

8. CONCLUSION AND REMARKS

A set of singularity-reduced boundary integral equations has been established for an isotropic, linearly elastic half-space containing arbitrary shaped cracks and subjected to various types of boundary conditions. The systematic regularization procedure proposed by Rungamornrat and Mear (2008a) along with results proposed by Li (1996) has been adopted to obtain the final completely regularized integral equations containing only weakly singular kernels. Another key feature of the developed integral equations is the automatic treatment of the free surface via the use of the fundamental solutions of the half-space with the same type of boundary conditions. This therefore avoids the discretization of the free surface in the solution procedure. A weakly singular symmetric Galerkin boundary element method has been implemented to numerically solve the weak-form traction boundary integral equation for the relative crack-face displacement and such information has then been used as the known data to determine the sum of the crack-face displacement from the weak-form displacement boundary integral equation by standard Galerkin technique. Special crack-tip elements have also been employed to enhance the approximation of the near-front field. The fracture data along the crack front such as the stress intensity factors and the T-stresses has been directly extracted from the relative and sum of crack-face displacement using the explicit formula.

Results from extensive numerical experiments and the comparison with several benchmarked cases have revealed that the proposed numerical procedure is highly accurate and computationally robust for the analysis of cracked half-spaces under various scenarios including arbitrary crack geometry, different boundary conditions and general loading conditions. Use of special crack-tip elements along the crack front has indicated that the stress intensity factors and T-stresses can be accurately captured using relatively very coarse meshes and this therefore renders the technique more suitable for linear fracture analysis than the standard finite element method which generally requires sufficiently fine mesh to capture the near-front field and experiences difficulty in treatment of an unbounded domain. While the proposed technique has been successfully implemented, it is still restricted to a half-space made of an isotropic linearly elastic material. The potential extension of the current work to treat a more general class of materials such as anisotropic solids and smart media is considered essential.

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