Reliability analysis of laterally loaded pile for an offshore wind turbine support structure

Gil-Lim Yoon[1], *Sun-Bin Kim[2], Jin-Hak Yi[1] and Jun-Hwan Lee[3]

[1], [2] *Coastal & Environmental Engineering Division, KIOST, Ansan 426-744, Korea*
[3] *School of Civil and Environmental Engineering, Yonsei Univ., Seoul 120-749, Korea*
[2] *ksb9677@kiost.ac.kr*

ABSTRACT

With an increasing demand of a renewable energy, new offshore wind turbine farms are being planned in some parts of the world. Foundation installations need a significant cost of the total budget of offshore wind turbine (OWT) projects. Hence, a cost reduction from foundation parts is a key when a cost-efficient designing of OWT budget. Mono-piles have been largely used, accounting about 78% of existing OWT foundations, because they are considered as a most economical alternative with a relatively shallow-water, less than 30m, depths. And OWT design codes such as IEC, GL, DNV, API, and Eurocode are being developed in a form of reliability based limit state design method. In this paper, reliability analysis using a response surface and simulation methods for an OWT mono-pile foundation were performed to investigate the sensitivities of mono-pile design parameters, and to find practical implications of reliability analysis.

1. INTRODUCTION

With an increasing demand of a renewable energy, several offshore wind farms are being planned around the world. It is well-known that offshore wind turbine (OWT) foundations take account of a significant part of the total budget. Hence, reduction of the foundation cost is a key to more cost-efficient wind energy. Two aspects of interest when designing an OWT foundation are the selection of type of foundation and design methodology. First, there have been developed various types of foundations for an OWT, including gravity-typed structure, mono-pile, jacket, tripods and suction bucket types. Most of those types of foundations are pile foundation except gravity-typed structure. Mono-pile is especially the most popular, accounting for 75~80 % of existing OWT foundations, by reason that those have been the most economical alternative with the relatively shallow-water depths (Doherty and Gavin, 2011). Secondly, design

---

[1] Principal Research Scientist
[2] Research Scientist
[3] Professor

methods are divided into two main approaches, in which one is deterministic approach and the other is probabilistic approach. Deterministic approach in general is based on criteria believed to be conservative. Then the trend is gradually changing toward probabilistic approach. Actually, the design codes for OWT such as EC, GL, DNV, API, ISO and Eurocode have been developed in the form of limit state design method based on reliability and probabilistic approach. On the one hand, one of the well-known methods in practice to analyze pile behaviors under lateral loadings is to model the pile as a vertical beam column supported by a set of springs. The p-y curves vary according to soil properties, pile dimension, depth, etc. However, due to the inherent uncertainties in nature, it is difficult to determine the load-displacement characteristics of the p-y curves with depth precisely in deterministic approach. The factor of safety obtained in a deterministic manner does not explicitly account for the uncertainties of load and resistance such as wind, wave and soil properties. Because of such uncertainties, it is necessary to adopt a probabilistic approach which can count on uncertainties in load and resistance when pile designing.

In this paper deals with the reliability analysis of mono-pile foundation for an OWT, which have planned to install at the test-bed in West-South coastal zone of Buan-Yeongkwang at the Yellow Sea of Korea. The 5 MW NREL wind converter was employed in the study and reliability analyses using response surface and Monte Carlo Simulation methods for the ultimate and serviceability limit states of the OWT mono-pile were performed. Comparison study of the reliability analysis with deterministic analysis is performed and practical implications of the findings are discussed.

2. PROBABILISTIC ANALYSIS METHOD

Reliability analysis has been applied to structural design and safety reassessment of the existing structures. The probability density function of the values of the performance function can be estimated by carrying out reliability analysis using the Monte Carlo simulation (MCS) and the first/second-order reliability methods (FORM/ SORM). MCS is a numerical process to evaluate the performance function through repeated calculation based on a large number of realizations of the random variables defining the function. A MCS starts with the generation of random numbers with respective prescribed probability distributions. Methods for generating a set of random numbers with well-known distributions are widely available. The accuracy of the probability of failure obtained through MCS will improve with the sample size which is number of random numbers generated for each distribution. The ordinary Monte Carlo method can be prohibitively costly for cases with very small failure probabilities, and where the deterministic analysis for each simulation trial is computationally intensive.

Reliability index approach is one of the most reliable computational methods for structural reliability. Practical difficulty or unnecessary hardship in computing probability of failure directly has led to the development of various approximation methods, of which the first-order reliability method (FORM) is considered to one of the most reliable computational methods. FORM is an analytical approximation in which the reliability index is interpreted as the minimum distance from the origin to the limit state surface in

standardized normal space and the most probable failure point (MPFP, design point) is searched using mathematical methods.

3. RESPONSE SURFACE METHODOLOGY

The FORM generally demands the values and partial derivatives of the limit state function (LSF) with respect to the design random variables. Such calculations can be performed efficiently when the LSF g(x') can be expressed in an explicit form or simple analytical form in terms of the design random variables x'. However, when the LSF is implicit, such calculations require additional efforts. A few approaches have been developed to cope with the problems with implicit LSF. One of the popular approaches is the response surface method (RSM). Response surface is the derived virtual surface which can be represented by the function of random variables. The surface is found by regression with limited responses from structural analysis and expressed in an explicit function of random variables. Then FORM is easily applied by using approximate response surface function. LSF in implicit form can be written as

$$g(X) = R(X_1, X_2, \ldots X_n) - S(X_1, X_2, \ldots X_n) \tag{1}$$

where R is the resistance, S is the loading function and X_i is the random variable.

First/Second order approximation of Eq. (1) can be expressed as

$$g'(X) = c_0 + c_1 X_1 + \cdots + c_n X_n \tag{2}$$

$$g''(X) = c_0 + \sum_{i=1}^{k} c_i X_i + \sum_{i=1}^{k} c_{ii} X_i^2 + \sum_{i<j} c_{ij} X_i X_j \tag{3}$$

where C_i is regression coefficient estimated by using structural responses.

The approximated function $g'(X)$ is a first-order model, when the response is a linear function of independent variables. When there is a curvature in the response surface, the first-order model is insufficient. A second-order model is useful in approximating a portion of the true response surface. The second-order model includes all the terms in the first-order model, plus all quadratic terms like $c_{ii} X_i^2$ and all cross product terms like $c_{ij} X_i X_j$.

It is important to select sampling points for the accuracy of approximation of response surface. There are many designs available for fitting a second-order model. The most popular one is the central composite design (CCD) and the other one is the Bucher-Bourgunnd (B-B) method. The CCD involves $2k$ the axial points, 2^k factorial points and 1 central point. While B-B method involves only the axial points and central point but not cross term of factorial points. In these methods, sampling points to evaluate the coefficients C_0, C_i, C_{ij} are possible combinations of X_i's. The sampling points are selected to be located at $\mu \pm f \cdot \sigma$, where μ and σ are the mean and the

standard deviation, respectively, and *f* is the axis point distance, which is a parameter determining the upper and lower limits in selection ranges.

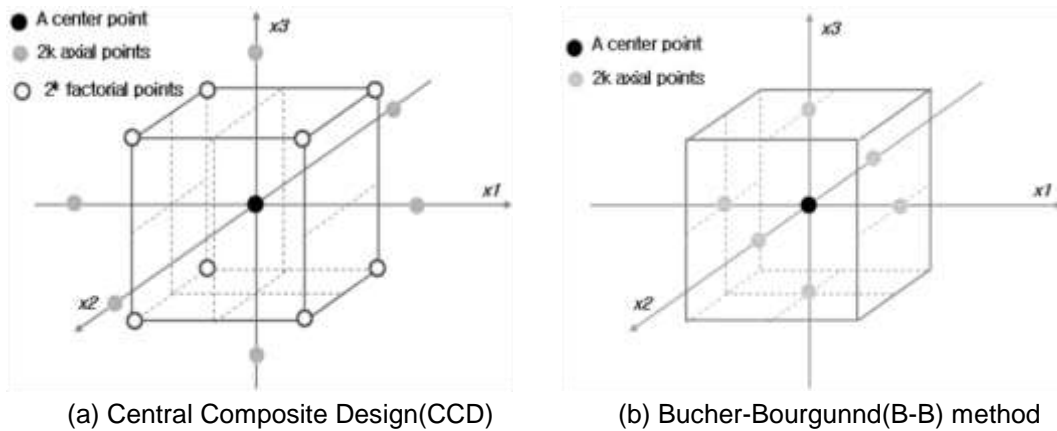(a) Central Composite Design(CCD)   (b) Bucher-Bourgunnd(B-B) method

Fig. 1. Experimental designs for fitting response surfaces

The probabilistic characteristics of an original limit state may not be properly represented by the response surface function evaluated from information at the sampling points in the vicinity of the mean values of basic random variables. To improve the accuracy of the response surface method, Bucher and Bourgunnd (1990) suggested an alternative process of selecting the sampling points. In the first step of this algorithm, the mean vector is selected as the center point. Then the response surface is used to find an estimate of the design point, X_D, on an interpolated limit state. In next step, the new center point is chosen on a straight line from the mean vector μ_X to X_D so that $g(x)=0$ at the new center point, X_M, from linear interpolation, i.e.,

$$X_M = \mu_X + (X_D - \mu_X)\frac{g(\mu_X)}{g(\mu_X) - g(X_D)} \tag{4}$$

This process is assumed to guarantee that the sampling points chosen according to the new center point include information on an original failure surface sufficiently. This method is also called the adaptive response surface method.

4. RELIABILITY ANALYSIS

4.1 Case study

This paper focuses on preliminary design of OWT foundation at the test-bed site of Buan-Yeongkwang in the Yellow Sea of Korea. Offshore wind turbine NREL 5.0MW OWT mono-pile type is referred for a comparison as shown in Fig. 2, which has a hub height of 87.6m and water depth of 15.0m (see Table 1). The combined load calculations at a seabed are shown in Table 2, which is based on DLCs 1.3, 1.4 and 6.2 of IEC 61400-3 standard. Ground conditions are also shown in Table 3, which were estimated using geotechnical report including SPT, CPT and unconfined & triaxial

compression tests at a test-bed site. Table 3 shows that material properties of seabed soils are for a total stress analysis, considering that seabed soils consist of low permeable clay layer.
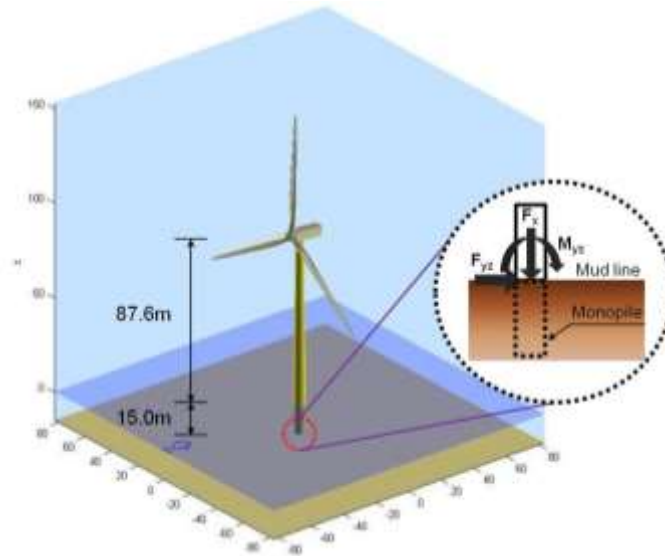

Fig. 2. 5MW monopile type OWT

Table 1. Dimensions of the reference OWT

Category	Turbine	Hub height (m)	Water depth (m)
Dimensions	NREL 5.0MW	87.6	15.0

Table 2. The combined loads at seabed

Category	F_x (kN)	F_{yz} (kN)	M_{yz} (kN·m)
Load	11,525.0	1,676.9	168,507.0

Table 3. Ground conditions and material properties of seabed soils

Soil layer		Depth (m)	Thickness (m)	Unit weight, γ_{sat} (kN/m^3)	Cohesion, c (kPa)	Internal friction angle, ϕ (°)
Clay	CH	0~5.0	5.0	17.0	20.00	-
	CL(1)	5.0~12.3	7.3	18.0	33.54	-
Sand	SM	12.3~23.0	10.7	19.0	16.63	31.59
Clay	CL(2)	23.0~30.0	17.0	18.0	60.00	-

It is important to quantify the uncertainties of loads such as gravity, wind, wave, current, and material properties in a probabilistic approach. Reliability analysis requires probability distribution function and variability of design parameters. In this paper, undrained shear strength including cohesion and internal friction angle, which are main design parameters of seabed soils, are defined as random variables. They are normally

distributed and coefficients of variance (*COV*) are estimated 0.26 (26%) and 0.063 (6.3%) as cohesion and internal friction angle, respectively, based on statistical analysis from site investigation results (Yoon et al., 2014).

Mono-pile foundation dimensions such as pile diameters, thickness and embeded pile length are referred by NREL 5.0MW. Table 4 presents dimensions for two types of steel mono-pile in terms of pile diameter 6.0 m and 7.0 m. The pile is modeled by beam elements with Young's modulus of $2.1{\times}10^8$ kPa and unit weight 77 kN/m^3. And surrounding soils are discretized by nonlinear springs to model soil-pile interaction. In this paper, t-z and q-z curves are based on API (2005), and p-y curves are based on API (2005) and Evans & Duncan (1992) to clay and sand layers, respectively. Element size was specified to be 0.1m.

Table 4. Monopile foundation dimensions

Category	Case 1	Case 2	Remarks
Pile diameter (m)	6.0	7.0	steel pile
Pile wall thickness (mm)	60.0	30.0	
Embeded pile length (m)	21.6	22.5	embedded in sand

Generally in the serviceability limit state design for the stability of the whole OWT structure under lateral loads such as wind, wave and current, etc., it shall be ensured that lateral deflection and rotational angle tolerances should not be exceeded. Accordingly, the major failure modes of mono-pile are considered as the lateral pile head displacement and rotational angle, and limit state functions (LSFs) can be expressed as follows

$$g_1 = \delta_a - \delta_{\max}(c_{u1}, c_{u2}, c_3, \phi_3) \tag{5}$$

$$g_2 = \theta_a - \theta_{\max}(c_{u1}, c_{u2}, c_3, \phi_3) \tag{6}$$

where δ_a and θ_a are the allowable lateral displacement and rotations of the pile head δ_{max} and θ_{max} are the lateral pile head displacement and rotational angle; c_{u1} is the undrained shear strength of clay_CH layer; c_{u2} is the undrained shear strength of clay_CL(1) layer; c_3 and ϕ_3 are the cohesion and internal friction angle of sand_SM layer. δ_{max} and θ_{max} are performance functions of random variables such as c_{u1}, c_{u2}, c_3 and ϕ_3 by numerical investigation results. δ_a and θ_a are considered as 1% of pile diameter and 0.3 degrees respectively (DNV, 2007; Kuo et al., 2008).

To convert Eqs. (5) and (6) to explicit one, g_1 and g_2 are expressed as a function of design random variables as follows

$$
\begin{aligned}
g_1' = {} & C_0 + C_1 c_{u1} + C_2 c_{u2} + C_3 c_3 + C_4 \phi_3 + C_5 c_{u1}^2 + C_6 c_{u2}^2 + C_7 c_3^2 + C_8 \phi_3^2 \\
& + C_9 c_{u1} c_{u2} + C_{10} c_{u1} c_3 + C_{11} c_{u1} \phi_3 + C_{12} c_{u2} c_3 + C_{13} c_{u2} \phi_3 + C_{14} c_3 \phi_3 \\
& + C_{15} c_{u1} c_{u2} c_3 + C_{16} c_{u1} c_{u2} \phi_3 + C_{17} c_{u1} \phi_3 + C_{18} c_{u2} c_3 \phi_3 + C_{19} c_{u1} c_{u2} c_3 \phi_3
\end{aligned}
\tag{7}
$$

$$g'_2 = C'_0 + C'_1 c_{u1} + C'_2 c_{u2} + C'_3 c_3 + C'_4 \phi_3 + C'_5 c^2_{u1} + C'_6 c^2_{u2} + C'_7 c^2_3 + C'_8 \phi^2_3$$
$$+ C'_9 c_{u1} c_{u2} + C'_{10} c_{u1} c_3 + C'_{11} c_{u1} \phi_3 + C'_{12} c_{u2} c_3 + C'_{13} c_{u2} \phi_3 + C'_{14} c_3 \phi_3$$
$$+ C'_{15} c_{u1} c_{u2} c_3 + C'_{16} c_{u1} c_{u2} \phi_3 + C'_{17} c_{u1} \phi_3 + C'_{18} c_{u2} c_3 \phi_3 + C'_{19} c_{u1} c_{u2} c_3 \phi_3 \qquad (8)$$

where g_1' and g_2'' are approximated functions of LSF; C_i and C_i' are the regression coefficients of response surface to be estimated from structural analysis.

Reliability analyses were conducted using in-house software "FROW" developed in KIOST (2012). CCD and B-B method are used to formulate LSFs to reliability analysis. Subsequently, adaptive response surface method is used to obtain optimized approximated functions. In which the initial axis point distance, $k_{initial}$, is applied to 5. And FORM is used to calculate a reliability index. Finally reliability indices computed by the RSM-FORM are compared with MCS results for verification. MCS with 100,000 trials were carried out for each LSF, in which output sample variance of LSF gives within 0.01%.

4.2 Analysis results

Table 5 shows lateral displacements and rotational angles at a pile head with the limit state failure modes, which were calculated in a deterministic manner. Table 6 and Fig. 3 present reliability indices (β) and probabilities of failure (P_f) computed by the RSM-FORM and MCS for each critical failure mode. P_f herein means the probability exceeding allowable values of design criteria. These values were also estimated explicitly to account for the uncertainties of soil properties, which make it possible to draw more reasonable decisions in pile design, compared to the deterministic approach.

It can be observed, in Case 1, lateral deflections are dominant failure mode, whereas for Case 2, rotational angles are dominant failure mode at pile head. The β by the RSM-FORM shows the relative small errors of 0.02~1.61% compared to MCS results.

Table 5. Lateral displacements & rotational angles of pile head

Category	Failure mode	Lateral Disp. (mm)		Rotation angle (°)	
		Computed	Allowable	Computed	Allowable
Case 1	Lateral Disp.	50.5	60.0	-	-
	Rotation angle	-	-	0.252	0.3
Case 2	Lateral Disp.	51.7	70.0	-	-
	Rotation angle	-	-	0.273	0.3

Table 6. Reliability index & probability of failure

Category	Failure mode	RSM-FORM		MCS	
		β	P_f	β	P_f
Case 1	Lateral Disp.	3.35	4.005×10^{-4}	3.34	4.200×10^{-4}
	Rotation angle	N/A	N/A	4.53	2.904×10^{-6}
Case 2	Lateral Disp.	N/A	N/A	5.91	1.739×10^{-9}
	Rotation angle	3.69	1.113×10^{-4}	3.62	1.500×10^{-4}

(a) Case 1 (Lateral Displacement)   (b) Case 2 (Rotational angle)
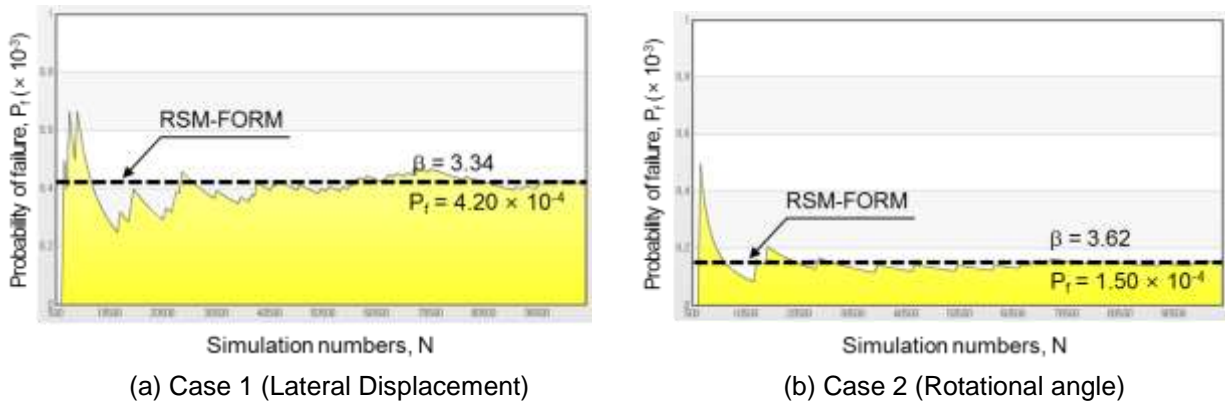
Fig. 3. Monte Carlo simulation (MCS) results

Sensitivity indices of design parameters of random variables in a reliability analysis were derived by the RSM-FORM. As shown in Fig. 4(a), internal friction angle of sand layer (ϕ_SM) is a governing factor in mono-pile lateral behaviors among random variables. Accordingly, Fig. 4(b) shows that as COV of internal friction angle increase, reliability indices tend to decrease. This means uncertainty of soil is greater, probability of failure or exceeding the serviceability limit state increase.
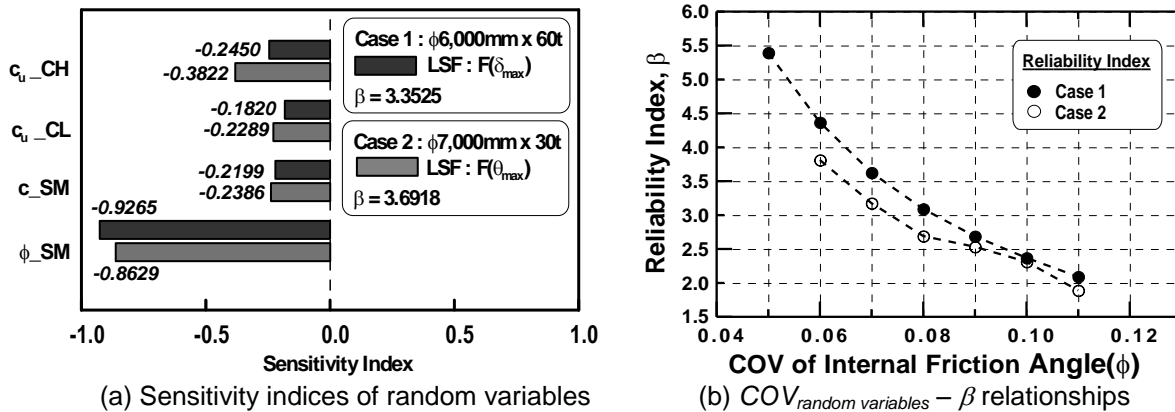

(a) Sensitivity indices of random variables   (b) $COV_{random\ variables} - \beta$ relationships

Fig. 4. Sensitivity analysis results for uncertainty of random variables

5. SUMMARY & CONCLUSIONS

Reliability analyses of laterally loaded pile for an NREL 5.0 MW offshore wind turbine support structure were performed to investigate the uncertainty and sensitivity of design parameters. Soil profile and the uncertainties of soil parameters were determined using geotechnical investigation at the test-bed in West-South coasts of Buan-Yeongkwang in Yellow Sea of Korea. First order reliability method with the response surface method (RSM-FORM) and the Monte Carlo simulation technique were used to the analyses. The pile analyses were also carried out by modeling a laterally loaded pile as a vertical beam supported by a series of discrete springs, each

of which has its own nonlinear load-displacement characteristics.

Numerical analyses indicate that reliability analysis enable to make more reasonable decisions in designing pile compared to the deterministic manner because engineers can find the criteria of probability exceeding a specified design requirement and which parameters govern in a limit states of pile behaviors.

Analyses by the RSM-FORM methods agree well with those of the Monte Carlo simulations, with slightly differences in a probability of failure. In addition, sensitivity analysis of the design variables indicate that internal friction angle of seabed sandy soil is the most dominant design factor for mono-pile lateral behaviors, and it show that as variability of the internal friction angle increase, reliability indices of mono-pile tend to decrease.

ACKNOWLEDGEMENTS

REFERENCES

API (2005), Recommended Practice for Planning, Design and Constructing Fixed Offshore Platforms-Working Stress Design, American Petroleum Institute Publishing Service, Washington D.C., 1-263.

Bucher, C. G. and Bourgunnd, U. (1990) "A fast and efficient response surface approach for structural reliability problems", *Structural Safety*, **7**, 57-66.

Doherty, P. and Gavin, K. (2011), "Laterally loaded monopile design for offshore wind farms", *Proceedings of the ICE - Energy*, **165**(1), 7-17.

DNV (2007), Design of Offshore Wind Turbine Structures, Offshore Standard DNV-OS-J101, Det Norske Veritas, Høvik, Norway, 153.

FROW (2012), Foundation RBD Program for Offshore Wind-Turbine, Korea Institute of Ocean & Science Technology, 2012, Korea (in Korean).

Kuo, Y., Achmus, M. and Kao, C. (2008), "Practical design considerations of monopile foundations with respect to scour", *Global WindPower*, Beijing, 29-31.

Yoon, G., Kim, S., Kwon, O. and Yoo, M. (2014), "Partial Safety Factor of Offshore Wind Turbine Pile Foundation in West-South Mainland Sea", *Journal of the Korean Society of Civil Engineering*, **34**(5), 1489-1504 (in Korean).