

## **Probabilistic fatigue life updating for bridges based on comprehensive structural monitoring data**

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### **ABSTRACT**

Fatigue failure is one of the main mechanisms of bridge failure. When a bridge is constructed, it should be designed such that it can survive a target period. However, the bridge strength degrades over its service life. For decision-making with respect to the effective maintenance of bridges, it is thus essential to predict their remaining fatigue life. However, it is a very challenging task because the fatigue life of a bridge should be predicted based on its current condition, and various sources of uncertainty exist. This paper presents a new approach for predicting the probabilistic fatigue life of bridges based on comprehensive structural monitoring data. The proposed method (1) predicts the probabilistic fatigue life by employing a finite element (FE) model that is updated at the global level using the ambient vibration under general passing vehicles and (2) updates the fatigue life based on crack detection data at the local level. The proposed method is applied to a numerical bridge example, and the impact of structural degradation on the fatigue life of a bridge is discussed.

### **1. INTRODUCTION**

Fatigue is one of the main causes of structural failure. In fact, many structural systems are subjected to the risk of fatigue-induced failure caused by repeated loading over their life cycle; this is an especially critical issue for bridges. Therefore, a bridge is designed to survive for a certain period of time when it is constructed. However, a bridge's status changes over its service life. It is thus essential to accurately predict the fatigue life in order to make decisions about effective bridge maintenance and retrofitting. However, this is a very challenging task for the following reasons: (1) fatigue life prediction for a bridge should be based on its current structural condition; (2) in practice, various types of structural monitoring data are obtainable; and (3) various sources of uncertainty exist, including material properties, anticipated vehicle loads, environmental conditions, and monitoring data.

Many studies have developed probabilistic methods of fatigue life prediction for

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bridges. However, in these studies, the fatigue life was predicted using the initial designs and visual inspection data of bridges; however, these approaches were limited by the fact that the current bridge condition could not be considered. For example, the degradation of material properties such as the changes in mass, stiffness, and damping ratio cannot be quantitatively considered in the fatigue life assessment. To overcome this issue and evaluate the probabilistic fatigue life of a bridge based on its current condition, various types of structural health monitoring (SHM) data have been introduced. SHM data has been proved to be useful for evaluating structural performances (Nagayama *et al.* 2007), and structural changes at that time can be rationally captured by using structural identification methods based on SHM data (Yi *et al.* 2012). However, in a few studies, the probabilistic fatigue life of a bridge was estimated using “comprehensive” SHM data on both the global structural behavior and the crack detection of local members.

This paper presents a new approach for the probabilistic fatigue life prediction of bridges based on comprehensive structural monitoring data. The proposed method (1) predicts the probabilistic fatigue life by employing a finite element (FE) model that is updated at the global level using the ambient vibration under general passing vehicles and (2) updates the fatigue life based on crack detection data at the local level.

## 2. PROBABILISTIC FATIGUE LIFE PREDICTION BASED ON SHM DATA

### 2.1 Limit-state Function Formulations for Fatigue Failure

To calculate the probability of fatigue failure, it is necessary to express the failure event of interest using the so-called limit-state function, which is an analytical function of random variables and deterministic parameters. Consider the following crack-growth model (Paris and Erdogan 1963):

$$\frac{da}{dN} = C(\Delta K)^m \quad (1)$$

where  $a$  denotes the crack length,  $N$  is the number of load cycles,  $C$  and  $m$  are the material parameters, and  $\Delta K$  denotes the range of the stress intensity factor. By Newman’s approximation (Newman and Raju 1981), the range of the stress intensity factor can be estimated as

$$\Delta K = S \cdot Y(a) \cdot \sqrt{\pi a} \quad (2)$$

where  $S$  denotes the range of the stress, and  $Y(a)$  is the geometry function. Upon substituting Eq. (2) into Eq. (1) and integrating it from the initial condition to the current time point, the relationship between the time duration and the current crack length is derived as

$$\int_{a^0}^a \frac{1}{[Y(a)\sqrt{\pi a}]^m} da = C \cdot N \cdot S^m = C \cdot v_0 \cdot T \cdot S^m \quad (3)$$

where  $a^0$  is the initial crack length;  $v_0$ , the loading frequency; and  $T$ , the time duration. If

a crack failure is defined as an event in which the crack length exceeds the critical crack length  $a^c$ , the time required for crack growth from  $a^0$  to  $a^c$  under stress  $S_0$ ,  $T_0$ , is then described as

$$T_0 = \frac{1}{Cv_0(S_0)^m} \int_{a^0}^{a^c} \frac{1}{[Y(a)\sqrt{\pi a}]^m} da \quad (4)$$

Then, the limit-state function for the failure of the member within a given time  $[0, T_s]$  is

$$g(\mathbf{X}) = T_0 - T_s = \frac{1}{Cv_0(S_0)^m} \int_{a^0}^{a^c} \frac{1}{[Y(a)\sqrt{\pi a}]^m} da - T_s \quad (5)$$

where  $\mathbf{X}$  denotes the vector of random variables representing the uncertainties in the parameters of the problem, including the material properties ( $C$ ,  $m$ ) and initial crack length ( $a^0$ ). In structural reliability analysis,  $g(\mathbf{X}) \leq 0$  generally indicates the occurrence of a failure event.

To calculate the probabilities of fatigue failure events using Eq. (5), a reliability analysis method should be employed. Numerous methods have been developed and adopted in various engineering disciplines (Haldar 2006). In this study, First Order Reliability Method (FORM) (Der Kiureghian 2005) is used, which is one of the representative methods for structural reliability analysis.

## 2.2 Fatigue Life Updating Based on SHM Data at Global Level

After the SHM data on the global behavior of a bridge is acquired, the bridge's initial FE model can be updated. Then, the stresses are changed and the time required for fatigue failure needs to be re-estimated accordingly. For this task, Lee and Cho (2015) developed a recursive formulation inspired by the work of Lee and Song (2014). The same formulation was employed in this study and summarized in this section.

If the FE model is updated at  $T_1^{up}$ , a recursive formulation of the time duration from that moment to the crack failure  $T_1$  is given as

$$T_1 = \frac{1}{Cv_0(S_1)^m} \int_{a^0}^{a^c} \frac{da}{[Y(a)\sqrt{\pi a}]^m} - \left(\frac{S_0}{S_1}\right)^m T_{up}^1 \quad (6)$$

where  $a_1$  and  $S_1$  respectively denote the crack length and stress at the moment that the FE model is updated. Through mathematical induction, a recursive formulation is derived for updating multiple FE models as

$$T_j = \frac{1}{Cv_0(S_j)^m} \int_{a^0}^{a^c} \frac{da}{[Y(a)\sqrt{\pi a}]^m} - \sum_{i=1}^{j-1} \left(\frac{S_i}{S_j}\right)^m T_{up}^i \quad (7)$$

where  $j$  and  $T_j$  denote the number of FE model updates and the time duration from the last SHM to the crack failure, respectively. The fatigue failure within a given time  $[0, T_s]$

is then described as

$$g(\mathbf{X}) = T_{up}^1 + T_{up}^2 + \dots + T_{up}^j + T_j - T_s \quad (8)$$

By using Eq. (8), the fatigue life of a bridge can be evaluated with multiple SHM and FE model updates. However, for simplicity, in this study, it is assumed that the SHM is only performed once (i.e.,  $j = 1$ ). As with Eq. (5), FORM is employed to calculate the failure probability using Eq. (8).

### 2.3 Fatigue Life Updating Based on Local Crack Detection Data

In addition to the SHM data on the global behavior of a bridge, crack detection data on local members is obtainable, and the fatigue life needs to be updated accordingly. Lee and Song (2014) recently developed a formula to update the fatigue life for aircraft based on local crack detection data. In this study, this same formula is introduced for bridges, and it is briefly explained in this section.

For a failure event of interest  $E_i$ , its probability  $P_i$  can be updated using local crack detection data as

$$P_{i,up} = P(E_i | IE_j) \quad (9)$$

where  $IE_j$  denotes the inspection event at the  $j$ -th member. This formulation can be extended to utilize multiple available inspection results to update the probability  $P_i$ , i.e.,

$$P_{i,up} = P(E_i | IE_j \cap IE_k \cap \dots \cap IE_l) \quad (10)$$

Inspection results are classified into two types of events: equality and inequality types, depending on whether a crack is detected (and measured) or not. Considering that inspections are generally performed at multiple locations, there are three possible combinations of inspection results: *inequality*, *equality*, and *mixed* cases (Jiao and Moan 1990).

First, an *inequality* event indicates that no crack is detected from inspection at a certain single location. There are two possible explanations for this case. First, a crack is too small to be detected. Second, although a relatively large crack actually exists at the inspected member, it may be missed owing to human error or the limitations of the detecting device. In either case, this event can be formulated as

$$IE_j : g_{j,no}(\mathbf{X}) = T_i - T_j^d(\mathbf{X}) < 0$$

$$\text{where } T_j^d = \frac{1}{C V_0 (S_j^0)} \int_{a_j^0}^{a^d} \frac{1}{[Y(a)\sqrt{\pi a}]^m} da \quad (11)$$

where  $T_j^d$  denotes the required time for the crack growth to a detectable crack size at the  $j$ -th member,  $a^d$  is the detectable crack size during the inspection process,  $a_j^0$  and  $S_j^0$  respectively are the initial crack length and the stress at the  $j$ -th member, and  $T_i$  denotes the time for which the member is inspected. The detectable crack size  $a^d$  is related to a specific inspection method and modeled as a random variable reflecting the

actual probability of detection.

For a single inequality event, the conditional probability in Eq. (9) can be calculated as

$$P_{i,up} = P(E_i | IE_j) = \frac{P(E_i \cap (g_{j,no}(\mathbf{X}) < 0))}{P(g_{j,no}(\mathbf{X}) < 0)} \quad (12)$$

Similarly, the conditional probability given multiple inequality events can be calculated as

$$\begin{aligned} P_{i,up} &= P(E_i | IE_j \cap \dots \cap IE_k) \\ &= \frac{P[E_i \cap (g_{j,no}(\mathbf{X}) < 0) \cap \dots \cap (g_{k,no}(\mathbf{X}) < 0)]}{P[(g_{j,no}(\mathbf{X}) < 0) \cap \dots \cap (g_{k,no}(\mathbf{X}) < 0)]} \end{aligned} \quad (13)$$

One can obtain the updated probabilities in Eqs. (12) and (13) by computing the probabilities in the numerator and denominator by component and system reliability analysis.

Second, an *equality* event indicates that a crack is detected and measured. The event is formulated as

$$\begin{aligned} IE_j : g_{j,yes}(\mathbf{X}) &= T_l - T_j^m(\mathbf{X}) = 0 \\ \text{where } T_j^m &= \frac{1}{C\nu_0(S_j^0)^m} \int_{a_j^0}^{a^m + \varepsilon^m} \frac{1}{[Y(a)\sqrt{\pi a}]^m} da = 0 \end{aligned} \quad (14)$$

where  $T_j^m$  denotes the required time for the crack growth to the measured crack size at the  $j$ -th member,  $a^m$  is the measured crack size, and  $\varepsilon^m$  is the measuring error. In the equation, it should be noted that  $T_j^m$  is equal to the inspection time  $T_l$ , which makes the formulation different from the limit-state formulations in Eq. (11). The conditional probability given the equality event is calculated as

$$P_{i,up} = P(E_i | IE_j) = \frac{P(E_i \cap (g_{j,yes}(\mathbf{X}) = 0))}{P(g_{j,yes}(\mathbf{X}) = 0)} \quad (15)$$

Compared to the terms in Eq. (12), the probability terms in Eq. (15) are more challenging to compute using structural reliability methods because the probabilities in both the numerator and the denominator are zero (Straub 2011). Through analytical derivations, Lee and Song (2014) transformed Eq. (15) to

$$P_{i,up} = \left[ \frac{\frac{\partial}{\partial \theta} P[E_i \cap (g_{j,yes} + \theta \leq 0)]}{\frac{\partial}{\partial \theta} P[(g_{j,yes} + \theta \leq 0)]} \right]_{\theta=0} \quad (16)$$

The updated probability in the equation can be calculated via numerical

differentiation. Similarly, the conditional probability given multiple equality events is formulated as follows (Madsen 1987):

$$P_{i,up} = P(E_i | IE_j \cap \dots \cap IE_k)$$

$$= \left[ \frac{\frac{\partial^n}{\partial \theta_1 \dots \partial \theta_n} P[E_i \cap (g_{j,yes} + \theta_1 \leq 0) \cap \dots \cap (g_{k,yes} + \theta_n \leq 0)]}{\frac{\partial^n}{\partial \theta_1 \dots \partial \theta_n} P[(g_{j,yes} + \theta_1 \leq 0) \cap \dots \cap (g_{k,yes} + \theta_n \leq 0)]} \right]_{\theta_1 = \dots = \theta_n = 0} \quad (17)$$

where  $n$  is the number of observed equality events. The updated probability  $P_{i,up}$  in Eq. (17) can also be calculated using the  $n$ -th order numerical differentiation. However, the equation needs to be used carefully, because such high-order numerical differentiation can create a significant error unless the probability calculations in the numerator and denominator are extremely accurate. Therefore, the equality case including multiple equality events will not be discussed in this study.

Lastly, for mixed cases, the simplest case involves a single inequality event and a single equality event. The updated probability is formulated as

$$P_{i,up} = \frac{P[E_i \cap (g_{j,no}(\mathbf{X}) < 0) \cap (g_{j,yes}(\mathbf{X}) = 0)]}{P[(g_{j,no}(\mathbf{X}) < 0) \cap (g_{j,yes}(\mathbf{X}) = 0)]} \quad (18)$$

This equation can be transformed to

$$P_{i,up} = \left[ \frac{\frac{\partial}{\partial \theta} P[E_i \cap (g_{j,no} < 0) \cap (g_{j,yes} + \theta \leq 0)]}{\frac{\partial}{\partial \theta} P[(g_{j,no} < 0) \cap (g_{j,yes} + \theta \leq 0)]} \right]_{\theta=0} \quad (19)$$

Finally, this formulation can be generalized for a mixed case with multiple inequality and equality inspection events as

$$P_{i,up} = \left\{ \frac{\partial^n}{\partial \theta_1 \dots \partial \theta_n} P[E_i \cap (g_{j,no} < 0) \cap \dots \cap (g_{k,no} < 0) \cap (g_{l,yes} + \theta_1 \leq 0) \cap \dots \cap (g_{l,yes} + \theta_n \leq 0)] / \frac{\partial^n}{\partial \theta_1 \dots \partial \theta_n} P[(g_{j,no} < 0) \cap \dots \cap (g_{k,no} < 0) \cap (g_{l,yes} + \theta_1 \leq 0) \cap \dots \cap (g_{l,yes} + \theta_n \leq 0)] \right\}_{\theta_1 = \dots = \theta_n = 0} \quad (20)$$

where  $n$  is the number of equality-type inspection events. It is noted that Eq. (20) is a combination of Eqs. (13) and (17). Because of the aforementioned difficulty in high-order numerical differentiation, the mixed case including multiple equality events will not be handled in this study.

### 3. NUMERICAL EXAMPLE: SAMSEUNG BRIDGE

#### 3.1 Numerical Example

To validate the FE model updating method proposed in Yi *et al.* (2007) and Lee and Cho (2015), extensive field tests were conducted on Samseung Bridge, which was constructed on the Jungbu Inland Expressway, Korea, in 2002. In this study, as a numerical example, the Samseung Bridge example is reused to evaluate its probabilistic fatigue life based on both the initial and the updated FE models.

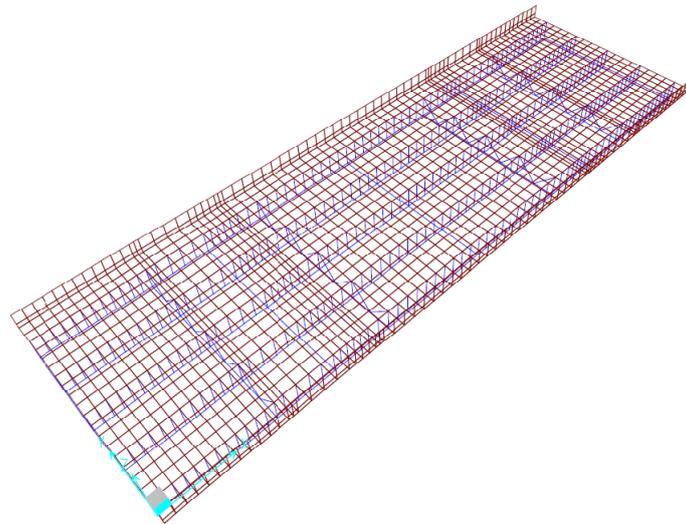
The Samseung Bridge is a single span, steel-plate girder bridge with a span length of 40 (m). It is composed of five main steel girders, floor beams, and a concrete slab, as shown in Fig. 1. Based on the design drawings of the bridge, an FE model for SAP2000 is constructed (with a span length of 38.8 m), as shown in Fig. 2. In the figure, the shell elements (in red) and frame elements (in blue) represent the concrete slab and steel girders, respectively. Five girders are labeled Girder 1 through Girder 5 from the bottom to the top.

#### 3.2 FE Model Updating Based on SHM Data

It is becoming increasingly important to monitor and evaluate long-term structural performance as well as structural integrity; this includes material degradation and cracks. For this purpose, many types of SHM systems have been developed and applied to various types of structures. Yi *et al.* (2007, 2012) recently presented a useful application of an instrumented SHM system for reliable seismic performance evaluation based on measured vibration data under ambient wind and traffic loadings. The procedure consists of (1) constructing the initial FE model of a target bridge based on its design drawings; (2) measuring the ambient vibration of the bridge under general traveling vehicles; (3) identifying modal parameters, including natural frequencies, mode shapes, and modal damping ratios from the measured acceleration data using an output-only modal identification method; (4) updating the linear structural parameters for an initial FE model using the identified modal parameters; and finally (5) performing the probabilistic analysis of interest by employing the updated FE model.



**Fig. 1** Front view (left) and bottom view (right) of Samseung Bridge (Yi *et al.* 2007)



**Fig. 2** FE model of Samseung Bridge for SAP2000

To find the optimal structural parameters for the initial FE model based on the measured modal properties of a bridge, a genetic algorithm (Kim and Yang 1995) is employed in this study. The objective function  $J$  in the optimization procedure represents the differences between the measured and the calculated natural frequencies, and the constraint equations are constructed based on the differences between the measured and the calculated mode shapes.

$$J = \sum_{i=1}^{N_m} \left\{ w_i \left( \frac{f_i^c - f_i^m}{f_i^m} \right) \right\}^2 \quad \text{subjected to } |\phi_{ji}^c - \phi_{ji}^m| \leq \varepsilon \quad (21)$$

where  $f_i$  is the  $i$ -th natural frequency, and  $\phi_{ji}$  denotes the  $j$ -th component of the  $i$ -th normalized mode shape  $\phi_i$ .  $w_i$  and  $\varepsilon$  are the weighting factors for the  $i$ -th mode and the admissible error bound for the mode shape, respectively. The superscripts “ $m$ ” and “ $c$ ” indicate the measured and calculated data, respectively. The details of the FE model updating procedure can be found in the following section and in Yi *et al.* (2007, 2012).

To update the FE model in Yi *et al.* (2007), a series of conventional load tests were conducted on the bridge in four different seasons: (1) August 2004, (2) December 2004, (3) July 2005, and (4) February 2006. Three heavy trucks with different weights—15, 30, and 40 tons—were used for loading. The truck weights were gauged before the test, and a vertical load equal to the gauged weight of the truck was added to calculate the deflection of the initial FE model. To measure the deflection of the bridge, three contact-type displacement transducers with connecting wires (OU displacement transducers) were installed beneath the centers of the three main girders. During the 3rd and 4th tests in July 2005 and February 2006, a laser vibrometer (OFV-505 Standard Optic Sensor Head and OFV-5000 Modular Controller, Polytec, Inc.) was also installed at the center of the third girder to validate the performance of the OU displacement transducers.

For ambient vibration tests, 21 accelerometers were installed on the bridge. Ambient vibrations were measured for 30 min at a sampling frequency of 200 Hz after each conventional load test with the trucks. The wind and traffic on the adjacent bridge were the main vibration sources during the ambient vibration tests. After ensuring that the high-frequency components of the acceleration (greater than 100 Hz) were very small, a low-pass filter with a cut-off frequency of 90 Hz was utilized. More details of the field test can be found in Yi *et al.* (2007).

The initial FE model was updated using the extracted modal properties. The downhill simplex method was employed as an updating algorithm, and SAP2000 was used to iteratively calculate the modal properties of the updated FE model. The objective function was constructed by using the differences between the measured and the estimated natural frequencies, and the constraint equations were considered to limit the differences between the measured and the estimated mode shapes.

If the number of updating parameters is excessively larger than the number of input points used for constructing the object function and constraint equations, the possibility of falling to a local minimum increases as the optimizing process proceeds. Therefore, model updating was processed in two steps to reduce the ill-posedness during the updating procedure. At first, the updating parameters were one equivalent spring constant at the supports, Young's modulus of the concrete slab, the 2nd moments of inertia for the five main girders, and the equivalent 2nd moments of inertia and torsional coefficients for nine floor beams. After the first step of model updating, 31 parameters were selected for the next model updating. These were spring constants at two supports, Young's modulus of the concrete slab, and the 2nd moments of inertia and torsional coefficients for the five main girders and nine floor beams. After updating the FE model, the natural frequencies of the initial FE model and updated FE model were compared to the measured ones, which showed that the natural frequencies of the updated FE model were closer to the measured values than those of the initial FE model.

### 3.3 Vehicle Load Model

To evaluate the fatigue life of a bridge, a fatigue load model should be determined. In many existing studies, Weigh in Motion (WIM) measurements of actual passing vehicles have been used to obtain the magnitude and frequency of fatigue loading. However, such a field test was not conducted on the Samseung Bridge. Thus, in this study, the fatigue load is modeled by using the vehicle load model (DL-24) of the Korea Highway Bridge Design Specification (KHBDS) (Ministry of Construction and Transportation 2005).

With this load model, FE analysis is performed for the initial and updated FE model to achieve the maximum stresses of the five girders. Because the stresses are obtained using static analyses, they are multiplied by an impact factor  $I$  in Eq. (22) to account for the dynamic effect of vehicle loads.

$$I = \frac{15}{40 + L} \leq 0.3 \quad (22)$$

where  $L$  is the span length of the bridge in meters. In the FE models for SAP2000, the

span length is assumed to be 38.8 (m). Thus, the impact factor is 1.19.

Table 1 lists the maximum stresses of the five girders from the initial and updated FE models. As shown in the table, the stress values from the updated model are smaller than those from the initial model. It is noticeable that the stress values are symmetrical with the center of Girder 3 in the initial FE model, reflecting its symmetry; however, it is slightly unsymmetrical with the updated model. This is because the structural parameters change during the FE model updating procedure. The table also shows that the stress values from the updated FE model are relatively small compared with those from the initial model. This is because the SHM test was conducted only for 2–4 years (in 2004–2006) after bridge construction (in 2002), which means that the bridge is still expected to be in a good condition.

<b>Stress (MPa)</b>	<b>Girder 1</b>	<b>Girder 2</b>	<b>Girder 3</b>	<b>Girder 4</b>	<b>Girder 5</b>
Initial FE model	18.24	20.77	20.03	20.77	18.24
Updated FE model	15.95	17.52	17.11	17.52	15.83

**Table 1** Girder stresses from the initial and updated FE models

### 3.4 Random Variables and Deterministic Parameters

In this example, the uncertainties of  $C$  in the Paris equation and initial crack length  $a^0$  are considered as random variables with mean values of  $2.18 \times 10^{-13}$  (mm/cycle/(MPa·mm)<sup>m</sup>) and 0.1 (mm), and coefficients of variation (C.O.V.s) of 0.2 and 0.1, respectively. The uncertainties of the stresses are also introduced using a load scale factor  $S$ , whose mean and C.O.V. are assumed to be 1.0 and 0.1, respectively. It is assumed that the initial crack length  $a^0$  follows an exponential distribution, whereas the others follow a lognormal distribution.

All of the random variables are assumed to be statistically independent of each other, except in the following cases: (1) between Paris equation parameter  $C$  of five girders (correlation coefficient: 0.6); and (2) between initial crack lengths ( $a^0$ ) of five girders (correlation coefficient: 0.6). The correlation coefficients of 0.6 indicate that Girders 1–5 were manufactured by the same manufacturer, and thus, their material properties are highly correlated. For more accurate results, however, the correlation values should be obtained from actual tests. The statistical properties of the random variables in this numerical example are summarized in Table 2.

In addition, the following deterministic parameters are used: half flange width ( $W$ ): 650 (mm); flange thickness: 30 (mm); critical crack length ( $a^c$ ): 30 (mm); average daily truck traffic (ADTT): 5000/day; time of global SHM test ( $T_{up}^1$ ): 4 years; time of local crack detection ( $T_l$ ): 4 years; detectable crack size ( $a^d$ ): 1.0 (mm). For the geometry function  $Y(a)$  in Eq. (2), the following function for I-beams is introduced from Wang *et al.* (2006):

$$Y = \frac{1.0 - 0.5\left(\frac{a}{W}\right) + 0.37\left(\frac{a}{W}\right)^2 - 0.044\left(\frac{a}{W}\right)^3}{\sqrt{1 - \frac{a}{W}}} \quad (23)$$

**Table 2** Statistical properties of random variables

(#: number, \*: standard deviation)

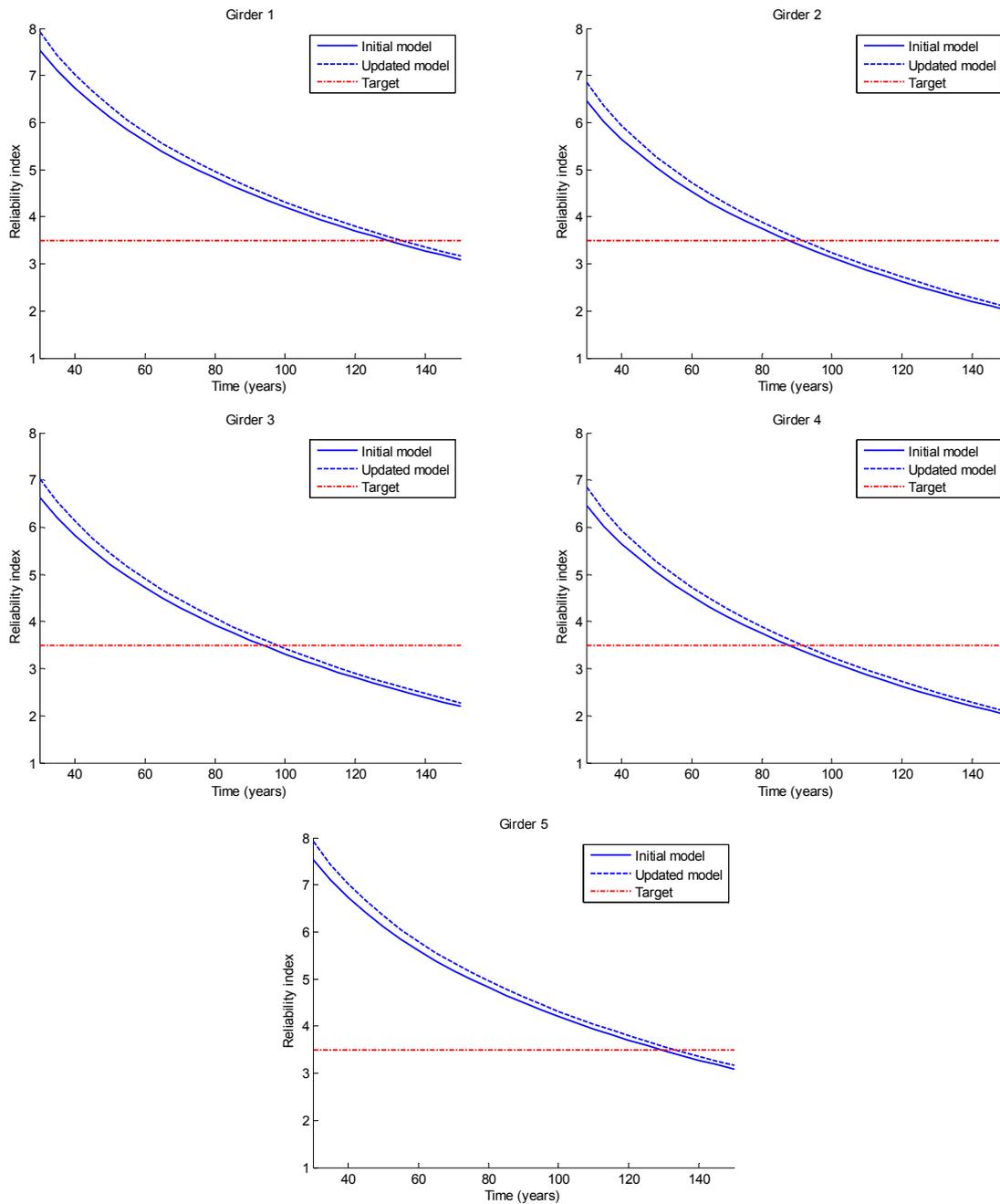
<b>Random variables (RVs)</b>	<b>Mean</b>	<b>Coefficient of variation</b>	<b>Distribution type</b>	<b>Number of RVs</b>
Paris law parameter (C)	$2.18 \times 10^{-13}$ (mm/cycle/(MPa·mm) <sup>m</sup> )	0.2	Lognormal	5
Initial crack length ( $a^0$ )	0.1 (mm)	0.1	Exponential	5
Detectable crack size ( $a^d$ )	1.0 (mm)	1.0	Exponential	# of inequality events
Measuring error ( $\varepsilon^m$ )	0	0.1*	Normal	# of equality events
Live load scale factor (S)	1	0.1	Lognormal	1

### 3.5 Analysis Results

First, the reliability indices with varying service times are plotted for the five girders and bridge system in Fig. 3. This figure shows that the level of the reliability indices is higher for the updated FE models. The American Association of State Highway and Transportation Officials (AASHTO) recommends, in the AASHTO Bridge Design Code, a target reliability index of 3.5 (i.e., a failure probability of  $2.33 \times 10^{-4}$ ) with a service life of 75 years for steel and prestressed concrete components. The fatigue lives of Girders 1–5 and the bridge system are estimated using the target reliability index (i.e., red lines in Fig. 3), as listed in Table 3. When using the updated FE model, the fatigue lives of the girders and bridge system are evaluated to be 4–5 years longer, and all of them meet the requirement of AASHTO, with fatigue lives longer than 75 years.

Using the crack detection data, the above fatigue life can be updated again. Table 4 lists the hypothetical inspection scenarios assumed to test the proposed method, and Table 5 shows the updated fatigue lives.

The fatigue lives in Scenario 1 are larger than the original lives because no crack is detected. The fatigue lives in Scenario are further increased. This is obviously because no crack is detected anywhere. When comparing Scenarios 3 and 4, the level of fatigue lives is evaluated to be lower when a large crack is detected (i.e., Scenario 4). However, the level of fatigue lives significantly increases in Scenario 5 because it is assumed that no crack is detected anywhere except for girder 2.



**Fig. 3** Failure probabilities using the initial and updated FE models

**Table 3** Updated fatigue lives based on global SHM data

<b>Fatigue life (years)</b>	<b>Girder 1</b>	<b>Girder 2</b>	<b>Girder 3</b>	<b>Girder 4</b>	<b>Girder 5</b>
Initial FE model	129	87	93	87	129
Updated FE model	133	92	98	92	133

**Table 4** Hypothetical inspection scenario

<b>Event Type</b>	<b>Scenario Number</b>	<b>Scenario Description</b>
Inequality	1	No crack is detected at girder 2
	2	No crack is detected anywhere
Equality	3	0.1 mm crack is found at girder 2
	4	0.5 mm crack is found at girder 2
Mixed	5	0.5 mm crack is found at girder 2, but nowhere else

**Table 5** Updated fatigue lives based on global and local SHM data

<b>Fatigue life (years)</b>	<b>Girder 1</b>	<b>Girder 2</b>	<b>Girder 3</b>	<b>Girder 4</b>	<b>Girder 5</b>
Scenario 1	140	102	104	97	140
Scenario 2	168	120	127	120	168
Scenario 3	137	98	102	95	137
Scenario 4	123	80	90	85	123
Scenario 5	159	108	120	112	159

### 3. CONCLUSIONS

In this study, a new approach was proposed for predicting the probabilistic fatigue life of a bridge based on comprehensive SHM data. The proposed method (1) predicts the probabilistic fatigue life by employing an FE model that is updated at the global level using the ambient vibration under general passing vehicles and (2) updates the fatigue life based on crack detection data at the local level. New limit-state formulations were derived to express the crack failure and predict the probabilistic fatigue life with updated stress values. These formulations allowed us to update the fatigue life of a bridge multiple times with global and local SHM data. To demonstrate the proposed method, it was applied to a numerical example of the Samseung Bridge, whose life updating based on global SHM data was already dealt with in a study. As a result, the fatigue failure probabilities of the five girders from the updated FE model were smaller than those from the initial FE model, because the level of stresses turned out to be relatively low with the updated model. As a result, the fatigue life values of the girders and bridge system were estimated to be 4–5 years longer, which means that the bridge is still in good condition. Furthermore, the impact of various inspection scenarios on fatigue life updating was investigated in the numerical example, such as the number of inspections and measured crack length. As a result, it was successfully shown that the proposed method enables us to predict the probabilistic fatigue life of a bridge based on its current condition by using the comprehensive structural monitoring data.

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