

## Failures of Slabs Having Low Flexural Steel Ratios Subjected to Punching Loads

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### ABSTRACT

This paper summarises our studies on the influence of low reinforcement ratios on the punching shear behaviour of the reinforced concrete slabs. When the flexural reinforcement ratio is relatively low, the modes of failures of the slabs become unclear as the slabs may fail in flexure first before punching failure can occur. In addition, the slabs do not necessarily have their failure loads lower or equal to the theoretical upper-bound failure loads as calculated using the Yield Line Theory. This phenomenon has been addressed by researchers but it has not been considered explicitly in codes of practice and no clear guideline is available on the amount of flexural reinforcement that is needed that can ensure ductile failure of a slab under punching load. The authors evaluated and verified the ACI 318 and Eurocode 2 punching shear formulas and various available methods with more than 300 slab data. The analyses show that both building codes overestimated the failure loads of slabs that have low flexural reinforcement ratios (i.e. lower than 0.5%). A simple method for calculating the punching shear strength of slab is derived and proposed. The proposed method considers, among other typical parameters, the effect of low reinforcement ratios. The prediction by the proposed method is shown to be conservative and consistent over wide ranges of design parameters.

**Keywords:** punching shear; low reinforcement ratio; yield line theory; reinforced concrete slabs

### 1. BACKGROUND

Reinforced concrete flat plate is a two-way floor system supported directly on columns without beam or column capital. Flat plate floor system is a popular choice in modern buildings, such as condominiums and office buildings. The system is also similar to

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industrial flat floor supported on piles as illustrated in Fig. 1. It is the most economical choice for column spans of up to 6 m. In general, flexural reinforcement ratios are would be in the range of 0.3 to 0.6% depending on the design loads. Typical values of the required flexural steel ratios for flat plate slabs at different design loadings are shown in Fig. 2.

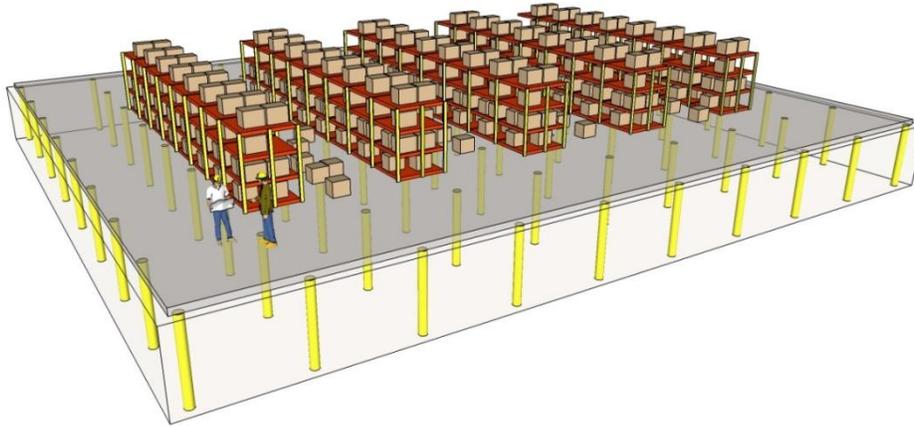


Fig. 1 Industrial flat floor supported on piles (similar to flat plate floor system).

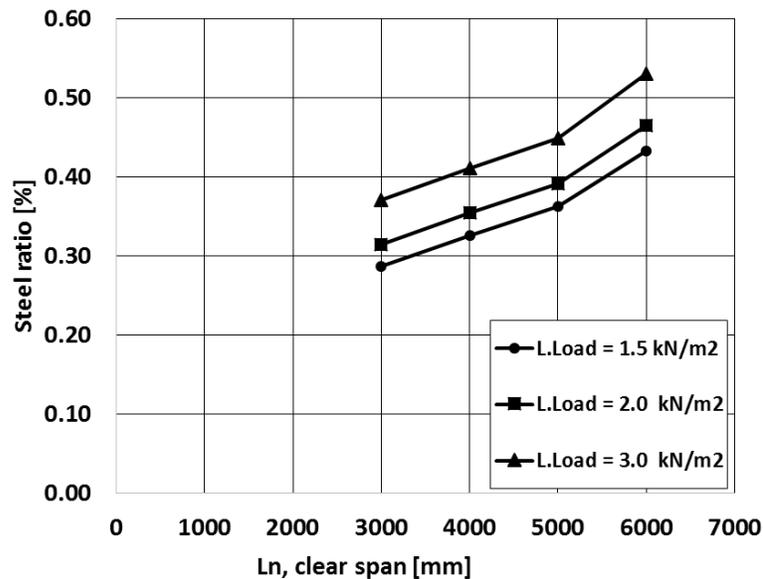


Fig. 2 Required flexural steel ratio of two-way slabs for various symmetric clear span, calculated using ACI 318 direct design method

When the flexural steel ratio of a slab is relatively low, the structural designer expects the failure mode of the slab to be ductile under punching load. Past experiments, however, have shown that brittle failures or punching shear failures can still occur and the slabs can have their actual values of failure loads to be higher than their theoretical failure loads as calculated using the Yield Line Theory (McHarg et al. 2000, Stein Timm. 2007, Peiris and Ghali 2012, Chanthabouala and Teng 2015). Provisions for the punching shear design of two way slabs in the design codes such as the ACI 318 and the Eurocode 2 (EC 2) have not sufficiently covered the influence of low reinforcement ratios on the punching failure loads. The ACI 318, however, considers the strength reduction factor due to the increase in the column aspect ratio and the effect of unsymmetrical bending like the case of edge and exterior slab-column connections. EC 2, not ACI 318, takes account of the effects of size (slab thickness) and flexural reinforcement ratio but it limits the flexural reinforcement ratio to only 2.0%. A higher reinforcement ratio is taken to be equal to 2%. The codes formulas for punching shear strengths are presented as follows.

$$\text{ACI 318: } V_c = \min \begin{cases} \frac{1}{6}(1 + 2/\beta) \sqrt{f'_c} b_o d \\ \frac{1}{12}(\alpha_s d/b_o + 2) \sqrt{f'_c} b_o d \\ \frac{1}{3} \sqrt{f'_c} b_o d \end{cases} \quad (1)$$

where  $V_c$  is the shear strength of a slab without shear reinforcement;  $\alpha_s$  is taken as 40 for interior slab;  $\beta$  is the ratio of the longer column edge to the shorter side column edge;  $b_o$  is the length of critical perimeter at a distance of  $0.5d$  away from the column face and has square corners for square columns and rounded corners for circular columns. Note that the strength reduction factor is not considered in the subsequent analysis.

$$\text{Eurocode 2: } V_c = 0.18k(100\rho f'_c)^{1/3} u_1 d \quad (2)$$

where  $k$  is a size effect coefficient ( $k = 1 + \sqrt{\frac{200}{d}}$  (mm) =  $1 + \sqrt{\frac{7.87}{d}}$  (in.)  $\leq 2.0$ ), and  $\rho$  is flexural reinforcement ratio (taken to be less than or equal to 0.02). The basic control perimeter  $u$  is taken at the distance of two times the effective depth  $2.0d$  from the column faces with round corners.

## 2. ANALYSIS OF SLAB DATA WITH LOW STEEL REBAR RATIO

In general, ACI 318 and EC 2 are conservative in estimating the punching shear strength of slabs. The studies of Teng et al. (2004), Teng and Tan (2005), Chanthabouala and Teng (2015) suggested that the design codes underestimate the punching shear strength of slabs by up to 30%. However, there is a considerable number of slabs that have their actual failure loads smaller than the codes predictions, which could lead to the undesirable designs.

The emphasis of this study is on slab specimens that have the flexural steel ratios,  $\rho$ , varying from 0.2 to 0.7%. All the slabs exhibited the flexural-punching failure, where ductile behaviours were observed before punching shear failures occurred abruptly at the peak loads. When the test-to-predicted ratio is smaller than 1.0, it means that the actual strength is overestimated and vice versa. Figure 3 shows that the ACI 318 equation overestimates the strength of the majority of slabs tested under punching loads, the overall average value of test-to-predicted ratio is 0.84 with the coefficient of variation (COV) of 0.31 and the minimum value of 0.38. The overall average of test-to-predicted ratio of EC 2 equation is 1.02 with the COV of 0.22. However, the prediction should not be considered as conservative, because Figure 4 shows that 50 % of the slabs have their test-to-predicted ratios being less than 1.0, and the minimum value of the ratio is 0.54. In short, it has been highlighted that the ACI 318 and EC 2 punching shear formulas overestimate the punching shear strength of slabs by up to 60%. Hence, even if the safety factors or strength reduction factors are to be considered, the codes formulas may still lead to the non-conservative designs of slabs that have low flexural reinforcements.

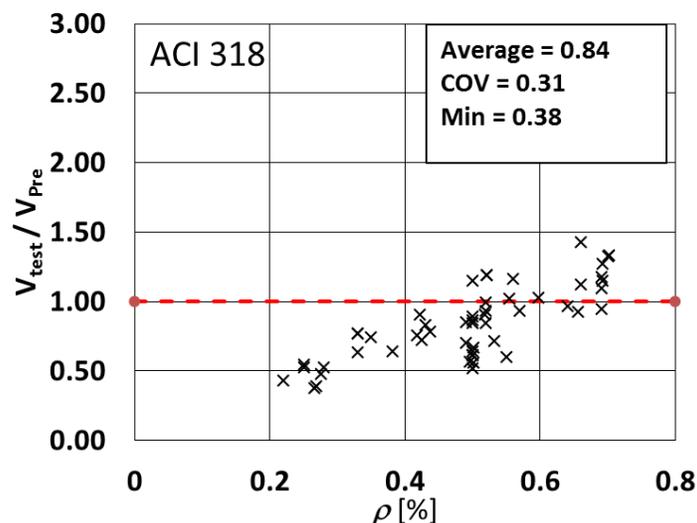


Fig. 3 Test-to- ACI 318 predicted ratio vs. flexural steel ratio up to 0.7%

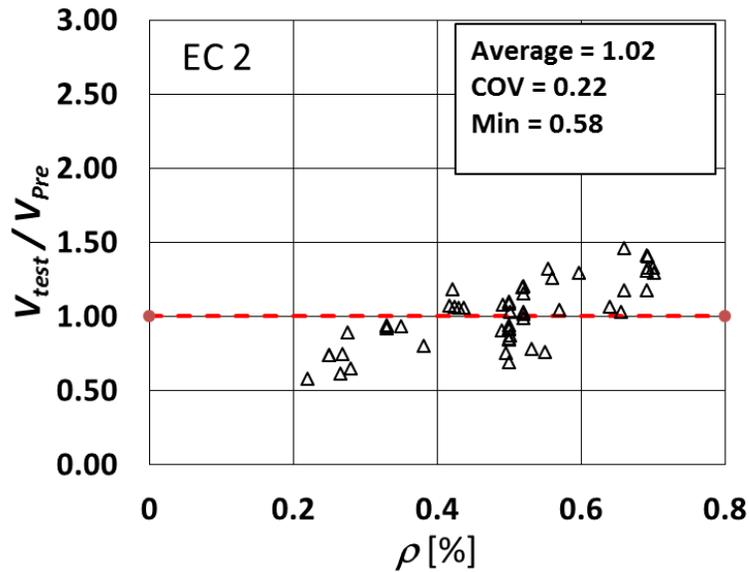


Fig. 4 Test-to- EC 2 predicted ratio vs. flexural steel ratio up to 0.7%

### 3. PROPOSED DESIGN EQUATION

A design method for calculating the punching shear strength of a slab is proposed and expressed in Eq. (3). The detailed derivation of the method is described in the works of Chanthabouala and Teng (2015). The limiting ratio for the flexural reinforcement,  $\mu$ , in Eq. (3) takes account of the effect of low flexural steel ratio. It can be used as an approximating tool to differentiate the flexural failure and punching shear failure modes of a slab.

$$V_u = \left[ 0.6\mu^{1/4} f_c'^{1/3} (100\rho)^{1/3} (1 + d/200)^{-1/3} \right] \gamma b_o d \quad (3)$$

where,

Limiting ratio,  $\mu = \rho / \rho_{ft}$

Limiting value of flexural steel ratio,  $\rho_{ft} = 0.6(100\rho)^{1/3} / \lambda$

Proportionality coefficient,  $\lambda = V_{flex} / \rho b_o d f_c'^{1/3}$

Flexural strength,  $V_{flex} = 8m_u$  or  $8 \rho f_y d^2 \left( 1 - 0.59 \frac{\rho f_y}{f_c} \right)$

Column aspect ratio reduction factor,  $\gamma = \left( \frac{b_1}{b_2} \right)^{1/4}$

Critical perimeter,  $b_o = 2(c_1 + c_2 + 2d)$

Short and long side of critical perimeter,  $b_1 = c_1 + d$ ;  $b_2 = c_2 + d$

Figure 5 shows the predicted strength of more than 300 data using proposed Eq. (3) plotted against the experimental strength. The overall average test-to-predicted ratio is 1.27, with the COV and the minimum value of 0.15 and 0.71, respectively (Shown in Fig. 6). Similarly to Fig. 3 and Fig. 4, figure 7 shows that the proposed method is able to perform consistently well in estimating the punching shear strength of slabs that have low flexural steel ratios. The average test-to-predicted ratio is 1.24 with the COV value and the minimum value of 0.14 and 0.86, respectively. Hence, the proposed method is shown to be an effective and conservative tool to calculate the punching shear strength of a slab with a low flexural steel ratio.

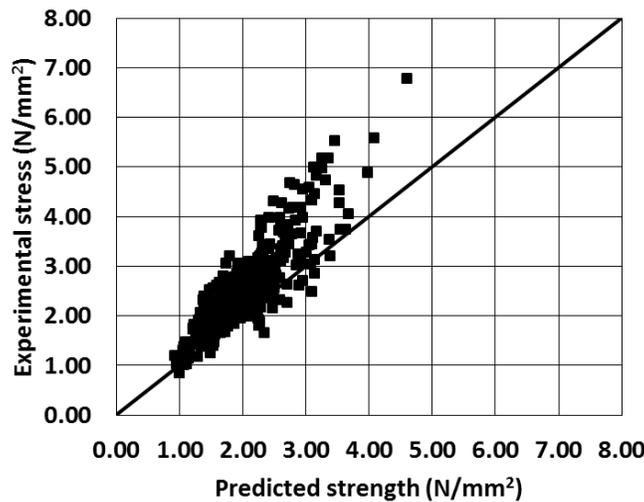


Fig.5 Proposed prediction vs. experimental strength

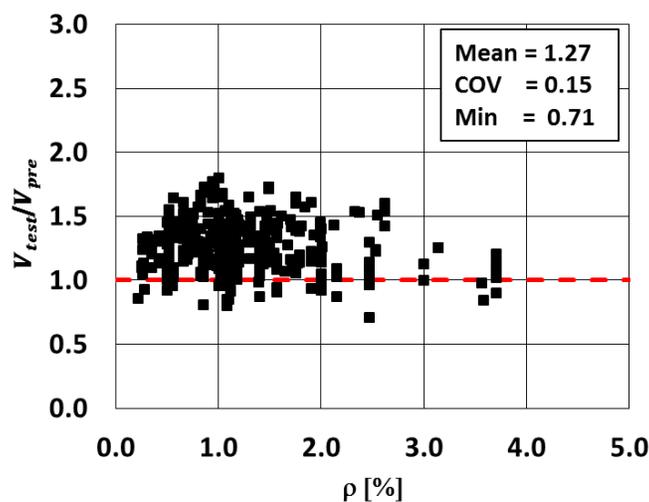


Fig. 6 Test-to-predicted ratio vs. flexural steel ratio for proposed method

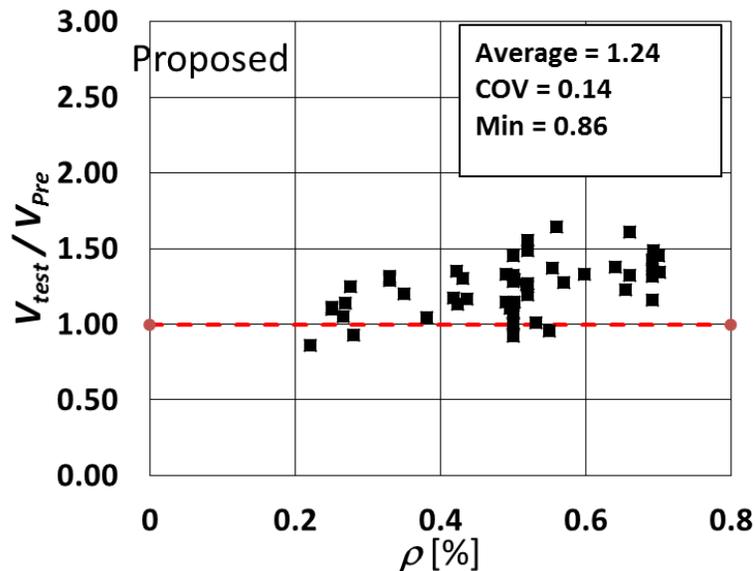


Fig. 7 Test-to- Proposed predicted ratio vs. flexural steel ratio up to 0.7%

Note: The slab data used in the analysis of this paper are from following references (Teng et al. 2004, Guandalini et al. 2009, Peiris and Ghali 2012, Chanthabouala and Teng 2015).

#### 4. CONCLUSIONS

In conclusion, this study can be summed up as follow:

1. The failure of the slab can become brittle even though the amount of the flexural reinforcement is low. The failure may be initiated by flexure because of the relative large deflection, however, the punching failure can occur suddenly at the peak load. Hence, Yield Line Theory may not necessarily be a conservative tool to predict the strength of a two way slab.
2. The analyses of more than 300 slab data show that ACI 318-11 and Eurocode 2 punching shear equations can be unsafe for the design of slabs with low amount of flexural reinforcements. In codes of practice, there has not been a clear guideline on the determination of flexural reinforcement ratios of a slab to ensure a ductile failure under punching load.
3. The proposed design method, Eq. (3) is conservative and consistent in predicting the punching shear strength of slabs which considers the effects of low flexural reinforcement ratios and other typical parameters.

## **ACKNOWLEDGEMENTS**

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