























$$\mu_{\theta} = \theta_u / \theta_y \quad (3)$$

where  $(\theta_u)$  is the ultimate plastic hinge rotation and  $(\theta_y)$  is the yield plastic hinge rotation.

### Displacement ductility $\mu_{\Delta}$ :

$$\mu_{\Delta} = \Delta_u / \Delta_y \quad (4)$$

where  $(\Delta_u)$  is the displacement of a structural element or of a whole structure at the ultimate state and  $(\Delta_y)$  is the displacement at yield limit.

Ashour (2000) mentioned that members with a displacement ductility in the range of 3 to 5 has adequate ductility and can be considered for structural members subjected to large displacement, such as sudden forces caused by earthquake.

All these conventional methods for ductility are not suitable for beams with FRP as FRP does not have a yield point (Patrick 2003). Researchers have proposed some new equations to quantify the ductility of concrete beams reinforced by FRP and by steel so that a comparison may be made between them. These modified ductility or deformability factors can be expressed as following.

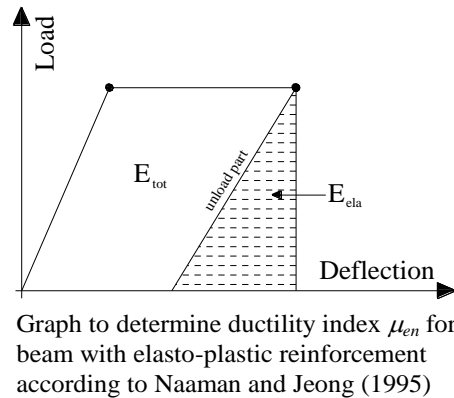
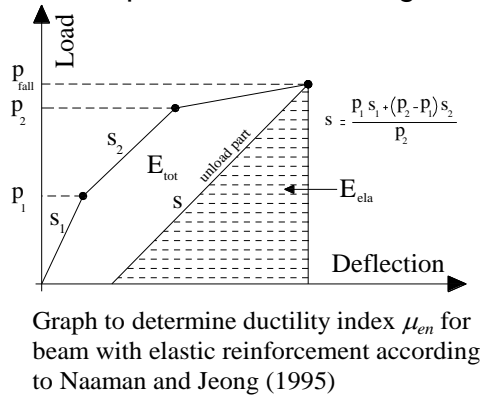


Fig. 8 Total and elastic energy under load-deflection curve (Naaman and Jeong 1995)

Naaman and Jeong (1995) proposed a ductility index  $\mu_{en}$  as:

$$\mu_{en} = 0.5 \left( \frac{E_{tot}}{E_{ela}} + 1 \right) \quad (5)$$

where  $(E_{tot})$  is the total energy under the load-deflection curve up to failure load (Fig. 8) and  $(E_{ela})$  is the elastic energy computed as the area of the triangle formed at failure load

by unloading the beam (Fig. 8). The development of this equation was based on the assumption that the concrete beams has fully elasto-plastic behaviour and is equally applicable for beams with FRP. ACI-440 (2001) defined the ductility index as the ratio of energy absorption (area under the moment-curvature curve) at ultimate strength of the section to the energy absorption at service level (ACI-440, 2001).

Abdelrahman, et. al. (1995) presented a deformability factor  $\mu$  as:

$$\mu = \Delta_u / \Delta_l \quad (6)$$

where  $(\Delta_u)$  is the ultimate deflection and  $(\Delta_l)$  is the equivalent deflection at uncracked section.

Mufti, et. al. (1996) and Jaeger, et. al. (1997) proposed an overall performance factor  $\mu_M$  as:

$$\mu_M = (M_u \phi_u / M_{0.001} \phi_{0.001}) \quad (7)$$

where  $(M_u)$  is the ultimate moment,  $(\phi_u)$  is the curvature at ultimate state,  $(\phi_{0.001})$  is the curvature at concrete strain of 0.001 at the outer most compression fibre and  $(M_{0.001})$  is the moment at a concrete strain of 0.001 at the outer most compression fibre (Fig. 9).

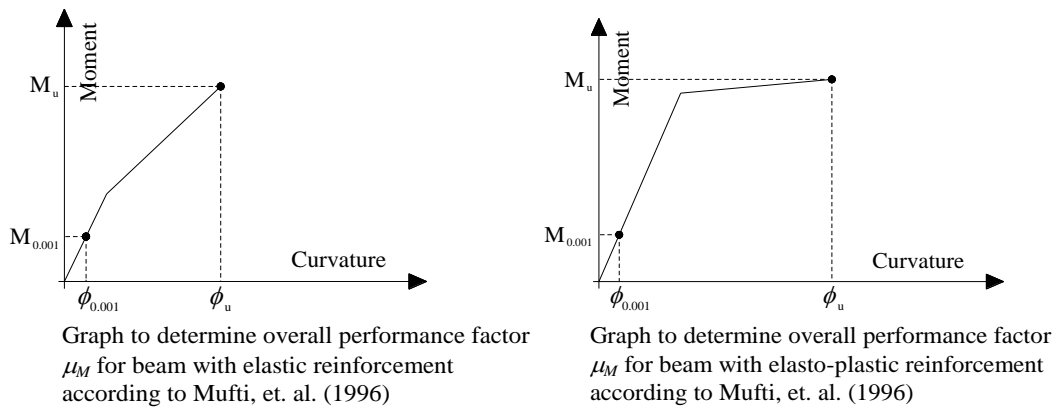


Fig. 9 Moment-curvature diagram for concrete beams in elastic and elasto-plastic behaviour

This model was developed for concrete beams with a rectangular cross-section and was based on a particular type of failure mode, namely, crushing of the concrete. The researchers claimed that under service load conditions, the concrete strain at the top compression fibre is about 0.001 for reinforced concrete beams.

Canadian Highway Bridge Design (CAN/CSA) (2000) Code has included provisions for fibre-reinforced structures where FRP is used as reinforcement and the Code includes a section “*Design for Deformability*”. The code mentioned that for concrete beams reinforced with FRP bars or grids, the overall performance factor ( $\mu_M$ ) must be at least 4.0 for rectangular sections and 6.0 for T-sections (Bakht, et. al. 2000).

## 7. DUCTILITY AND DEFORMABILITY INDICES

Depending on the conventional, modified ductility and deformability indices that mentioned in previous section as well as the experimental results, the comparison between the ductility and deformability indices for the flexural cracked tested group beams are summarized in Table 4.

Table 4 Ductility and deformability indices for the flexural tested group beams

Group Beam number	GB1	GB2	GB3	GB4	GB5	GB6
Type of Reinforcement	GFR	GFR	GFR	SRFT	SRFT	SRFT
Reinforcement ratio [%]	0.50	0.75	1.00	0.50	0.75	1.00
Conventional Ductility						
Displacement ductility, $\mu_{\Delta}$	NA	NA	NA	3.39	3.08	1.92
Rotation ductility, $\mu_{\theta}$	NA	NA	NA	2.91	2.23	1.58
Curvature ductility, $\mu_{\phi}$	NA	NA	NA	5.15	3.34	2.47
Modified Ductility						
Ductility index, $\mu_{en}$	1.540	1.321	1.261	3.680	6.810	14.330
Deformability factor, $\mu$	16.11	18.23	27.45	93.83	61.22	44.37
Performance factor, $\mu_M$	7.501	6.798	5.901	7.179	6.021	5.744

NA: Not Applicable

For the conventional ductility (Table 4), the ductility of steel reinforced beams with low reinforcement ratios is less than that with high reinforcement ratio, while the conventional ductility equations are not applicable for GF reinforced beams.

As a comparison between the different modified ductility and deformability indices mentioned in Table 4, the deformability factor ( $\mu$ ) and the performance factor ( $\mu_M$ ) show logical and acceptable trend as conventional ductility for steel reinforced beams, while the ductility index ( $\mu_{en}$ ) and the performance factor ( $\mu_M$ ) show similar trend for the ductility of GF reinforced beams. The performance factor ( $\mu_M$ ) plays as indicator for the ductile behaviour when it compared to the limit value that were mentioned in Canadian Highway Bridge Design (CAN/CSA) Code (2000), which must be at least 4.0 for rectangular sections, therefore, the performance factor ( $\mu_M$ ) shows that all the tested GF reinforced beams behave as ductile beams, although they contain brittle materials such as GFR and

concrete. This may be due to the high deformation capacity of these beams before their failure, which present enough warranty before failure.

## **8. CONCLUSION**

The conventional ductility for steel reinforced concrete beams showed that when the reinforcement ratio increased the ductility of the beams decreased specially in the case of curvature ductility.

By comparing the three modified ductility equations for steel reinforced concrete beams, they showed various trends. In the case of ductility index, the ductility increased by increasing the steel reinforcement ratio, which is not meet the trend of conventional ductility. However, the deformability factor and the performance factor for steel reinforced beams are closed to conventional ductility values especially in the case of deformability factor. Thus, the deformability factor may be used as a comparison index for both steel reinforced beams and GF reinforced beams.

The GF reinforced beams reach their peak moment capacity just before failing. However, it should be stated that the GF reinforced beams are capable of exhibiting deformation characteristics comparable to that of steel reinforced beams before failure, the only difference being that they are unable to sustain peak capacities for long before failing. This can be attributed to the elastic ideal plastic behaviour of steel and a purely elastic behaviour of GFR rods.

In General, the GFR concrete beams showed ductile behaviour less than that of steel reinforced concrete beams. The ductile behaviour of GF reinforced beams enhanced by increasing the GF reinforcement ratio as the deformability factor showed. This may be referring to the increasing in the deformation capacity before failure for the beams that reinforced with high GF reinforcement ratio.

The high deformation of GF reinforced beams can be attributed to the rods being capable of fairly large strains before reaching an ultimate strength of 1000 MPa that will not occurred, where the GFR designed strength is less than 360 MPa (Table 1). It can be stated that at high reinforcement ratio, the difference between the maximum deflections values prior to failure for GF reinforced beams and steel reinforced beams will be more than that at low reinforcement ratio. The deformability factor shows that, the tested GF reinforced beams behave as ductile beams, although they contain brittle materials such as GFR and concrete. This may be due to the high deformation capacity of these beams before their failure, which present enough warranty before failure.

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