

## **An efficient method to choose basic vectors for equivalent static wind loads of long span roofs**

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### **ABSTRACT**

Long span roofs are characterized with multimode buffeting responses, and multiple objectives for equivalent static wind loads. Through the analysis of the equivalent static wind loads calculation methods of long span roofs, we found that, one key is how to choose the basic vectors. In this paper, the generalized resuming forces will be defined, then POD analysis is conducted to obtain the intrinsic modes. Generalized resuming forces are the quasi static forces of the structure at each time, So if the intrinsic modes of generalized resuming forces are chosen as the basic vectors, these basic vectors will be more effective than the others. With constraint equations, a least-squares method is employed for calculating the combination factors of these basic vectors. The universal equivalent static wind loads thus obtained may reflect the characteristics of wind induced responses.

### **1. INTRODUCTION**

The equivalent static wind loads theory research begins with the high-rise structures. So far, there are three main methods, including Gust loading factors (GLF,1967), inertial wind loads(IWL,1988), load-response-correlation (LRC,1992). However, these methods can only be applied to a single response, and not to the multiple responses for long span roofs. To solve this problem, some scholars have carried out the related research. Katsumura(2007) choose the intrinsic mode of fluctuating wind loads as the basic vectors, to find the most reasonable coefficients of combination for different responses; Hu(1992) calculates several equivalent static wind loads corresponding to some key responses, and choose those wind loads as the basic vectors, then the universal equivalent static wind loads is obtained by a least-squares method.

In this paper, the generalized resuming forces will be defined, then POD analysis is conducted to obtain the intrinsic modes. Generalized resuming forces are the quasi static force of the structure at each time, So if the intrinsic modes of generalized resuming forces are chosen as the basic vectors, these basic vectors will be more

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effective than the others. With constraint equations, a least-squares method is employed for calculating the combination factors of these basic vectors.

## 2. BASIC VECTORS AND EQUATIONS

According to the three components method, the fluctuating wind loads reflect the structural background responses, and the inertial forces reflect the resonance responses of the structure. If some basic vectors include both the background responses and the resonance responses, these basic vectors will be more effective than others.

In this paper, the generalized resuming forces are defined as follows,

$$\{p_e(t)\} = \{p(t)\} - [M]\{\ddot{y}(t)\} - [C]\{\dot{y}(t)\} \quad (1)$$

Where,  $\{p(t)\}$  is the external wind loads vector,  $M$   $C$  is the mass matrix, damping matrix of the structure,  $\{y(t)\}$  is the dynamic displacement vector.

As can be seen, generalized resuming forces are composed of 3 parts. The first part is the fluctuating wind loads, the second part is the inertial wind loads, and the third part is the damping forces. Generally speaking, for some types of long-span roofs, only one part need to be chosen, but most types of long-span roofs need to focus the fluctuating wind loads and the inertial wind loads on the same time.

According to the dynamic responses equation of structure under wind loads, the generalized resuming forces can be calculated as follows,

$$\{p_e(t)\} = \{p(t)\} - [M]\{\ddot{y}(t)\} - [C]\{\dot{y}(t)\} = [K]\{y(t)\} \quad (2)$$

Where,  $K$  is the stiffness matrix of the structure. As the dynamic displacement vector can be obtained by wind vibration calculation, then the generalized resuming forces also can be obtained.

If the generalized resuming forces is decomposed by Proper Orthogonal Decomposition method (POD, 1999), the intrinsic modes of generalized resuming forces are obtained. The intrinsic modes and eigenvalues are calculated as follows,

$$[C_{pp}]\{G\}_k = \lambda_k \{G\}_k \quad (3)$$

Where,  $[C_{pp}]$  is the Covariance matrix of generalized resuming forces,  $\{G\}_k$  and  $\lambda_k$  are intrinsic modes and eigenvalues. By selecting the first  $n$  dominant intrinsic modes, the universal equivalent static wind loads are expressed as follows,

$$\{F_e\} = [c_1\{G\}_1 + c_2\{G\}_2 + \dots + c_n\{G\}_n] = [F_0]\{c\} \quad (4)$$

Where,  $\{c\}$  is the combination coefficient vector,  $[F_0]$  is load distribution matrix by basic vectors. For the equivalent targets, the combination coefficient vector  $\{c\}$  needs to meet,

$$[\beta]\{F_e\} = [\beta][F_0]\{c\} = \{\hat{r}\} \quad (5)$$

Where,  $[\beta]$  is the influence line matrix,  $\{\hat{r}\}$  is the extreme dynamic response vector.

The universal equivalent static wind loads can be obtained by solving the Eq. (5), according to the relations between the dominant intrinsic number  $n$  and equivalent targets number  $m$ , the solution can be divided into the following three conditions,

- 1) when  $n = m$ , there is only one solution for the equation,
- 2) when  $n > m$ , there are infinite viable solutions for the equation,
- 3) when  $n < m$ , there is no solution for the equation, but there is the only one least-squares solution for the equation.

### 3. CONSTRAINT EQUATIONS

The equivalent static wind loads are not deeply discussed in the Eq. (5), so the distribution of the equivalent static wind loads is often unreasonable. The equivalent static wind loads may reach hundreds or even thousands Pa, and the distribution will be particularly concentrated.

In this paper, the constraint equations are introduced to solve this problem, the values of the equivalent static wind loads will be constrained by the equations. The method is, the absolute value of the equivalent static wind loads is no more than the value of the corresponding extreme generalized restoring forces. The constraint equations can be expressed as follows,

$$\left\{ \begin{array}{l} |F_{e1}| \leq \hat{p}_{e1} \\ |F_{e2}| \leq \hat{p}_{e2} \\ \dots \\ |F_{en}| \leq \hat{p}_{en} \end{array} \right. \quad (6)$$

Where,  $\hat{p}_{e1}, \hat{p}_{e2}, \dots, \hat{p}_{en}$  are the extreme generalized restoring forces.

After the constraint of the Eq. (6), the value of the equivalent static wind loads can be well controlled, and the distribution is reasonable.

In summary, the main steps of this analysis method can be expressed as follows,

- 1) The generalized restoring forces are decomposed by POD method, and the intrinsic modes are obtained,
- 2) Set up the Eq. (5) and Eq. (6), calculate the optimal combination coefficient vector  $\{c\}$ ,
- 3) Calculate the universal equivalent static wind loads according to Eq. (4).

## 4. PROJECT

### 4.1 Wind tunnel test and preliminary analysis

The long span roof is a novel, unique shell structure. The long span is 143m, the short span is 80m, the height is 24m. The test was carried out in the Southwest Jiaotong University Wind Engineering Research Center XNJD-3 industrial wind tunnel, shown in Fig. 1.

The finite element model of the long span roof is shown in Fig. 2. By using the wind pressure from the test, the dynamic responses of the structure are analyzed, and the extreme responses of the equivalent targets are obtained.



Fig.1 Model in wind tunnel

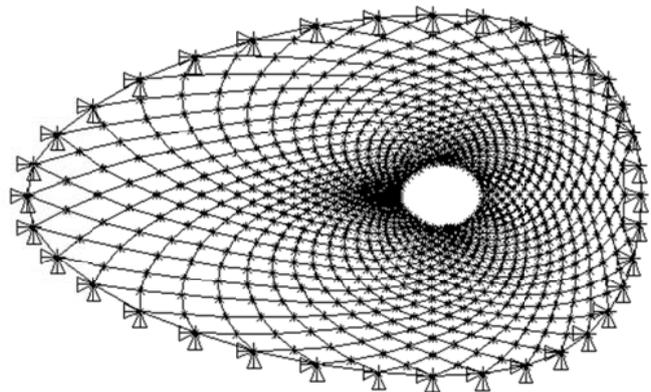


Fig.2 Finite element model

### 4.2 Wind Equivalent static wind loads analysis

In order to display the efficiency of the basic vectors, two types of basic vectors will be chosen to calculate the wind equivalent static wind loads. One is the intrinsic modes of the fluctuating wind loads, the other is the intrinsic modes of the generalized resuming forces in this paper.

When the intrinsic modes of the fluctuating wind loads are chosen as the basic vectors, the equivalent static wind loads is calculated by using the Eq. (5) and Eq. (6), then the static responses are obtained. Fig. 3(a) shows the distribution of the universal equivalent static loads, Fig. 3(b) shows the static responses under the equivalent static wind loads and the extreme responses of dynamic analysis.

As can be seen from the Figure, most of the target responses are in good agreement, but there are also large deviations in some target responses. The distribution of the equivalent static wind loads is not particularly reasonable, some partial positions are concentrated.

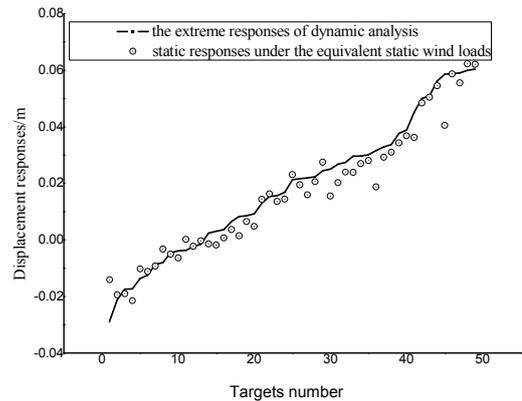
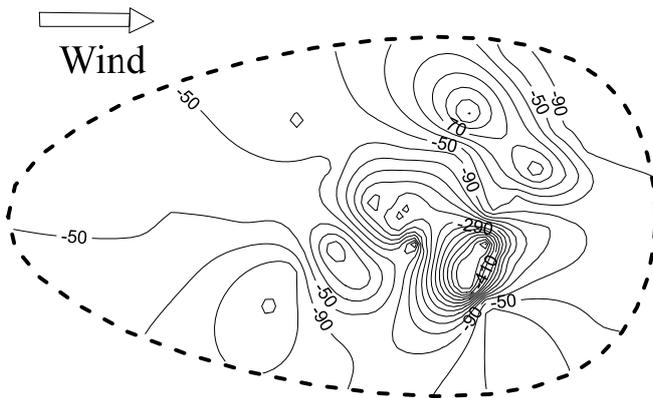


Fig.3(a) Universal ESWL distribution (pa)

Fig.3(b) Displacement responses

Fig.3 The results when the intrinsic modes of the fluctuating wind loads are chosen as the basic vectors

When the intrinsic modes of the generalized resuming forces are chosen as the basic vectors, also the equivalent static wind loads is calculated by using the Eq. (5) and Eq. (6), then the static responses are obtained. Fig. 4(a) shows the distribution of the universal equivalent static loads, Fig. 4(b) shows the static responses under the equivalent static wind loads and the extreme responses of dynamic analysis.

As can be seen from the Figure, almost all the target responses are in good agreement, and the distribution of the equivalent static wind loads is reasonable. These basic vectors for equivalent static wind loads of long span roofs are more efficient than above.

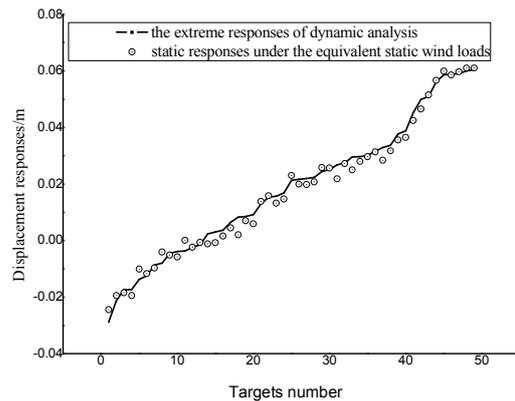
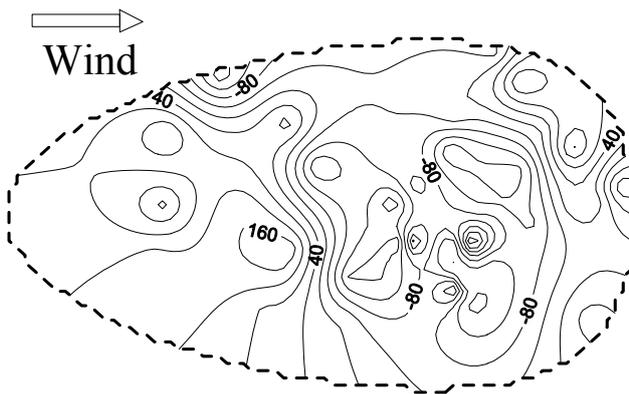


Fig.4(a) Universal ESWL distribution (pa)

Fig.4(b) Displacement responses

Fig.4 The results when the intrinsic modes of the generalized resuming forces are chosen as the basic vectors

## **5. CONCLUSIONS**

In this paper, an efficient method to choose basic vectors for equivalent static wind loads of long span roofs is presented, the main conclusions are as follows,

1) Generalized resuming forces are the quasi static force of the structure at each time, So if the intrinsic modes of generalized resuming forces are chosen as the basic vectors, these basic vectors are more effective than the others.

2) The optimal combination coefficients of these basic vectors are obtained by a least-squares method, the universal equivalent static wind loads can be obtained. In addition, the distribution of the equivalent static wind loads is reasonable by introducing the constraint equations.

3) The equivalent static wind loads of a long span roof is analyzed by this method, the validity of the method is verified through case analysis.

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