$$\mu_\theta = \theta_u / \theta_y \tag{3}$$

where (θ_u) is the ultimate plastic hinge rotation and (θ_y) is the yield plastic hinge rotation.

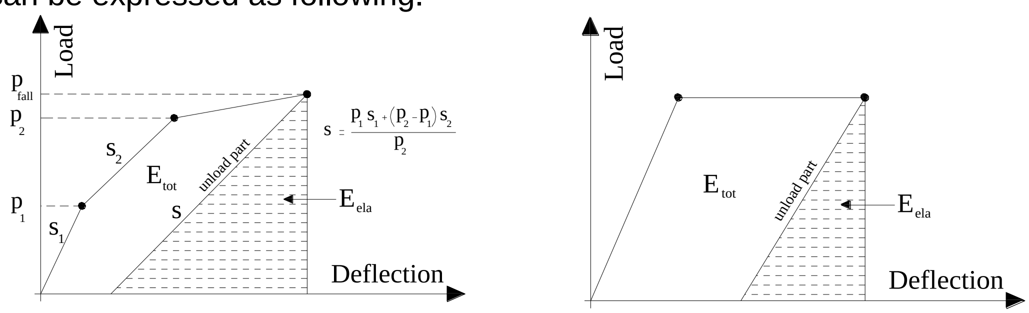
Displacement ductility μ_Δ :

$$\mu_\Delta = \Delta_u / \Delta_y \tag{4}$$

where (Δ_u) is the displacement of a structural element or of a whole structure at the ultimate state and (Δ_y) is the displacement at yield limit.

Ashour (2000) mentioned that members with a displacement ductility in the range of 3 to 5 has adequate ductility and can be considered for structural members subjected to large displacement, such as sudden forces caused by earthquake.

All these conventional methods for ductility are not suitable for beams with FRP as FRP does not have a yield point (Patrick 2003). Researchers have proposed some new equations to quantify the ductility of concrete beams reinforced by FRP and by steel so that a comparison may be made between them. These modified ductility or deformability factors can be expressed as following.



Graph to determine ductility index μ_{en} for beam with elastic reinforcement according to Naaman and Jeong (1995)

Graph to determine ductility index μ_{en} for beam with elasto-plastic reinforcement according to Naaman and Jeong (1995)

Fig. 8 Total and elastic energy under load-deflection curve (Naaman and Jeong 1995)

Naaman and Jeong (1995) proposed a ductility index μ_{en} as:

$$\mu_{en} = 0.5\left(\frac{E_{tot}}{E_{ela}} + 1\right) \tag{5}$$

where (E_{tot}) is the total energy under the load-deflection curve up to failure load (Fig. 8) and (E_{ela}) is the elastic energy computed as the area of the triangle formed at failure load by unloading the beam (Fig. 8). The development of this equation was based on the assumption that the concrete beams has fully elasto-plastic behaviour and is equally applicable for beams with FRP. ACI-440 (2001) defined the ductility index as the ratio of energy absorption (area under the moment-curvature curve) at ultimate strength of the section to the energy absorption at service level (ACI-440, 2001).

Abdelrahman, et. al. (1995) presented a deformability factor μ as:

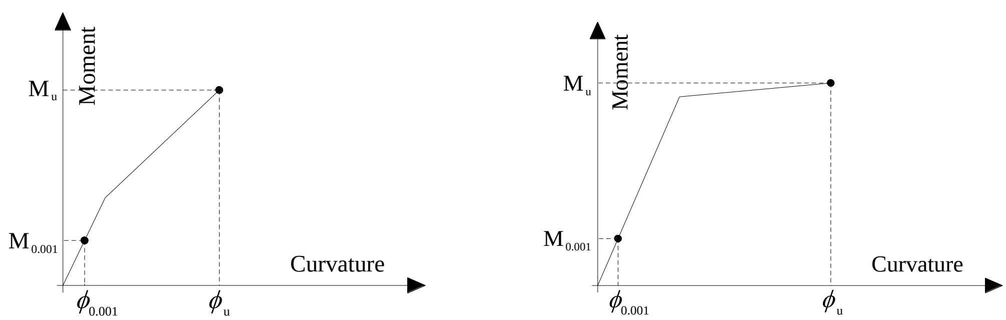
$$\mu = \Delta_u / \Delta_l \tag{6}$$

where (Δ_u) is the ultimate deflection and (Δ_l) is the equivalent deflection at uncracked section.

Mufti, et. al. (1996) and Jaeger, et. al. (1997) proposed an overall performance factor μ_M as:

$$\mu_M = (M_u\ \phi_u / M_{0.001}\ \phi_{0.001}) \tag{7}$$

where (M_u) is the ultimate moment, (ϕ_u) is the curvature at ultimate state, ($\phi_{0.001}$) is the curvature at concrete strain of 0.001 at the outer most compression fibre and ($M_{0.001}$) is the moment at a concrete strain of 0.001 at the outer most compression fibre (Fig. 9).



Graph to determine overall performance factor μ_M for beam with elastic reinforcement according to Mufti, et. al. (1996)

Graph to determine overall performance factor μ_M for beam with elasto-plastic reinforcement according to Mufti, et. al. (1996)

Fig. 9 Moment-curvature diagram for concrete beams in elastic and elasto-plastic behaviour

This model was developed for concrete beams with a rectangular cross-section and was based on a particular type of failure mode, namely, crushing of the concrete. The researchers claimed that under service load conditions, the concrete strain at the top compression fibre is about 0.001 for reinforced concrete beams.

Canadian Highway Bridge Design (CAN/CSA) (2000) Code has included provisions for fibre-reinforced structures where FRP is used as reinforcement and the Code includes a section "*Design for Deformability*". The code mentioned that for concrete beams reinforced with FRP bars or grids, the overall performance factor ($_{M}$) must be at least 4.0 for rectangular sections and 6.0 for T–sections (Bakht, et. al. 2000).

7. DUCTILITY AND DEFORMABILITY INDICES

Depending on the conventional, modified ductility and deformability indices that mentioned in previous section as well as the experimental results, the comparison between the ductility and deformability indices for the flexural cracked tested group beams are summarized in Table 4.

Table 4 Ductility and deformability indices for the flexural tested group beams

Group Beam number	GB1	GB2	GB3	GB4	GB5	GB6
Type of Reinforcement	GFR	GFR	GFR	SRFT	SRFT	SRFT
Reinforcement ratio [%]	0.50	0.75	1.00	0.50	0.75	1.00
Conventional Ductility						
Displacement ductility, μ_{Δ}	NA	NA	NA	3.39	3.08	1.92
Rotation ductility, μ_{θ}	NA	NA	NA	2.91	2.23	1.58
Curvature ductility, μ_{ϕ}	NA	NA	NA	5.15	3.34	2.47
Modified Ductility						
Ductility index, μ_{en}	1.540	1.321	1.261	3.680	6.810	14.330
Deformability factor, μ	16.11	18.23	27.45	93.83	61.22	44.37
Performance factor, μ_M	7.501	6.798	5.901	7.179	6.021	5.744

NA: Not Applicable

For the conventional ductility (Table 4), the ductility of steel reinforced beams with low reinforcement ratios is less than that with high reinforcement ratio, while the conventional ductility equations are not applicable for GF reinforced beams.

As a comparison between the different modified ductility and deformability indices mentioned in Table 4, the deformability factor (μ) and the performance factor (μ_M) show logical and acceptable trend as conventional ductility for steel reinforced beams, while the ductility index (μ_{en}) and the performance factor (μ_M) show similar trend for the ductility of GF reinforced beams. The performance factor (μ_M) plays as indicator for the ductile behaviour when it compared to the limit value that were mentioned in Canadian Highway Bridge Design (CAN/CSA) Code (2000), which must be at least 4.0 for rectangular sections, therefore, the performance factor (μ_M) shows that all the tested GF reinforced beams behave as ductile beams, although they contain brittle materials such as GFR and concrete. This may be due to the high deformation capacity of these beams before their failure, which present enough warranty before failure.

8. CONCLUSION

The conventional ductility for steel reinforced concrete beams showed that when the reinforcement ratio increased the ductility of the beams decreased specially in the case of curvature ductility.

By comparing the three modified ductility equations for steel reinforced concrete beams, they showed various trends. In the case of ductility index, the ductility increased by increasing the steel reinforcement ratio, which is not meet the trend of conventional ductility. However, the deformability factor and the performance factor for steel reinforced beams are closed to conventional ductility values especially in the case of deformability factor. Thus, the deformability factor may be used as a comparison index for both steel reinforced beams and GF reinforced beams.

The GF reinforced beams reach their peak moment capacity just before failing. However, it should be stated that the GF reinforced beams are capable of exhibiting deformation characteristics comparable to that of steel reinforced beams before failure, the only difference being that they are unable to sustain peak capacities for long before failing. This can be attributed to the elastic ideal plastic behaviour of steel and a purely elastic behaviour of GFR rods.

In General, the GFR concrete beams showed ductile behaviour less than that of steel reinforced concrete beams. The ductile behaviour of GF reinforced beams enhanced by increasing the GF reinforcement ratio as the deformability factor showed. This may be referring to the increasing in the deformation capacity before failure for the beams that reinforced with high GF reinforcement ratio.

The high deformation of GF reinforced beams can be attributed to the rods being capable of fairly large strains before reaching an ultimate strength of 1000 MPa that will not occurred, where the GFR designed strength is less than 360 MPa (Table 1). It can be stated that at high reinforcement ratio, the difference between the maximum deflections values prior to failure for GF reinforced beams and steel reinforced beams will be more than that at low reinforcement ratio. The deformability factor shows that, the tested GF reinforced beams behave as ductile beams, although they contain brittle materials such as GFR and concrete. This may be due to the high deformation capacity of these beams before their failure, which present enough warranty before failure.

9. REFERANCES

Abdelrahman A., (1995), *Serviceability of concrete beams Prestressed by Carbon Fiber Reinforced Plastics Tendons*, PhD. thesis, Univ. of Manitoba, Canada.

Abdelrahman A., Tadro G., and Rizkalla S.H. (1995), "Test Model for the First Canadian Smart Highway Bridge", *ACI Structural Journal*, 92(4), pp. 451-458.

Almusallam T.H., (1997), "Analytical Prediction of Flexural Behaviour of Concrete Beams Reinforced with FRP bars", *Journal of Rheology*, Vol. 31 (7), pp. 640–657.

American Concrete Institute, (2001), *Guide for Design and Construction of Concrete Reinforced with FRP Bars*, ACI-440.1R-01, Detroit.

American Concrete Institute, (1999), *Building Code Requirements for Reinforced Concrete*, ACI-318-99, Detroit.

Ashour S.A., (2000), "Effect of Compressive Strength and Tensile Reinforcement Ratio on Flexural Behaviour of High-Strength Concrete Beams", *Engineering Structures J.*, 22(5), pp. 413-423.

Bachmann H., (1992), *Dynamische Belastingen, Earthquake Actions on Structures*, constructief ontwerp, Reprint from Cement, No. 11.

Bakht B., G. Al-Bazi, N. Banthia, M. Cheung, M. Erki, M. Faoro, A. Machida, A.A. Mufti, K.W. Neale, and G. Tadros, (2000), "Canadian Bridge Code Provisions for Fibre Reinforced Structures, J. of Composite Construction", 4(1), pp. 3-15.

Canadian Highway Bridge Design Code: CAN/CSA-S6-00, (2000), *A National Standard of Canada*, CSA.

Daia Zwicky, (2013), "Bond and Ductility: A Theoretical Study on the Impact of Construction Details – Part 1": *Basic Considerations. Advances in Concrete Construction J.*, Vol. 1(1), pp.103-119.

Dong J.F., Wang Q.Y. and Guan Z.W., (2012), "Structural Behavior of RC Beams Externally Strengthened with FRP Sheets under Fatigue and Monotonic Loading", *Eng. Struct.*, 41: 24-33. DOI:10.1016/j.engstruct.2012.03.024.

Egyptian Code of Practice: ECP 203, (2007), *Design and Construction for Reinforced Concrete Structures*, Ministry of Building Construction, Research Center for Housing Building and Physical Planning, Cairo, Egypt.

Faza S., and Gangarao H.V.S., (1991), "Bending Response of Beams Reinforced with FRP Rebars for Varying Concrete Strengths", *Advanced Composite Materials in Civil Engineering Structures*, pp. 262–270.

Francis T.K. Au, Cliff C.Y. Leung, and Albert K.H. Kwan, (2011), "Flexural ductility and deformability of reinforced and prestressed concrete sections", *Computers and Concrete J.*, Vol. 8(4), pp. 473-489.

Huanzi, W. and A. Belarbi, (2011), "Ductility Characteristics of Fiber-Reinforced-Concrete Beams Reinforced with FRP rebars". *Journal of Constr. Buil. Mater*., 25:2391-2401. DOI:10.1016/j.conbuildmat. 2010.11.040.

Jaeger L.G., Mufti A. and Tadros G., (1997), "The Concept of the Overall Performance Factor in Rectangular Section Reinforced Concrete Beams", *Proceeding of 3rd International Symposium On Non-metallic (FRP) Reinforcement for Concrete Structures, Japan Concrete Institute*, Tokyo, Vol. 2, pp. 551-558.

Mahalingam M., Rao R.P.N. and Kannan S., (2013), "Ductility Behavior Fiber Reinforced Concrete Beams Strengthed with Externally Bonded Glass Fiber Reinforced Polymer Laminates", *American Journal of Applied Sciences*, 10 (1), 2013, pp. 107-111.

Martinola, G., A. Meda, G.A. Plizzari and Z. Rinaldi, (2010), "Strengthening and Repair of RC Beams with Fiber Reinforced Concrete". *Cem. Concr. Comp.*, 32: 731-739. DOI: 10.1016/j.cemconcomp.2010.07.001.

Materschläger A. and Bergmeister K., (1998), *Fiber Added Concrete*, 2nd Int. PhD Symposium in Civil Engineering, Budapest.

Mufti A., Newhook J.P. and Tadros G., (1996), "Deformability Versus Ductility in Concrete Beams with FRP Reinforcement", *Proceeding of Advanced Composite Materials in Bridges and Structures*, Québec, pp. 189-199.

Naaman A.E. and Jeong S.M., (1995), "Structural Ductility of Concrete Beams Prestressed with FRP Tendons. Non-metallic (FRP) Reinforcement for Concretes Structures", *Taerwe, L. ed.*, E.&FN Spoon, London, pp. 379-401.

Nanni A., Bakis C. and Boothby T.E., (1995), "The Methods of FRP-Concrete systems subjected to mechanical loads: State-of-the-Art review", *J. Reinf. Plast. Compos.*, Vol. 14, pp. 524-558.

A. Nanni, (1993), "Flexural Behavior and Design of RC Members using FRP Reinforcement", *Structural Engineering Journal*, Vol. 119 (11), pp. 3344–3359.

Park R., (1992), "Capacity Design of Ductile RC Building Structures for Earthquake Resistance", *Structural Engineering Journal*, Vol. 70 (16/18), pp. 279–289.

Patrick X.W., (2003), "Flexural Behavior and Deformability of Fibre Reinforced Polymer Prestressed Concrete Beams", *J. of Composites for Construction*, Vol.7 (4).

Sadaatmanesh H. and Ehsani M.R., (1991), "Fiber Composite Bar for Reinforced Concrete Construction", *J. Cryst. Growth*, Vol. 25, pp. 188–203.

Satoh K., Kodama K. and Ohki H., (1991), "A study on the bending behaviour of repaired concrete beams using fiber reinforced plastic (FRP) and polymer mortar", *Proceeding of ACI International Conference, Evaluation and Rehabilitation of Concrete Structures and Innovations in Design*, Detroit, Mich., pp. 1017–1031.

Theriault M. and Benmokrane B., (1998), "Effects of FRP Reinforcement Ratio and Concrete Strength on Flexural Behavior of Concrete Beams", *Journal of Composites for Construction*, Vol. 2 (1), pp. 7–16.

Thiagarajan G., (2003), "Experimental and Analytical Behaviour of Carbon Fiber-Based Rods as Flexural Reinforcement", *Journal of Composites for Construction*.