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Modal Response Characteristics of Seismic Controlled Multi-Story Shear Building Using Apparent Mass Dampers

*Kohju Ikago[1)]

[1)] *International Research Institute of Disaster Science, Tohoku University, Aramaki-aza, Aoba 6-6, Sendai, Japan*
[1)] ikago@irides.tohoku.ac.jp

ABSTRACT

There has been considerable research on the use of an apparent mass device that generates inertia force proportional to the relative acceleration between two nodes; this device is referred to as an inerter or dynamic mass.

The authors have developed a seismic control system by arranging viscous damping elements in parallel and supporting spring elements in series with an apparent mass to obtain a system similar to a tuned mass damper or dynamic vibration absorber. This seismic control system is known as a tuned viscous mass damper (TVMD) system.

The modal response characteristics of a TVMD system are mathematically described in this paper. The TVMD system developed in this research uses secondary apparent masses arranged such that their distribution is proportional to that of the primary stiffness of a structure. The fundamental modes of the undamped primary system are preserved upon adding a secondary system in the seismic control system. In addition, the participation mode vectors of a multiple degrees-of-freedom TVMD controlled system can be obtained by combining the participation mode vectors of an uncontrolled primary system with those of a reduced two-degrees-of-freedom controlled system. The results of this study aim to help practicing structural designers to understand the modal response characteristics of a seismic control system.

1. INTRODUCTION

Recently, in Japan, apparent mass dampers with a ball-screw amplifying mechanism have been effectively applied as seismic control devices for high-rise buildings. The inertial force of the mass damper is generated by an apparent mass that is dependent on inter-story relative acceleration. Translational motion input to the apparent mass damper is converted into high-speed rotational motion using a damper

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flywheel to generate a large inertial resistance torque. The torque is then amplified again when converted back to a translational force. Thus, a large apparent mass (about several thousand times as large as the actual mass) is produced by the ball-screw amplifying mechanism. By arranging viscous damping elements in parallel and supporting spring elements in series with the apparent mass, a system similar to a tuned mass damper (TMD) or a dynamic vibration absorber (DVA) is obtained. In this system, referred to as a tuned viscous mass damper (TVMD) system, the deformation of the viscous elements is amplified by a secondary vibration system consisting of an apparent mass and its supporting spring to provide large energy dissipation. Saito *et al*. (2008) noted that when the secondary apparent mass distribution is proportional to the distribution of the primary stiffness in a TVMD system, the primary modes of the undamped system are preserved. However, a theoretical explanation of this phenomenon has not been provided.

One of the most important steps in the design of a structure is to understand its modal response characteristics. Unfortunately, recent advancements in computational technology have obscured the importance of this design aspect. Particularly in Japan, most practicing structural designers prefer direct-integration time-history analysis for examining the response characteristics of building structures.

In this paper, we discuss a TVMD seismic control system in which the secondary apparent masses are arranged such that their distribution is proportional to that of the primary stiffness. We found that this configuration preserves the fundamental modes of the undamped primary system upon adding the secondary system. This suggests that an accurate approximation of the maximum seismic response of a TVMD system can be obtained using the undamped primary modes obtained through a spectrum modal analysis.

2. APPARENT MASS DAMPER

2.1 Tuned Mass Damper

Although McNamara (1979) proved that TMD systems are effective for reducing wind-induced vibrations, Kaynia *et al*. (1981) stated that they are ineffective for seismic vibrations. The maximum value of the additional mass ratio in the study conducted by Kaynia *et al*. was 0.02. Larger additional mass ratios are unrealistic because of the large effective mass of a building as the primary system. However, it can be expected that a TMD with a large enough apparent mass produced by a mass amplifying mechanism could effectively dampen the effects of seismic vibrations on a building.

2.2 Amplified Apparent Mass Obtained by a Ball-screw Mechanism

Sone *et al*. (1998) proposed a concept for obtaining a large apparent mass using a combination of a pendulum and lever. Arakaki *et al*. (1999a, 1999b) successfully developed a viscous damper with a large apparent mass using a flywheel and a ball-screw amplifying mechanism.

The damper developed by Arakaki *et al*. was subsequently improved and implemented as an actual damper called as a rotary damping tube (RDT) (Fig. 1). Although the device has been installed in many buildings in Japan, the small apparent mass effect in the RDT device was not utilized. The RDT subsequently was redesigned into a more efficient apparent mass damper called as an inertial rotary damping tube (*i*RDT) (Fig. 2). In an *i*RDT, an external cylinder is used as a flywheel to produce a large inertia force, whereas the internal cylinder with a smaller diameter produces a relatively small inertia force.

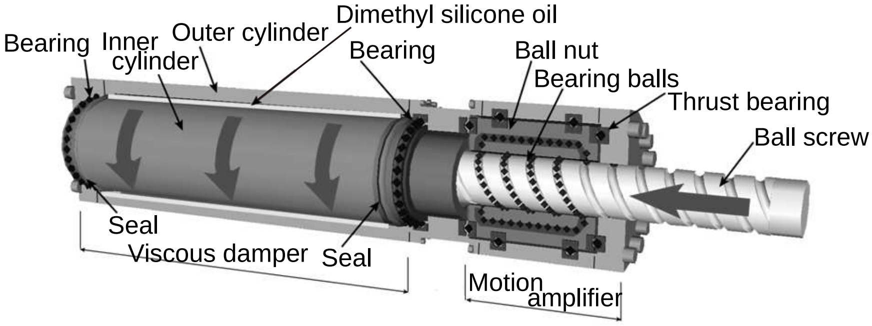


Fig. 1 Rotary damping tube (RDT)

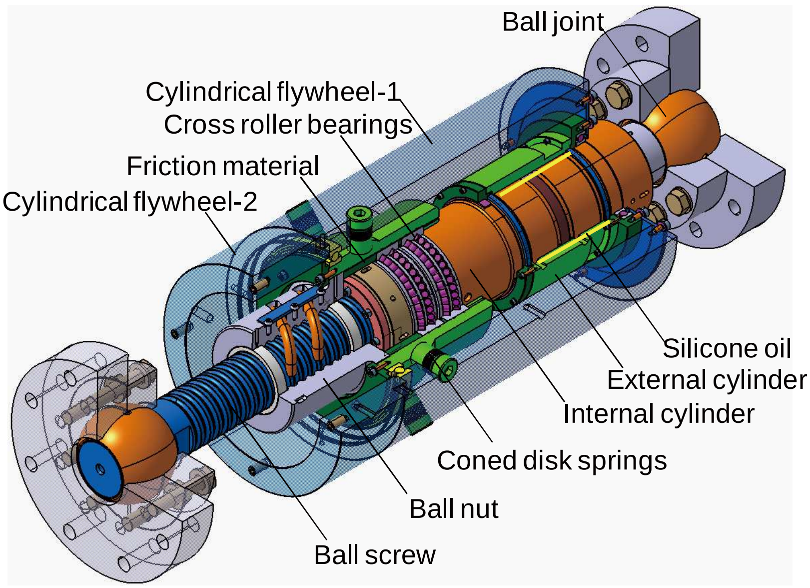


Fig. 2 Inertial rotary damping tube (*i*RDT)

3. CONTROL OF NATURAL FREQUENCY USING APPARENT MASS

Multiple research campaigns have investigated the usefulness of an inertia force generated by the relative acceleration between two nodes. Smith (2002) defined an inerter as a two-node mechanical device that generates an inertial force proportional to the relative acceleration between two nodes. The basic concept of a dynamic mass defined by Furuhashi and Ishimaru (2008) is the same. In this paper, such a device is referred to as an apparent mass.

Fig. 3 shows an n-story shear building incorporating apparent mass devices. The terms m_j and k_j are the mass and stiffness of the jth story of the primary structure, respectively. x_j is the displacement of the jth floor relative to the ground. m_{dj} and x_{dj} are the apparent mass incorporated into the jth story to generate an inertial force proportional to the relative acceleration between the adjacent stories and the inter-story drift of the jth story, respectively.

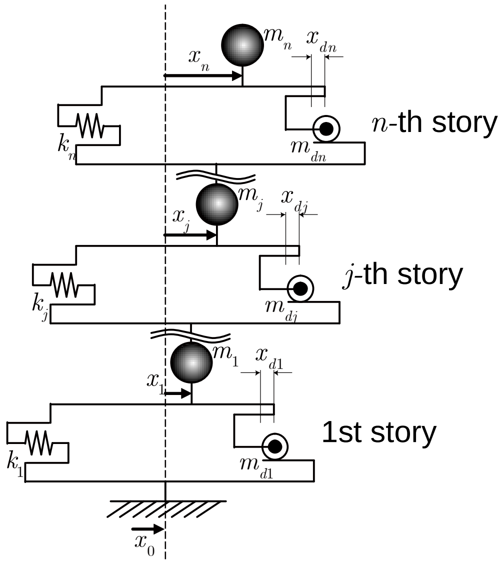


Fig. 3 n-story shear building containing apparent masses

The equation of motion of the structure (Furuhashi and Ishimaru 2008) is

$$\mathbf{M}\ddot{\boldsymbol{x}} + \mathbf{K}_{\mathrm{P}}\boldsymbol{x} = -\mathbf{M}_{\mathrm{P}}\boldsymbol{r}\ddot{x}_0 \tag{1}$$

where

$$\mathbf{M} = \mathbf{M}_{\mathrm{P}} + \mathbf{M}'_{\mathrm{D}},\ \boldsymbol{x} = \{x_1, x_2, \cdots, x_n\}^T,\ \boldsymbol{r} = \{1, 1, \cdots, 1\}^T \tag{2}$$

$$\mathbf{M}_{\mathrm{P}} = \begin{bmatrix} m_1 & 0 & \cdots & & 0 \\ 0 & m_2 & & & \vdots \\ \vdots & & \ddots & & \\ & & & & 0 \\ 0 & \cdots & & 0 & m_n \end{bmatrix} \tag{3}$$

$$\mathbf{M}'_{\mathrm{D}} = \begin{bmatrix} m_{d1} + m_{d2} & -m_{d2} & \cdots & & 0 \\ -m_{d2} & m_{d2} + m_{d3} & & & \vdots \\ \vdots & \ddots & \ddots & \ddots & 0 \\ & & -m_{dn-1} & m_{n-1} + m_{dn} & -m_{dn} \\ 0 & \cdots & 0 & -m_{dn} & m_{dn} \end{bmatrix} \tag{4}$$

$$\mathbf{K}_{\mathrm{P}} = \begin{bmatrix} k_1 + k_2 & -k_2 & \cdots & 0 \\ -k_2 & k_2 + k_3 & -k_3 & \vdots \\ 0 & & \ddots & 0 \\ \vdots & -k_{n-1} & k_{n-1} + k_n & -k_n \\ 0 & \cdots & -k_n & k_n \end{bmatrix} \tag{5}$$

Furuhashi and Ishimaru (2008) pointed out that the eigenvectors of this system are identical to those of the undamped primary system if the apparent masses m_{dj} are arranged such that they are proportional to the primary stiffness distribution k_j, that is,

$$m_{dj} = \alpha k_j \quad (j = 1, 2, \cdots, n) \tag{6}$$

Let ${}_k\boldsymbol{u}$ be the kth eigenvector of the undamped primary system. Then, the following equation holds:

$$\mathbf{K}_{\mathrm{P}\,k}\boldsymbol{u} = {}_k\lambda \mathbf{M}_{\mathrm{P}\,k}\boldsymbol{u} \tag{7}$$

where the eigenvalue ${}_k\lambda$ equals the square of the circular natural frequency for the kth mode ${}_k\omega$.

$${}_k\lambda = {}_k\omega^2 \tag{8}$$

Because the matrices $\mathbf{M}_{\mathrm{P}}$ and $\mathbf{K}_{\mathrm{P}}$ are symmetrical, the eigenvectors are orthogonal.

$${}_k\boldsymbol{u}^T \mathbf{M}_{\mathrm{P}\,s}\boldsymbol{u} = 0,\ {}_k\boldsymbol{u}^T \mathbf{K}_{\mathrm{P}\,s}\boldsymbol{u} = 0 \quad (k \neq s) \tag{9}$$

The quadratic forms of the mass and stiffness matrices give the eigenvalue

$${}_k\lambda = {}_k\omega^2 = \frac{{}_k\boldsymbol{u}^T \mathbf{K}_{\mathrm{P}\,k}\boldsymbol{u}}{{}_k\boldsymbol{u}^T \mathbf{M}_{\mathrm{P}\,k}\boldsymbol{u}} \tag{10}$$

The orthogonality of the eigenvectors for the undamped primary system ${}_k\boldsymbol{u}$ also holds for the mass matrix of the system containing apparent masses.

$${}_k\boldsymbol{u}^T \mathbf{M}\,{}_s\boldsymbol{u} = {}_k\boldsymbol{u}^T (\mathbf{M}_{\mathrm{P}} + \mathbf{M}'_{\mathrm{D}})\,{}_s\boldsymbol{u} = {}_k\boldsymbol{u}^T \mathbf{M}_{\mathrm{P}\,s}\boldsymbol{u} + \alpha \cdot {}_k\boldsymbol{u}^T \mathbf{K}_{\mathrm{P}\,s}\boldsymbol{u} = 0 \quad (k \neq s) \tag{11}$$

Thus, incorporating the apparent masses $m_{dj} = \alpha k_j$ into the structure preserves the original modes of the undamped system.

The k th circular frequency for the system containing apparent masses ${}_k\hat{\omega}$ is

$${}_k\hat{\omega}^2 = \frac{{}_k\boldsymbol{u}^T \mathbf{K}_{\mathrm{P}\,k}\boldsymbol{u}}{{}_k\boldsymbol{u}^T \mathbf{M}_k \boldsymbol{u}} = \frac{{}_k\boldsymbol{u}^T \mathbf{K}_{\mathrm{P}\,k}\boldsymbol{u}}{{}_k\boldsymbol{u}^T \mathbf{M}_{\mathrm{P}\,k}\boldsymbol{u} + \alpha \cdot {}_k\boldsymbol{u}^T \mathbf{K}_{\mathrm{P}\,k}\boldsymbol{u}} = \frac{{}_k\omega^2}{1+\alpha \cdot {}_k\omega^2} \tag{12}$$

Thus, incorporating the apparent masses $m_{dj} = \alpha k_j$ decreases the k th natural frequency by

$$\sqrt{\frac{1}{1+\alpha \cdot {}_k\omega^2}} \tag{13}$$

4. SHEAR BUILDINGS CONTAINING TVMD

Saito *et al*. (2008) proposed a TMD-like seismic control device constructed by attaching viscous elements in parallel and supporting springs in series with an apparent mass. Ikago, Saito, and Inoue (2012) named the device a tuned viscous mass damper and compared its seismic control performance with a viscous damper and viscous mass damper to show its advantages.

This section describes the modal response characteristics of TVMD seismic control systems.

4.1 Single Degree-of-Freedom (SDOF) Shear Building Containing TVMD

Fig. 4 shows an analytical model for a shear building containing a TVMD.

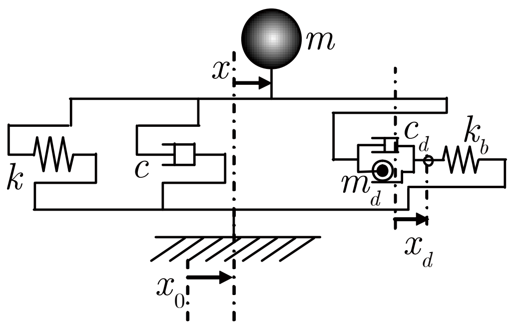


Fig. 4 SDOF shear building containing TVMD

The terms m, c, and k are the mass, damping coefficient, and stiffness of primary system, respectively. The terms m_d, c_d, and k_b are the apparent mass, damping coefficient, and supporting spring stiffness of the secondary system, respectively. The terms x, x_0, and x_d represent the primary displacement relative to the ground, ground displacement, and secondary mass displacement, respectively.

The equation of motion for this system is

$$\begin{bmatrix} m & 0 \\ 0 & m_d \end{bmatrix}\begin{Bmatrix} \ddot{x} \\ \ddot{x}_d \end{Bmatrix} + \begin{bmatrix} c & 0 \\ 0 & c_d \end{bmatrix}\begin{Bmatrix} \dot{x} \\ \dot{x}_d \end{Bmatrix} + \begin{bmatrix} k+k_d & -k_d \\ -k_d & k_d \end{bmatrix}\begin{Bmatrix} x \\ x_d \end{Bmatrix} = -\begin{bmatrix} m & 0 \\ 0 & m_d \end{bmatrix}\begin{Bmatrix} 1 \\ 0 \end{Bmatrix}\ddot{x}_g \tag{14}$$

Dividing both sides of Eq.(14) by m yields

$$\begin{aligned} \ddot{x} + 2h\omega_0\dot{x} + (1+\eta)\omega_0^2 x - \eta\omega_0^2 x_d &= -\ddot{x}_0 \\ \mu\ddot{x}_d + 2\mu\beta h_d\omega_0\dot{x}_d - \eta\omega_0^2 x + \eta\omega_0^2 x_d &= 0 \end{aligned} \tag{15}$$

where

$h = \dfrac{c}{2\sqrt{mk}}$, $\omega_0 = \sqrt{\dfrac{k}{m}}$, $\mu = \dfrac{m_d}{m}$, $\beta = \dfrac{\omega_d}{\omega_0}$, $\omega_d = \sqrt{\dfrac{k_b}{m_d}}$, $h_d = \dfrac{c_d}{2\sqrt{m_d k_b}}$, and $\eta = \dfrac{k_b}{k} = \mu\beta^2$

For the TVMD, the optimum design to minimize the peak displacement amplification factor (Saito *et al*. 2008) is employed, where the frequency ratio β, the damping ratio h_d, and the stiffness ratio η for a given mass ratio μ are obtained as follows:

$$\beta = \beta^{\circ} = \frac{1-\sqrt{1-4\mu}}{2\mu}, h_d = h_d^{\circ} = \frac{\sqrt{3(1-\sqrt{1-4\mu})}}{4}, \eta = \eta^{\circ} = \mu(\beta^{\circ})^2 \tag{16}$$

Because this system is non-proportionally damped, the eigenvalues and eigenvectors for this two-degrees-of-freedom (2-DOF) system are two pairs of complex conjugates, and we can assume that ${}_{\ell}\overline{\lambda}, {}_{\ell}\overline{\lambda}^*$ and ${}_{\ell}\overline{\boldsymbol{u}} = \{{}_{\ell}\overline{u}, {}_{\ell}\overline{u}_d\}^T$, ${}_{\ell}\overline{\boldsymbol{u}}^* = \{{}_{\ell}\overline{u}^*, {}_{\ell}\overline{u}_d^*\}^T$ are the ℓth conjugate pair of eigenvalues and eigenvectors.

Thus, the following equations hold for $\ell = 1,2$:

$$\begin{aligned} {}_{\ell}\overline{\lambda}^2\, {}_{\ell}\overline{u} + 2h\omega_{0\ell}\,{}_{\ell}\overline{\lambda}\,{}_{\ell}\overline{u} + (1+\eta^{\circ})\omega_{0\ell}^2\,{}_{\ell}\overline{u} - \eta^{\circ}\omega_{0\ell}^2\,{}_{\ell}\overline{u}_d &= 0 \\ \mu\,{}_{\ell}\overline{\lambda}^2\, {}_{\ell}\overline{u}_d + 2\mu\beta^{\circ}h_d^{\circ}\omega_{0\ell}\,{}_{\ell}\overline{\lambda}\,{}_{\ell}\overline{u}_d - \eta^{\circ}\omega_{0\ell}^2\,{}_{\ell}\overline{u} + \eta^{\circ}\omega_{0\ell}^2\,{}_{\ell}\overline{u}_d &= 0 \end{aligned} \tag{17}$$

$$\begin{aligned} {}_{\ell}\overline{\lambda}^{*2}\, {}_{\ell}\overline{u}^* + 2h\omega_{0\ell}\,{}_{\ell}\overline{\lambda}^*\,{}_{\ell}\overline{u}^* + (1+\eta^{\circ})\omega_{0\ell}^2\,{}_{\ell}\overline{u}^* - \eta^{\circ}\omega_{0\ell}^2\,{}_{\ell}\overline{u}_d^* &= 0 \\ \mu\,{}_{\ell}\overline{\lambda}^{*2}\, {}_{\ell}\overline{u}_d^* + 2\mu\beta^{\circ}h_d^{\circ}\omega_{0\ell}\,{}_{\ell}\overline{\lambda}^*\,{}_{\ell}\overline{u}_d^* - \eta^{\circ}\omega_{0\ell}^2\,{}_{\ell}\overline{u}^* + \eta^{\circ}\omega_{0\ell}^2\,{}_{\ell}\overline{u}_d^* &= 0 \end{aligned} \tag{18}$$

4.2 Multiple Degrees-of-Freedom (MDOF) Shear Building Containing TVMDs

TVMD seismic control systems selectively dampen a specified mode with a small change in the primary eigenvectors from the original modes of the undamped system

(Ikago *et al*. 2012a, 2012b). This helps a practicing structural designer understand the seismic response of the seismic control system in terms of modal responses.

The inherent damping elements c_j of the primary system, and the damping elements c_{dj} and supporting springs k_{bj} of the secondary system are added to an undamped system, as shown in Fig. 3, to obtain the TVMD seismic control system shown in Fig. 5.

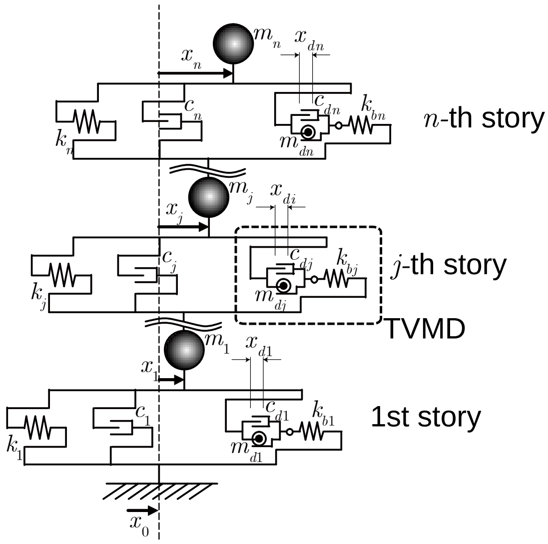


Fig. 5 n-story shear building containing TVMDs

Here, the free vibration of an MDOF shear building containing TVMDs is considered. The equation of motion for the system can be expressed as follows (Ikago *et al*. 2011):

$$\tilde{\mathbf{M}}\ddot{\tilde{\boldsymbol{x}}} + \tilde{\mathbf{C}}\dot{\tilde{\boldsymbol{x}}} + \tilde{\mathbf{K}}\tilde{\boldsymbol{x}} = \mathbf{0} \tag{19}$$

where

$$\tilde{\boldsymbol{x}} = \begin{Bmatrix} \boldsymbol{x} \\ \boldsymbol{x}_d \end{Bmatrix} \tag{20}$$

$$\tilde{\mathbf{M}} = \begin{bmatrix} \mathbf{M}_\mathrm{P} & \mathbf{O} \\ \mathbf{O} & \mathbf{M}_\mathrm{D} \end{bmatrix}, \tilde{\mathbf{C}} = \begin{bmatrix} \mathbf{C}_\mathrm{P} & \mathbf{O} \\ \mathbf{O} & \mathbf{C}_\mathrm{D} \end{bmatrix} \tag{21}$$

$$\tilde{\mathbf{K}} = \begin{bmatrix} \mathbf{K}_\mathrm{P} + \mathbf{K}_\mathrm{B11} & \mathbf{K}_\mathrm{B12} \\ \mathbf{K}_\mathrm{B21} & \mathbf{K}_\mathrm{B22} \end{bmatrix} \tag{22}$$

$$\mathbf{C}_\mathrm{D} = \begin{bmatrix} c_{d1} & 0 & \cdots & & 0 \\ 0 & c_{d2} & & & \vdots \\ \vdots & & \ddots & & \\ & & & & 0 \\ 0 & \cdots & & 0 & c_{dn} \end{bmatrix} \tag{23}$$

$$\mathbf{K}_0 = \begin{bmatrix} k_1 & 0 & \cdots & & 0 \\ 0 & k_2 & & & \vdots \\ \vdots & & \ddots & & \\ & & & & 0 \\ 0 & \cdots & & 0 & k_n \end{bmatrix}, \mathbf{K}_\mathrm{B} = \begin{bmatrix} k_{b1} & 0 & \cdots & & 0 \\ 0 & k_{b2} & & & \vdots \\ \vdots & & \ddots & & \\ & & & & 0 \\ 0 & \cdots & & 0 & k_{bn} \end{bmatrix} \tag{24}$$

$$\mathbf{K}_\mathrm{B11} = \mathbf{T}^T\mathbf{K}_\mathrm{B}\mathbf{T}, \mathbf{K}_\mathrm{B12} = -\mathbf{T}^T\mathbf{K}_\mathrm{B}, \mathbf{K}_\mathrm{B21} = \mathbf{K}_\mathrm{B12}^T, \mathbf{K}_\mathrm{B22} = \mathbf{K}_\mathrm{B} \tag{25}$$

$$\boldsymbol{x}_d = \{x_{d1}, x_{d2}, \cdots, x_{dn}\}^T \tag{26}$$

$$\mathbf{C}_\mathrm{P} = \frac{2h}{{}_1\omega}\mathbf{K}_\mathrm{P} \tag{27}$$

and $\mathbf{T}$ is the coordinate transformation matrix that transforms relative displacement into inter-story drift.

$$\mathbf{T} = \begin{bmatrix} 1 & & & \\ -1 & 1 & & \\ & \ddots & \ddots & \\ & & -1 & 1 \end{bmatrix} \tag{28}$$

Assuming that Eq. (6) holds for the TVMD seismic control system and that the TVMD is tuned to the first mode, the mass ratio μ is defined as the ratio of effective mass of the secondary masses to that of the primary masses for the first mode:

$$\mu = \frac{{}_1\boldsymbol{u}^T\mathbf{M}'_\mathrm{D}\,{}_1\boldsymbol{u}}{{}_1\boldsymbol{u}^T\mathbf{M}_\mathrm{P}\,{}_1\boldsymbol{u}} = \frac{\alpha\cdot{}_1\boldsymbol{u}^T\mathbf{K}_\mathrm{P}\,{}_1\boldsymbol{u}}{{}_1\boldsymbol{u}^T\mathbf{M}_\mathrm{P}\,{}_1\boldsymbol{u}} = \alpha\cdot{}_1\omega^2 \tag{29}$$

and c_{dj} and k_{bj} are determined such that they are tuned to the first mode:

$$c_{dj} = 2\beta^\circ h_{d1}^\circ \omega m_{dj}, \quad k_{bj} = (\beta^\circ)^2{}_1\omega^2 m_{dj} \tag{30}$$

Thus,

$$\mathbf{M}_\mathrm{D} = \alpha\mathbf{K}_0 \tag{31}$$

$$\mathbf{C}_\mathrm{D} = 2\beta^\circ h_d^E\cdot{}_1\omega\mathbf{M}_\mathrm{D} = 2\alpha\beta^\circ h_d^\circ\cdot{}_1\omega\mathbf{K}_0 = \frac{2\mu\beta^\circ h_d^\circ}{{}_1\omega}\mathbf{K}_0 \tag{32}$$

$$\mathbf{K}_\mathrm{B} = (\beta^\circ\cdot{}_1\omega)^2\mathbf{M}_\mathrm{D} = \alpha(\beta^\circ\cdot{}_1\omega)^2\mathbf{K}_0 = \mu(\beta^\circ)^2\mathbf{K}_0 = \eta^\circ\mathbf{K}_0 \tag{33}$$

4.3 Equivalent 2-DOF System

An n-story shear building containing TVMDs can be reduced to a 2-DOF system using the vectors $\{_k\boldsymbol{u}\}$ of the original modes of the undamped system. For the primary system, the equivalent mass ${}^kM_p$, the equivalent stiffness ${}^kK_p$, and the equivalent inherent damping coefficient ${}^kC_p$ for the kth mode are

$$ {}^kM_p = {}_k\boldsymbol{u}^T\mathbf{M}_{\mathrm{P}\,k}\boldsymbol{u},\ {}^kK_p = {}_k\boldsymbol{u}^T\mathbf{K}_{\mathrm{P}\,k}\boldsymbol{u},\ {}^kC_p = {}_k\boldsymbol{u}^T\mathbf{C}_{\mathrm{P}\,k}\boldsymbol{u} \tag{34} $$

For the secondary system, the equivalent apparent mass ${}^kM_d$ for the kth mode is

$$ {}^kM_d = {}_k\boldsymbol{u}^T\mathbf{M}'_{\mathrm{D}\,k}\boldsymbol{u} = m_{d1}\cdot{}_ku_1^2 + \sum_{j=2}^{n} m_{dj}({}_ku_j - {}_ku_{j-1})^2 \tag{35} $$

The secondary mass ratio for the kth mode ${}^k\mu$ can be defined as

$$ {}^k\mu = \frac{{}^kM_d}{{}^kM_p} \tag{36} $$

For this study, it was assumed that the TVMD was tuned to the first mode. Substituting $\mu = {}^1\mu$ into Eq.(16), the following equations can be defined:

$$ {}^1\beta^{\mathrm{o}} = \frac{1-\sqrt{1-4^1\mu}}{2^1\mu},\quad {}^1h_d^{\mathrm{o}} = \frac{\sqrt{3(1-\sqrt{1-4^1\mu})}}{4},\quad {}^1\eta^{\mathrm{o}} = {}^1\mu({}^1\beta^{\mathrm{o}})^2 \tag{37} $$

Thus, the equivalent supporting spring stiffness ${}^kK_b$ and the equivalent damping coefficient ${}^kC_d$ for the kth mode of the secondary system are

$$ {}^kK_b = {}_k\boldsymbol{u}^T\mathbf{K}_{\mathrm{B11}\,k}\boldsymbol{u} = {}^1\eta^{\mathrm{o}}\,{}^kK_p \tag{38} $$

$$ {}^kC_d = {}_k\boldsymbol{u}^T\mathbf{T}^T\mathbf{C}_{\mathrm{D}}\mathbf{T}_k\boldsymbol{u} = \frac{2^1\mu^1\beta^{\mathrm{o}}\,{}^1h_d^{\mathrm{o}}}{{}_1\omega}\,{}^kK_p \tag{39} $$

Here, we can define the ℓth conjugate pairs of eigenvalues ${}^k_\ell\overline{\lambda}, {}^k_\ell\overline{\lambda}^*$, eigenvectors ${}^k_\ell\overline{\boldsymbol{u}} = \{{}^k_\ell\overline{u}, {}^k_\ell\overline{u}_d\}^T$, ${}^k_\ell\overline{\boldsymbol{u}}^* = \{{}^k_\ell\overline{u}^*, {}^k_\ell\overline{u}_d^*\}^T$, and their participation factors ${}^k_\ell\overline{\nu}, {}^k_\ell\overline{\nu}^*$, respectively, where $\ell = 1, 2$.

4.4 Eigenvectors of an MDOF Shear Building Containing TVMDs

Because a TVMD seismic control system provides non-proportional damping and it is assumed that the system is under-damped, the eigenvectors are $2n$ pairs of complex conjugate vectors, which can be expressed by combining the original primary participation mode ${}_k\nu\,{}_k\boldsymbol{u}$, where ${}_k\nu$ is the participation factor for the kth uncontrolled primary mode, and the complex conjugate pair modes of the reduced 2-DOF system equivalent to the MDOF system containing TVMDs.

The combination of the original kth mode and ℓth complex conjugate pair modes of the equivalent reduced 2-DOF system for the original k th mode yields $r(=n(\ell-1)+k)$ th complex conjugate pair modes. Because the undamped primary system has n modes and the equivalent 2-DOF system has two complex conjugate pair modes, the combination of modes yields $2n$ pairs of complex conjugate modes.

$$
{}_r\tilde{\boldsymbol{u}} = \begin{Bmatrix} {}^k_\ell\overline{u}\,{}_k\boldsymbol{u} \\ {}^k_\ell\overline{u}_d\mathbf{T}\,{}_k\boldsymbol{u} \end{Bmatrix}, \quad {}_r\tilde{\boldsymbol{u}}^* = \begin{Bmatrix} {}^k_\ell\overline{u}^*\,{}_k\boldsymbol{u} \\ {}^k_\ell\overline{u}_d^*\mathbf{T}\,{}_k\boldsymbol{u} \end{Bmatrix}, \quad \begin{cases} k=1,2,\cdots,n \\ \ell=1,2 \\ r=n(\ell-1)+k \end{cases} \tag{40}
$$

The rth conjugate pair of participation factors ${}_r\tilde{\nu}$ and ${}_r\tilde{\nu}^*$ for the mode vectors ${}_r\tilde{\boldsymbol{u}}$ and ${}_r\tilde{\boldsymbol{u}}^*$, respectively, are obtained by the following combination:

$$
{}_r\tilde{\nu} = {}^k_\ell\overline{\nu}\,{}_k\nu, \quad {}_r\tilde{\nu}^* = {}^k_\ell\overline{\nu}^*\,{}_k\nu \tag{41}
$$

From the definition of the participation factors,

$$
\sum_{k=1}^{n} {}_k\nu\,{}_k\boldsymbol{u} = \boldsymbol{r} \tag{42}
$$

$$
\sum_{\ell=1}^{2}\left[{}^k_\ell\overline{\nu}\begin{Bmatrix} {}^k_\ell\overline{u} \\ {}^k_\ell\overline{u}_d \end{Bmatrix} + {}^k_\ell\overline{\nu}^*\begin{Bmatrix} {}^k_\ell\overline{u}^* \\ {}^k_\ell\overline{u}_d^* \end{Bmatrix}\right] = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{43}
$$

Thus, the sum of ${}_r\tilde{\nu}\,{}_r\tilde{\boldsymbol{u}}$ and ${}_r\tilde{\nu}^*\,{}_r\tilde{\boldsymbol{u}}^*$ for all $r=1,2,\cdots,2n$ yields the influence coefficient vector of the TVMD controlled system.

$$
\begin{aligned}
\sum_{r=1}^{2n}\left({}_r\tilde{\nu}\,{}_r\tilde{\boldsymbol{u}} + {}_r\tilde{\nu}^*\,{}_r\tilde{\boldsymbol{u}}^*\right) &= \sum_{k=1}^{n}\sum_{\ell=1}^{2} {}_k\nu\left[{}^k_\ell\overline{\nu}\begin{Bmatrix} {}^k_\ell\overline{u}\,{}_k\boldsymbol{u} \\ {}^k_\ell\overline{u}_d\mathbf{T}\,{}_k\boldsymbol{u} \end{Bmatrix} + {}^k_\ell\overline{\nu}^*\begin{Bmatrix} {}^k_\ell\overline{u}^*\,{}_k\boldsymbol{u} \\ {}^k_\ell\overline{u}_d^*\mathbf{T}\,{}_k\boldsymbol{u} \end{Bmatrix}\right] \\
&= \sum_{k=1}^{n} {}_k\nu\begin{Bmatrix} {}_k\boldsymbol{u} \\ \mathbf{0} \end{Bmatrix} = \begin{Bmatrix} \boldsymbol{r} \\ \mathbf{0} \end{Bmatrix}
\end{aligned} \tag{44}
$$

Substituting Eqs.(7),(8),(31),(32),(33) and $\tilde{\boldsymbol{x}} = {}_r\tilde{\boldsymbol{u}} e^{\tilde{\lambda} t},\ {}_r\tilde{\boldsymbol{u}}^* e^{\tilde{\lambda} t}$ into Eq.(19) yields

$$\begin{aligned} \{\tilde{\lambda}^2\, {}^k_\ell\overline{u} + 2\,{}_k h\,{}_k\omega\tilde{\lambda}\, {}^k_\ell\overline{u} + (1 + {}^1\eta^{\mathrm{o}})\,{}_k\omega^2\, {}^k_\ell\overline{u} - {}^1\eta^{\mathrm{o}}\,{}_k\omega^2\, {}^k_\ell\overline{u}_d\}\mathbf{M}_{\mathrm{P}\,k}\boldsymbol{u} e^{\tilde{\lambda} t} = 0 \\ \{{}^1\mu\tilde{\lambda}^2\, {}^k_\ell\overline{u}_d + 2\,{}^1\mu\,{}^1\beta^{\mathrm{o}}\,{}^1h^{\mathrm{o}}_d\,{}_k\omega\tilde{\lambda}\, {}^k_\ell\overline{u}_d - {}^1\eta^{\mathrm{o}}\,{}_k\omega^2\, {}^k_\ell\overline{u} + {}^1\eta^{\mathrm{o}}\,{}_k\omega^2\, {}^k_\ell\overline{u}_d\}\mathbf{K}_0\mathbf{T}\,{}_k\boldsymbol{u} e^{\tilde{\lambda} t} = 0 \end{aligned} \tag{45}$$

$$\begin{aligned} \{\tilde{\lambda}^{*2}\, {}^k_\ell\overline{u}^* + 2\,{}_k h\,{}_k\omega\tilde{\lambda}^*\, {}^k_\ell\overline{u}^* + (1 + {}^1\eta^{\mathrm{o}})\,{}_k\omega^{2}\, {}^k_\ell\overline{u}^* - {}^1\eta^{\mathrm{o}}\,{}_k\omega^2\, {}^k_\ell\overline{u}_d^*\}\mathbf{M}_{\mathrm{P}\,k}\boldsymbol{u} e^{\tilde{\lambda} t} = 0 \\ \{{}^1\mu\tilde{\lambda}^{*2}\, {}^k_\ell\overline{u}_d^* + 2\,{}^1\mu\,{}^1\beta^{\mathrm{o}}\,{}^1h^{\mathrm{o}}_d\,{}_k\omega\tilde{\lambda}^*\, {}^k_\ell\overline{u}_d^* - {}^1\eta^{\mathrm{o}}\,{}_k\omega^2\, {}^k_\ell\overline{u}^* + {}^1\eta^{\mathrm{o}}\,{}_k\omega^2\, {}^k_\ell\overline{u}_d^*\}\mathbf{K}_0\mathbf{T}\,{}_k\boldsymbol{u} e^{\tilde{\lambda} t} = 0 \end{aligned} \tag{46}$$

where

$${}_k h = \frac{{}_k\omega}{{}_1\omega} h \tag{47}$$

The coefficients for $\mathbf{M}_{\mathrm{P}\,k}\boldsymbol{u} \neq \mathbf{0}$ and $\mathbf{K}_0\mathbf{T}\,{}_k\boldsymbol{u} \neq \mathbf{0}$ in Eqs. (45) and (46) are identical to those in the eigenvalue equation for the reduced 2-DOF system derived by the uncontrolled primary k th mode.

Thus, it can be proved that the eigenvectors assumed in Eq. (40) satisfy Eqs. (45) and (46).

If ${}_r\tilde{\lambda}$ and ${}_r\tilde{\lambda}^*$ are defined as the eigenvalues for Eqs. (45) and (46), the r th fundamental circular frequency ${}_r\tilde{\omega}$ and damping ratio ${}_r\tilde{h}$ for the TVMD controlled system can be obtained as follows:

$${}_r\tilde{\omega} = \left|{}_r\tilde{\lambda}\right| = \left|{}_r\tilde{\lambda}^*\right| \tag{48}$$

$${}_r\tilde{h} = -\frac{\mathrm{Re}[{}_r\tilde{\lambda}]}{\left|{}_r\tilde{\lambda}\right|} = -\frac{\mathrm{Re}[{}_r\tilde{\lambda}^*]}{\left|{}_r\tilde{\lambda}^*\right|} \tag{49}$$

5. ANALYTICAL EXAMPLE

A 2-DOF shear building containing TVMDs is employed to illustrate the modal response characteristics of TVMD seismic control system. The specification of the 2-DOF primary structure is listed in Table 1.

Table 1 Specification of primary structure

Floor	m_i [t]	k_i [kN/m]
1	1	3
2	1	2

The participation mode vectors of the primary system are real valued as depicted in Fig. 6.

(a) First mode ($k=1$)

(b) Second mode ($k=2$)

Fig. 6 Participation vectors of uncontrolled primary system

Table 2 lists the specifications for the reduced 2-DOF system obtained using uncontrolled primary modes. The TVMD system is designed to be tuned to the first primary mode. According to Eq.(37), the mass ratio ${}^{1}\mu = 0.05$ results in ${}^{1}h_d$=0.141 and ${}^{1}\eta$=0.056.

Table 2 Specifications for the reduced 2-DOF systems

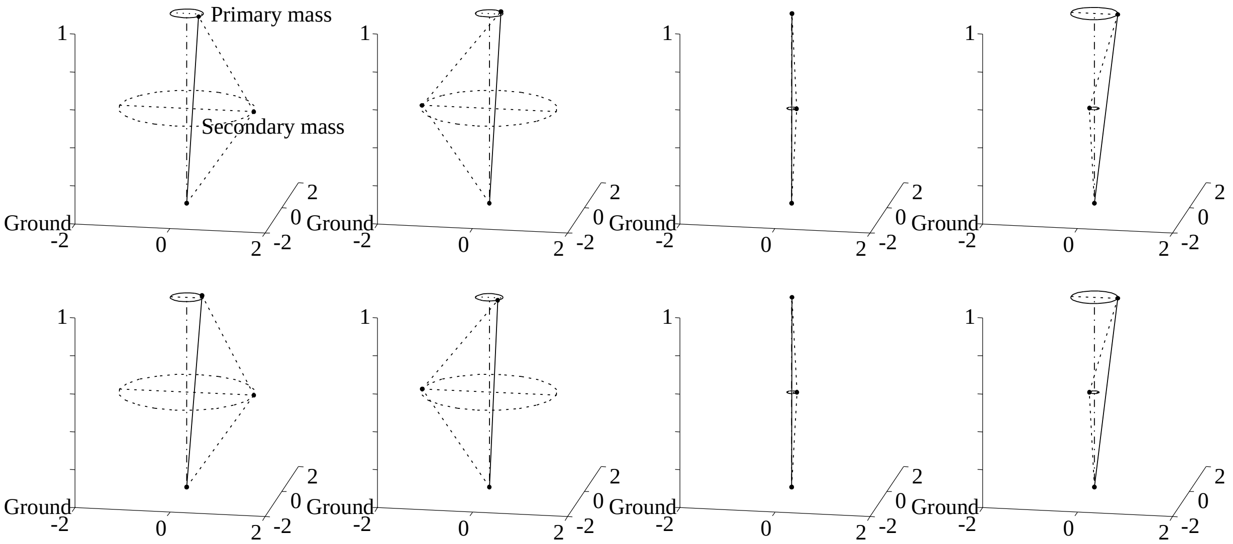
k	*Primary system*		*Secondary system (TVMD)*		
	$\dfrac{{}^{k}M_p}{\sum m_i}$	${}_{k}\omega = \sqrt{\dfrac{{}^{k}K_p}{{}^{k}M_p}}$	${}^{k}\mu = \dfrac{{}^{k}M_d}{{}^{k}M_p}$	${}^{k}h_d = \dfrac{{}^{k}C_d}{2\sqrt{{}^{k}M_d\,{}^{k}K_d}}$	${}^{k}\eta = \dfrac{{}^{k}K_b}{{}^{k}K_p}$
1	0.9	1.00	0.05	0.141	0.056
2	0.1	$\sqrt{6}$	0.30		

Table 3 Fundamental circular frequencies and damping ratios for the TVMD controlled system

ℓ	k	${}_{r}\tilde{\omega}$	${}_{r}\tilde{h}$
1	1	0.94	0.072
	2	1.02	0.144
2	1	1.13	0.072
	2	2.53	0.008

The 2-DOF system obtained using the uncontrolled primary first mode yields two conjugate pairs of participation mode vectors, as shown in Fig. 7(a) and (b). In a similar manner, the 2-DOF system obtained using uncontrolled primary second mode yields two conjugate pairs of participation mode vectors, as shown in Fig. 7(c) and (d).

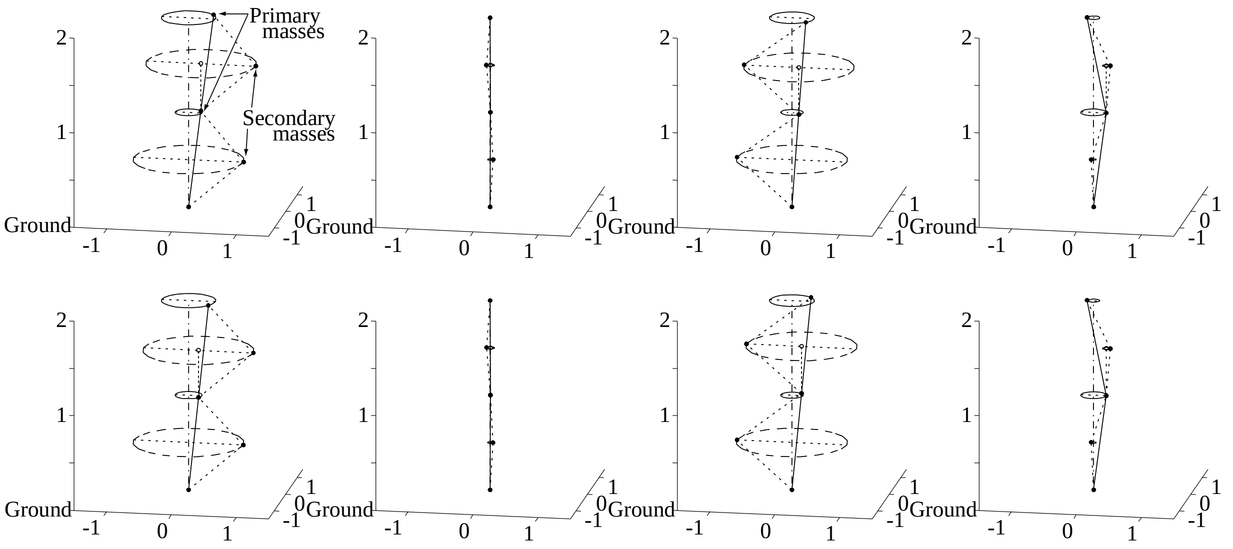
Combination of the participation mode vectors of undamped primary system (Fig. 6) and the participation mode vectors of the reduced 2-DOF systems (Fig. 7) obtains participation mode vectors of the TVMD controlled system, as shown in Fig. 8.



(a) First conjugate pair modes ($\ell = 1$) (b) Second conjugate pair modes ($\ell = 2$) (c) First conjugate pair modes ($\ell = 1$) (d) Second conjugate pair modes ($\ell = 2$)

Participation vectors of Reduced 2 DOF system (k = 1) Participation vectors of Reduced 2 DOF system (k = 2)

Fig. 7 Participation vectors of reduced 2-DOF systems



(a) First conjugate pair modes $r = 1$ $(k = 1, \ell = 1)$ (b) Second conjugate pair modes $r = 3$ $(k = 2, \ell = 1)$ (c) Third conjugate pair modes $r = 2$ $(k = 1, \ell = 2)$ (d) Fourth conjugate pair modes $r = 4$ $(k = 2, \ell = 2)$

Fig. 8 Participation vectors of TVMD seismic control system (Eqs.(40),(41))

6. CONCLUSION

The advantage of a TVMD seismic control system is that it can perform selective damping to a specified mode with little change in the original modes of the undamped system. Using this system, a practicing structural engineer can design a seismic control system on the basis of modal response control.

Although many analytical studies have confirmed that adding a TVMD system almost never changes the original modes of the undamped system, the reason for this phenomenon has not yet been theoretically explained. Thus, this paper discusses the modal response characteristics of a TVMD seismic control system in which the secondary apparent masses are arranged such that their distribution is proportional to the primary stiffness. It was found that the fundamental modes of the undamped primary system remained unchanged after adding the secondary system only when the secondary mass distribution is proportional to the primary stiffness distribution.

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