response of a finite element-based governing equation, the first step is conventionally to conduct the modal reduction. After modal reduction, mass, damping and stiffness matrices become diagonal matrices, and $\left[\bar{M}\right]$ becomes an identity matrix, $\left[\bar{C}\right] = diag\left[2\xi_1\omega_1 \quad 2\xi_2\omega_2 \quad \ldots \quad 2\xi_n\omega_n\right]$, $\left[\bar{K}\right] = diag\left[\omega_1^2 \quad \omega_2^2 \quad \ldots \quad \omega_n^2\right]$.

Therefore, the modal output vector $\{\eta\}$ is decoupled and for a single-degree-of-freedom system, the harmonic response can be obtained as follows.

$$\eta_i(t) = \frac{\left|\bar{f}_i\right| / \omega_i^2}{\left\{\left(1-\left(\omega/\omega_i\right)^2\right)^2 + \left(2\xi_i\,\omega/\omega_i\right)^2\right\}^{1/2}}\cos(\omega t - \phi), \quad \phi_i = \tan^{-1}\left(\frac{2\xi_i\,\omega/\omega_i}{1-\left(\omega/\omega_i\right)^2}\right) \qquad (5)$$

where the magnitude of η_i represents the amplitude of frequency response of *i*th modal and ϕ_i is the phase delay.

Therefore, the displacement output of a multi-degree-of-freedom system can be obtained by superposing each modal output and the frequency response can be obtained.

3. NUMERICAL RESULTS

The numerical example adopts a symmetrically layered cross ply laminate ($[0/90]_{4s}$) with one actuator and three sensors surface bonded, as shown in Fig. 1. The finite element mesh consists of 60×10 elements in length and width which is sufficient to obtain accurate response. Numerical results are obtained by 100V sinusoidal load. The material properties of composite laminate are given below.

E_1 = 372GPa, E_2 = 4.12GPa, G_{12} = 3.99GPa, v_{12} = 0.275, v_{23} = 0.42, ρ = 1788.5kg/m^3

The actuator and sensors have same mechanical and electrical properties using PZT-5H and material properties are given below.

E = 62 GPa, ρ = 7500kg/m^3, v = 0.31, d_{31} = 274x10^{-12}m/V, b_{11} = 14.41nf/m

To study the influence of delamination, there delamination locations are considered and marked as D0, D3, D6, which stand for the delamination location at the midplane, the third interface from the midplane and the sixth interface from the midplane, respectively.

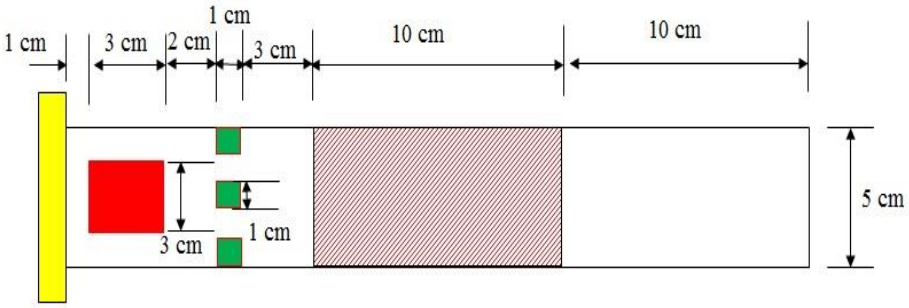

Fig. 1 Geometry configuration of smart composite laminate with single delamination at layer interface

Fig. 2 and Fig. 3 show the frequency response curves of tip displacement and sensor one output for healthy and delaminated laminates. As expected, delamination influences the dynamic nature of structure. The existence of delamination probably reduces the stiffness of laminate and cause possible change of natural frequencies in turn. This phenomenon can be clearly found in Fig. 2 and Fig. 3. In the tip displacement response curve, it presents frequency shift in each natural frequency. Small shift of natural frequencies is observed at the low natural frequencies, while at higher natural frequencies, large shift of natural frequencies is observed. Thus, the existence of delamination significantly influences the higher natural frequencies of laminate. Moreover, for three delamination locations, D0 case presents largest reduction of natural frequencies compared with D3 and D6 cases, which implies the largest damage effect to the natural frequencies.

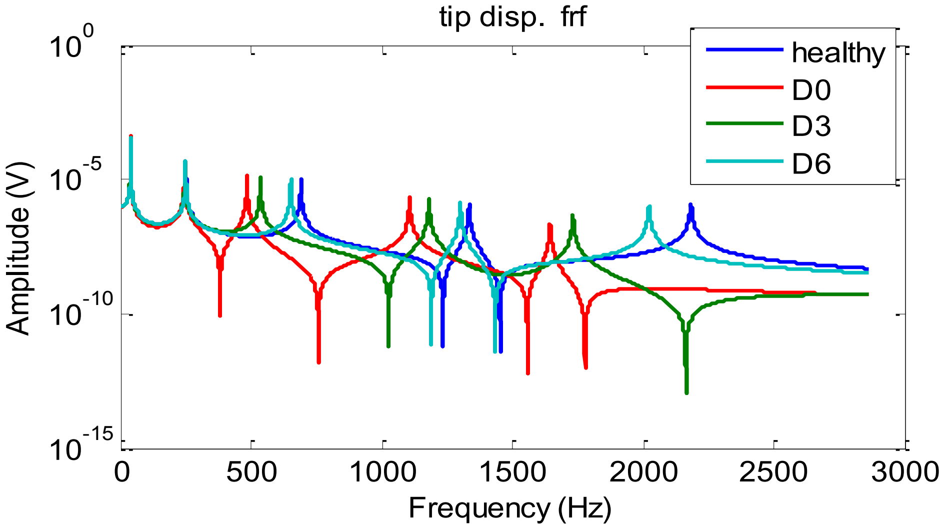

Fig. 2 Frequency response of tip displacement for healthy and delaminated laminates

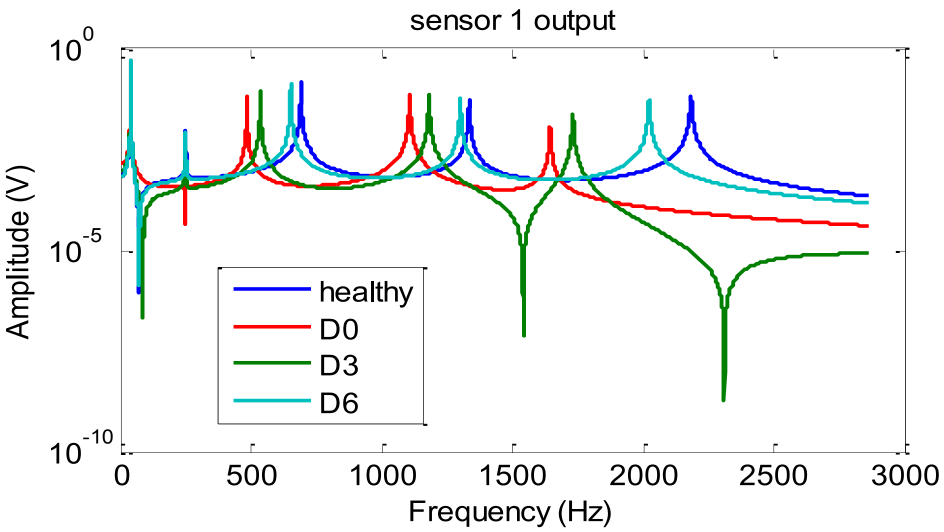

Fig. 3 Frequency response of sensor one output for healthy and delaminated laminates

4. CONCLUSIONS

This paper proposed an analytical approach to investigate the frequency response of smart composite laminate with single delamination at ply interface. This approach well predicted the delamination effect to the dynamic characteristics of smart composite laminates. It was found that the midplane delamination has largest effect to the shift of natural frequencies, especially to the higher natural frequencies. Although the proposed approach is not capable of predicting the delamination location and size, it is possible to conduct postprocesses by using intelligent inverse algorithm such as Neural Networks, Genetic Algorithm and other system identification methods to identify the severity of damage. Thus, the proposed method presents its bright potential in structural health monitoring of delaminated smart composite laminates.

ACKNOWLEDGEMENT: This research was supported by Basic Science Research Program through the National Research Foundation of Korea (NRF) funded by the Ministry of Education, Science and Technology (NRF-2011-0021720).

REFERENCES

Chattopadhyay, A., Thornburgh, R. and Ghoshal, A. (2003), "Transient analysis of smart structures using a coupled piezoelectric-mechanical theory." *Proceedings of the IUTAM Symposium,* Japan.

Cho, M. and Kim, J. S. (1997), "Bifurcation buckling analysis of delaminated composites using global-local approach." *AIAA J.*, **35**(10), 1673-1676.

Kim, H. S., Chattopadhyay, A. and Ghoshal, A. (2003), "Dynamic analysis of composite laminates with multiple delamination using improved layerwise theory." *AIAA J.*, **41**(9), 1771-1779.

Kim, H. S., Ghoshal, A., Kim, J. and Choi, S. B. (2006), "Transient analysis of delaminated smart composite structures by incorporating the fermi-dirac distribution function." *Smart Mater. Struct.*, **15**(2), 221-231.