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Reducing the membrane locking in the MITC4 shell element

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ABSTRACT

A method to reduce membrane locking in the 4-node MITC shell element is presented. To reduce membrane locking in bending dominated shell problems when shell geometries are warped, the membrane strain of the MITC4 shell element is substituted by that obtained using two triangular geometries. With an additional modification on numerical integration, a new MITC4 shell element is developed to alleviate membrane locking without increasing computational cost. The proposed element passes the patch and zero energy mode tests. In various shell problems, improved convergence behaviors are shown compared to the original MITC4 shell element.

1. INTRODUCTION

For decades, much research effort has been devoted in developing more effective finite element method for the analysis of shell structures. Since shell structures are efficient for thin and curved geometries, reliable shell finite element method should converge well to analytical solutions regardless of shell thickness and curvature. On the other hand, it is very hard to meet all basic requirements of finite elements while also showing good convergence behavior in any case.

The MITC (Mixed Interpolation of Tensorial Component) method has been successfully used for shell elements for improved accuracy while satisfying all the basic requirements (Bucalem and Bathe, 1993, Lee and Bathe, 2004, Lee et. al., 2014). In this paper, we focus on the improvement of the 4-node quadrilateral MITC4 shell element, which has been widely used for various applications.

The MITC4 element (Dvorkin and Bathe, 1984), one of the most successful 4-node shell element, is such example. This element satisfies all the basic requirements and converges well in most of analysis cases except when distorted mesh is used in

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bending problems of curved structure (Jeon et. al., 2014). This is caused from membrane locking. In this work, we present a method to resolve this problem (Ko et. al., 2015).

2. PROPOSED METHOD

The geometry and displacement of the 4-node quadrilateral shell element are interpolated by

$$\vec{x}(r,s,t) = \sum_{i=1}^{4} h_i(r,s)\vec{x}_i + \frac{t}{2}\sum_{i=1}^{4} a_i h_i(r,s)\vec{V}_n^i \, , \tag{1}$$

with $h_1(r,s) = \frac{1}{4}(1-r)(1-s)$, $h_2(r,s) = \frac{1}{4}(1+r)(1-s)$, $h_3(r,s) = \frac{1}{4}(1+r)(1+s)$, $h_4(r,s) = \frac{1}{4}(1-r)(1+s)$,

$$\vec{u}(r,s,t) = \sum_{i=1}^{4} h_i(r,s)\vec{u}_i + \frac{t}{2}\sum_{i=1}^{4} a_i h_i(r,s)(-\vec{V}_2^i \alpha_i + \vec{V}_1^i \beta_i) \, , \tag{2}$$

which are used for element geometry and covariant transverse shear strain of the element.

In order to reduce membrane locking, we adopt geometry and displacement interpolation of internal triangular elements

$$\vec{x}_T(r,s,t) = \sum_{i=1}^{3} h_{T,i}(r,s)\vec{x}_{T,i} + \frac{t}{2}\sum_{i=1}^{3} a_i h_{T,i}(r,s)\vec{V}_{n,T}^i \tag{3}$$

with $h_{T,1}(r,s) = r$, $h_{T,2}(r,s) = s$, $h_{T,3}(r,s) = 1 - r - s$,

$$\vec{u}_T(r,s,t) = \sum_{i=1}^{3} h_{T,i}(r,s)\vec{u}_i + \frac{t}{2}\sum_{i=1}^{3} a_i h_{T,i}(r,s)(-\vec{V}_{2,T}^i \alpha_i + \vec{V}_{1,T}^i \beta_i) \, . \tag{4}$$

The new covariant in-plane strain of shell element is expressed as

$$e_{ij}(r,s,t) = \frac{1}{2}\left(\frac{\partial \vec{x}_T}{\partial r_i} \cdot \frac{\partial \vec{u}_T}{\partial r_j} + \frac{\partial \vec{x}_T}{\partial r_j} \cdot \frac{\partial \vec{u}_T}{\partial r_i} \right), \tag{5}$$

where the base vectors are defined by

$$\vec{g}_{T,r} = \frac{\partial \vec{x}_T}{\partial r}, \quad \vec{g}_{T,s} = \frac{\partial \vec{x}_T}{\partial s}, \quad \vec{g}_{T,t} = \left. \frac{\partial \vec{x}}{\partial t} \right|_{(0,0)} = \frac{1}{2}\sum_{i=1}^{4} a_i h_i(0,0)\vec{V}_n^i \, .$$

We calculate the membrane strain over two triangles depicted in Fig. 1. The in-plane strain is coupled with the transverse shear strain of the MITC4 shell element for each 2 by 2 Gauss points, see Fig. 1.

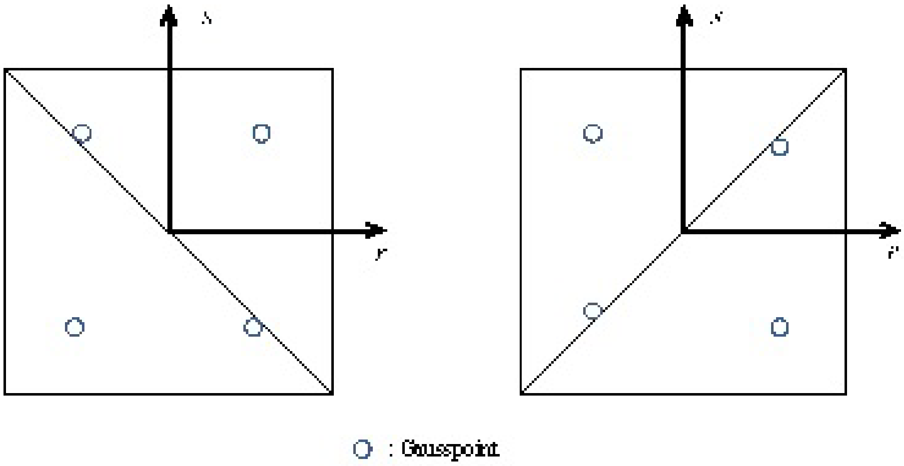

Fig. 1 Division of a quadrilateral element into triangles and assignment of Gauss points

In order to fully satisfy the in-plane patch test, we multiply factor to each in-plane strain under numerical integration. Let two Jacobian matrices be

$$\mathbf{J} = \begin{vmatrix} \vec{g}_r \\ \vec{g}_s \\ \vec{g}_t \end{vmatrix} \quad \text{and} \quad \mathbf{J}_T = \begin{vmatrix} \vec{g}_{T,r} \\ \vec{g}_{T,s} \\ \vec{g}_{T,t} \end{vmatrix}, \tag{6}$$

in which

$$\vec{g}_r = \frac{\partial \vec{x}}{\partial r}, \quad \vec{g}_s = \frac{\partial \vec{x}}{\partial s}, \quad \vec{g}_t = \frac{\partial \vec{x}}{\partial t}. \tag{7}$$

The factor has the following form.

$$c = \sqrt{\frac{1}{4}\frac{|\mathbf{J}_T|}{|\mathbf{J}|}}. \tag{8}$$

3. PERFORMANCE

The proposed method passes zero energy mode tests and patch tests. One drawback is that the proposed method cannot represent spatially isotropic condition.

We test its performance through a numerical study. Let us consider the bending problem of hyperboloid shell shown in Fig. 2, where most of the load is carried by membrane action if the boundary condition is free.

$$x^2 + z^2 = 1 + y^2; \quad y \in [-1, \quad 1]. \tag{9}$$

For the given problem, loading is defined by

$$p(\theta) = p_0 \cos(2\theta). \tag{10}$$

Due to symmetry, the analyses are performed using one-eighth of the structure corresponding to the shaded region ABCD in Fig. 2. To investigate bending dominated case, the free boundary condition is imposed: $u_z = \beta = 0$ along BC, $u_x = \beta = 0$ along AD, and $u_y = \alpha = 0$ along DC.

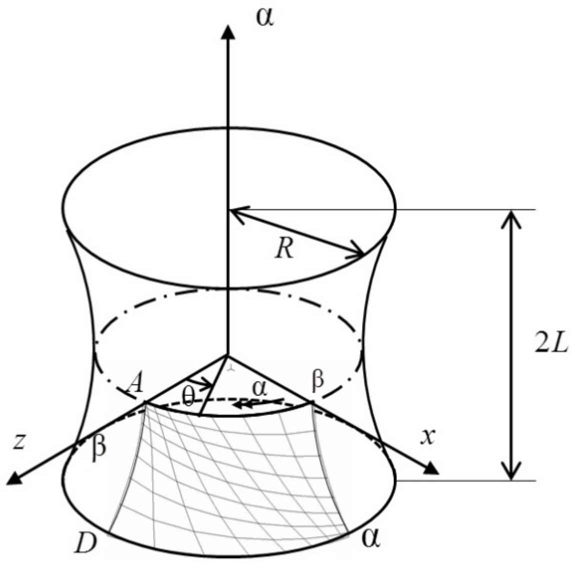

Fig. 2 Hyperboloid shell problem with distorted mesh ($E = 2.0 \times 10^{11}$, $\nu = 1/3$ and $p_0 = 1.0$).

In order to measure errors in the finite element solutions, we use s-norm (Hiller and Bathe 2003), where it is measured with regard to mesh refinement *h* and the reference solution is obtained by the 72 x 72 mesh of the MITC9 (Bucalem and Bathe 1993, Bathe et. al., 2003) elements. The result is presented in Fig. 3.

The numerical examples have illustrated that the proposed finite elements could be very useful for analyzing bending of shell structures which has both mesh distortion and nonzero curvature.

4. CONCLUSIONS

In this study, an efficient method to reduce membrane locking of MITC4 element was developed. Although the method does not satisfy the isotropic condition, it exactly passes patch tests and zero energy mode tests. If we further specify mesh algorithm where triangular divisions are automatically determined, this method is also able to represent nearly the isotropic condition.

The great potential of the proposed method is that it can effectively reduce membrane locking of the MITC4 shell element with no additional computational cost. Development of shell finite elements which further satisfy the isotropic condition can be the future research.

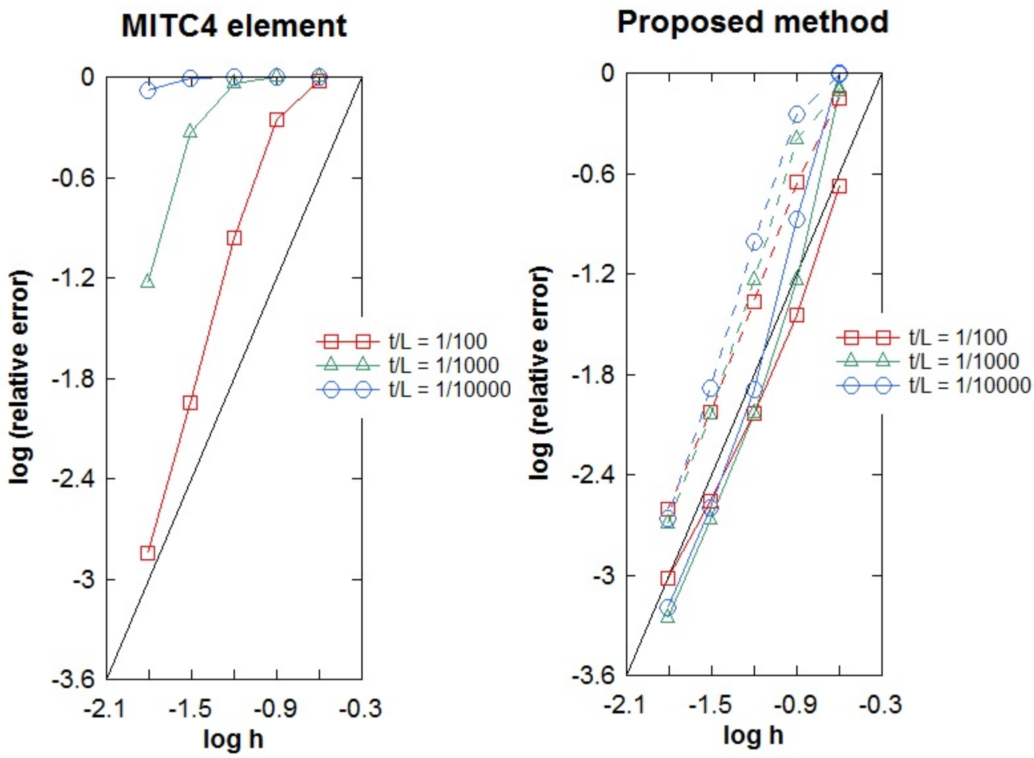

Fig. 3 Convergence curves for the free hyperboloid shell problem with distorted mesh.

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