

## **Method to Synchronize the Dynamic Properties between Existing Bridge and Analysis Model**

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### **ABSTRACT**

Dynamic property synchronization between existing bridge and design model is an important process before analyzing an actual bridge because the difference causes wrong estimation of the responses. This study assumes that they have same mode shapes if the natural frequencies are slightly different. From this assumption of the same mode shapes, estimation method of mass and stiffness variation is suggested. The method uses a least square method to solve a simultaneous polynomial equation transformed from the one of free vibration problem. Overall structural property variations can be estimated from natural frequencies only. If the proposed method is adopted to real-time measurement system, it is expected to increase the maintenance efficiency.

### **1. INTRODUCTION**

In the design phase, an analysis model of a bridge has tendencies to assess the stiffness as weak and to consider the load as large in order to ensure the safety of the structure. Although the tendencies guarantee the structural safety, but that can be a main cause of presenting a difference between the bridge and the model. It needs a process to synchronize the dynamic properties before the maintenance purpose dynamic analyses that evaluate the wind, earthquake, or vehicle induced vibration. Generally used synchronizing method is manual tuning method which depends on subjective judgment of technician. It needs a new method to replacing the MTM.

There are many studies have been conducted related to the damage detection. These studies have been tried to develop an algorithm or a process that assesses damage to the bridge from the information of measured dynamic properties of the bridge. Chou studied the damage detection method(Chou 2001) using static measurement data, bases on the genetic algorithm. This kind of damage detection methods generally have an assumption that the undamaged bridge and the analysis model have exactly equal dynamic properties. Most of the previous studies have been

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focusing at the algorithm to estimate a degree of damage of a structure. The study was not conducted for the synchronization between existing bridge and analysis model. This study suggests the algorithm which analogizes the stiffness and the mass difference between the bridge and the model unrelated to whether the bridge damaged or not using only the measured natural frequencies. The proposed algorithm was verified by numerical experiments using a simple beam model and applied to a real bridge models.

## 2. PROBLEM DEFINITION

The eigenvalue problems of the analysis model and existing bridge is Eq. (1a), (1b).

$$(\mathbf{K} - \lambda_i^2 \mathbf{M}) \mathbf{v}_i = \mathbf{0} \quad (1a)$$

$$(\mathbf{K}' - \lambda_i'^2 \mathbf{M}') \mathbf{v}_i' = \mathbf{0} \quad (1b)$$

where  $\mathbf{K}$ ,  $\mathbf{M}$  is stiffness as mass matrix of the analysis model,  $\lambda_i$ ,  $\mathbf{v}_i$  is natural frequency and eigenvector,  $i$  is the order of the eigenvalues, and the hyphen mark means existing bridge's properties.

Because it has a small natural frequencies difference between the bridge and the model, the bridge and the model are regarded to have an equal mode shapes (Kim 2013). Regulated equation Eq. (3) is obtained by substituting Eq. (2) into Eq. (1b). Substituting Eq. (1a) into Eq. (3) and eliminate the infinitesimal terms then the residual is presented like Eq. (4).

$$\mathbf{K}' = \mathbf{K} + \mathbf{k}, \quad \mathbf{M}' = \mathbf{M} + \mathbf{m}, \quad \lambda_i' = \lambda_i + \varepsilon_i, \quad \mathbf{v}_i = \mathbf{v}_i' \quad (2)$$

$$\left\{ \mathbf{K} - \lambda_i^2 \mathbf{M} + \mathbf{k} - \lambda_i^2 \mathbf{m} - 2\lambda_i \varepsilon_i \mathbf{M} - 2\lambda_i \varepsilon_i \mathbf{m} - \varepsilon_i^2 \mathbf{M} - \varepsilon_i^2 \mathbf{m} \right\} \mathbf{v}_i = \mathbf{0} \quad (3)$$

$$\left\{ \mathbf{k} - \lambda_i^2 \mathbf{m} - 2\lambda_i \varepsilon_i \mathbf{M} \right\} \mathbf{v}_i = \mathbf{R}_i \approx \mathbf{0} \quad (4)$$

where  $\mathbf{M}$ ,  $\lambda_i$ ,  $\mathbf{v}_i$  are known values obtained from the results of free vibration,  $\varepsilon_i$  can be calculated from the measuring data of the existing bridge. Therefore the problem on the dynamic property synchronization of the bridge could be solved by determining the stiffness and mass matrices  $\mathbf{k}$ ,  $\mathbf{m}$  that obtained by solving the simultaneous Eq. (4) to be satisfied the minimum residual condition. Introduce the term  $\mathbf{r}_i$  as  $2\lambda_i \varepsilon_i \mathbf{M} \mathbf{v}_i$  then Eq. (4) is represented to Eq. (5), (6). Moreover Eq. (6) can be expanded as Eq. (9), (10) to a 4 more-degree of freedom structure.

$$\mathbf{k} \mathbf{v}_i - \lambda_i^2 \mathbf{m} \mathbf{v}_i - \mathbf{r}_i = \mathbf{R}_i \quad (5)$$

$$\left\{ \begin{array}{ccc} k_{11} & k_{12} & k_{13} \\ & k_{22} & k_{23} \\ \left[ \begin{array}{c} \text{sym.} \\ & k_{33} \end{array} \right] \end{array} \right\} - \lambda_i^2 \left\{ \begin{array}{ccc} m_{11} & m_{12} & m_{13} \\ & m_{22} & m_{23} \\ \left[ \begin{array}{c} \text{sym.} \\ & m_{33} \end{array} \right] \end{array} \right\} \left\{ \begin{array}{c} v_{i1} \\ v_{i2} \\ v_{i3} \end{array} \right\} - \mathbf{r}_i = \mathbf{R}_i \quad (6)$$

$$R(\mathbf{k}, \mathbf{m}) = \frac{1}{n\lambda_i^2} \sum_{i=1}^n \sqrt{\frac{\mathbf{R}_i \cdot \mathbf{R}_i}{(2\lambda_i \mathbf{M} \mathbf{v}_i) \cdot (2\lambda_i \mathbf{M} \mathbf{v}_i)}} \quad (7)$$

$$k_{m1} \cdot v_{i1} + k_{m2} \cdot v_{i2} + k_{m3} \cdot v_{i3} - \lambda_i^2 (m_{m1} \cdot v_{i1} + m_{m2} \cdot v_{i2} + m_{m3} \cdot v_{i3}) - r_{im} = R_{im} \quad (8)$$

$$\sum_n k_{mn} v_{in} - \lambda_i^2 \sum_n m_{mn} v_{in} - r_{im} = R_{im} \quad (9)$$

Although the simplified simultaneous equation Eq. (6) has infinite number of solution, only a few cases, that have a certain range variation of the stiffness and mass, has the mechanical meaning. This method presents solutions varies with initial value, Eq. (8).

$$(\mathbf{k}_0 - \lambda_i^2 \mathbf{m}_0) \mathbf{v}_i = \mathbf{r}_i \quad (8)$$

Then using Newton-Raphson to modify  $\mathbf{k}$ ,  $\mathbf{m}$  to have infinitesimal value of  $R$ . Determine the combinations of mass and stiffness by performing the method repeatedly.

### 3. SIMPLE BEAM MODEL ESTIMATION

It is expected to the suggested method could assess the variation of stiffness and mass from the measured data of existing bridge but if the differences between an existing bridge and an analysis model are large, then the error of the method will be larger. The suggested method is verified by performing numerical experiment using simple beam model. The simple beam model has 10m width, 50m length, elastic modulus is 2.0e8kN/m<sup>2</sup>, mass density is 0.6580kN/g, torsional constant is 0.4035m<sup>4</sup>, 2<sup>nd</sup> order section moments  $I_y$ ,  $I_z$  is 0.10416m<sup>4</sup>, 41.67m<sup>4</sup>, respectably. The finite element model is constituted that has 11 nodes and 10 frame elements. The free vibration results are listed in the Table 1.

Table 1 Modal properties of the analysis model

Mode	Nat. Freq. (Hz)	Direction	RMS Disp. (mm)	Peak Factor
1	0.500	Vertical	181.111	3.537
2	2.000	Vertical	4.486	3.890
4	4.500	Vertical	0.673	4.076
3	3.429	Torsional	0.479	3.992
7	10.000	Lateral	0.163	4.210

From the generated analysis model, calculate the natural frequencies of the structure which is modified slightly. This suggested synchronization method was verified by comparing the analysis results with the numeric experimental one.

#### 4. EXISTING BRIDGE APPLICATION

In case of the method has been adopted an existing bridge, some other variation properties can be occurred, It is verified what the structural property variation can be assessed by the method.

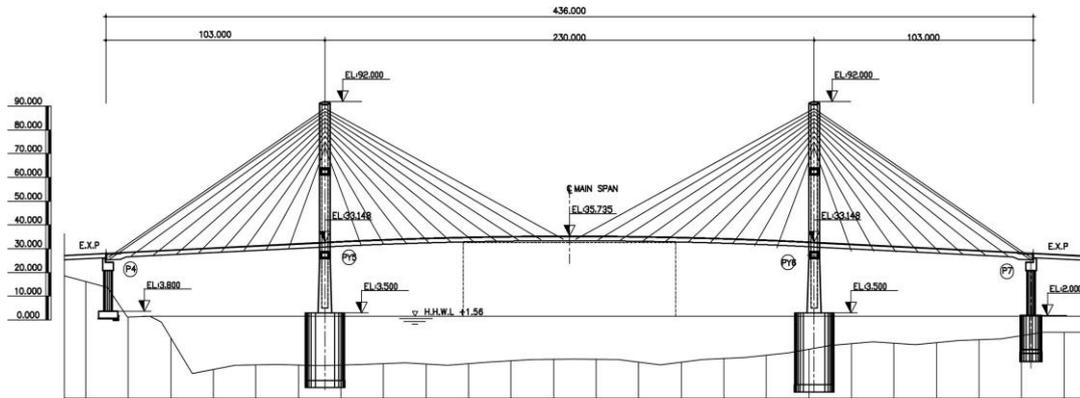


Fig. 1 Longitudinal profile of existing bridge model

As a result of the application of the method to the bridge model, the credibility of the method is 21% less than the simple beam's one. Nevertheless the analysis model dependence, suggested method will help the maintenance technician to do quantitative decision.

#### 5. CONCLUSION

The object of this study is suggest the method to synchronize the dynamic properties between an existing bridge and an analysis model using only measured natural frequencies. The method on the process of developing is an open problem which must be adopt another type of conserving method. In future study, the results from the conserving method will be analyzed.

#### REFERENCES

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