

## **A new model updating technique based on kriging surrogate model for bridge structures**

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### **ABSTRACT**

Model updating techniques are frequently used for bridge performance evaluation. In conventional model updating techniques, a precise initial analytical model is decisive to obtain accurate results. However, a precise model consumes excessive calculation time. In this study, a new model updating technique based on a surrogate model is presented to settle an issue of the excessive calculation time. A kriging surrogate model was selected as a result of considering various kinds of surrogate models. In order to improve accuracy of the kriging surrogate model, a sequential sampling method was used. The performance and efficiency of proposed technique were verified by lab-scale bridge model, which is a scale down model of high way bridge. The proposed technique showed good performance in the calculation time, comparing with conventional model updating techniques.

### **1. Introduction**

Civil infrastructure such as high-rise buildings and long-span bridges are increasing gradually. These structures are generally designed to endure for decades. However, the performance of these structures decreases gradually due to fatigue, corrosion, natural hazards, etc. In order to maintain their serviceability and prevent structure failure, continuous structure performance evaluation and reinforcement are necessary. For the structure performance evaluation, the finite element (FE) model of target structure is necessary. Considering that current design and assessment procedures do not have any quantitative linkage to actual existing structures (Catbas et al. 2013), the process to link the FE model with the corresponding existing structure is needed by using FE model updating.

In the model updating process, the accuracy and reliability of a FE model are decided by degree of precision of a FE model. Using a sophisticated FE model, more

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accurate and reliable result can be obtained. However, the sophisticated FE model consumes excessive calculation time. The excessive calculation time can be a burden to user.

In order to solve computational burden of the sophisticated FE model, a surrogate model has been suggested recently. The surrogate model is a computational modeling technique with a mathematical construction of input and output relation. It is also called response surface model. It can replace model updating process with the response surface (Jones, 2001). By using the response surface, updating parameter of FE model can be obtained in a relatively short calculation time.

In this study, a new model updating technique, based on a kriging surrogate model is presented to reduce the excessive calculation time. As a result of considering various kinds of surrogate models, kriging surrogate model was selected. In order to minimize the calculation time, a sequential sampling method was used. The performance and efficiency of proposed technique were verified by lab-scale bridge model, which is a scale down model of high way bridge. The proposed technique showed good performance in the calculation time, comparing with conventional model updating techniques.

## 2. Theory

The kriging surrogate model is a way of modeling the function as a realization of a stochastic process with mean  $\mu$  and variance  $\sigma^2$ . It is based on a spatial correlation among the value of the function. Based on the spatial correlation, the kriging basis with  $k$  dimensions can be expressed as

$$\psi^{ij} = \exp(-\sum_{p=1}^k \theta_p \|x_p^i - x_p^j\|) \quad (1)$$

where “ $p$ ” denotes the dimension of sample  $x$  and “ $i$ ” and “ $j$ ” indicate  $i$ -th and  $j$ -th sample. The accuracy and curvature of the kriging model are determined by the variables, included in the basis function.

For constructing kriging surrogate model, correlation of function is defined by distance of each samples. The variance, obtained from the correlation of function can be expressed by follow equations:

$$\text{corr}[y(x^i), y(x^j)] = \exp(-\sum_{p=1}^k \theta_p \|x_p^i - x_p^j\|) = \Psi \quad (2)$$

$$\text{Cov}[Y, Y] = \sigma^2 \text{corr}(Y, Y) = \sigma^2 \Psi \quad (3)$$

Because the value of the function is the realization of the stochastic process, estimation of parameters such as  $\mu$  and  $\sigma^2$  is necessary. The Eq. (4) expresses logarithmic likelihood function.

$$\ln(L) \approx -\frac{n}{2} \ln(\sigma^2) - \frac{1}{2} \ln|\Psi| - \frac{(Y-1\mu)^T \Psi^{-1} (Y-1\mu)}{2\sigma^2} \quad (4)$$

By evaluating the prediction value, surrogate model can be constructed with linear

combination form.

The sequential sampling method is an adaptive sampling, until target accuracy of the Kriging model is achieved. The proposed sequential sampling strategy is based on an expected improvement (EI(x)). The expected improvement (EI) approach is the criteria to evaluate how much improvement of current best value  $y_{best}$  is expected if a new sample is obtained (Jones et al. 2001). It is shown as

$$EI(x) = (y_{best} - \hat{y}(x))\Phi\left(\frac{y_{best} - \hat{y}(x)}{\hat{s}(x)}\right) + \hat{s}(x)\phi\left(\frac{y_{best} - \hat{y}(x)}{\hat{s}(x)}\right) \quad (6)$$

where  $\Phi(x)$  and  $\phi(x)$  are the Gaussian cumulative distribution function and probability density function, respectively. Using the EI, additional sample, which can improve the accuracy of surrogate model is selected. Fig. 1 shows the process of constructing the surrogate model with the sequential sampling method.

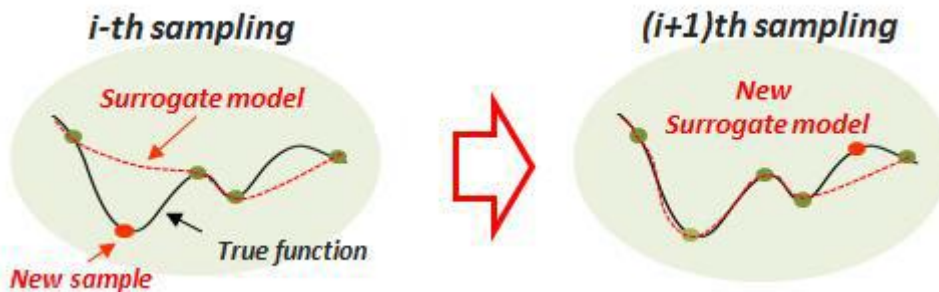


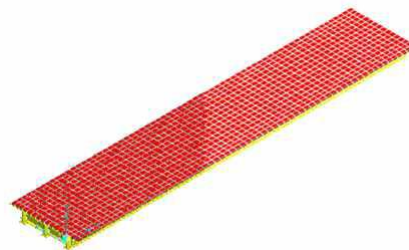
Fig. 1 Construction of the surrogate model with sequential sampling method

### 3. Experimental Validation

In order to validate the proposed method, experimental validation was conducted with lab scale model and its FE model. Fig. 2 shows the lab scale model and its FE model, constructed by using SAP2000.



(a) Lab scale model



(b) FE model

Fig. 2 Lab scale and FE model

Young's modulus of girder and deck were selected as parameter of surrogate model. The kriging surrogate model was constructed by measuring the natural frequencies and deflection of experimental model. Except the samples which are used to construct the surrogate model, additional 300 samples were selected to validate the performance of the surrogate model. For the 300 sample points, prediction values from the surrogate

model and measured values from FE model were compared. The  $R^2$  and RMSE of the 300 sample points are shown in Fig. 3. The kriging surrogate model showed quite similar results, comparing with FE model analysis results.

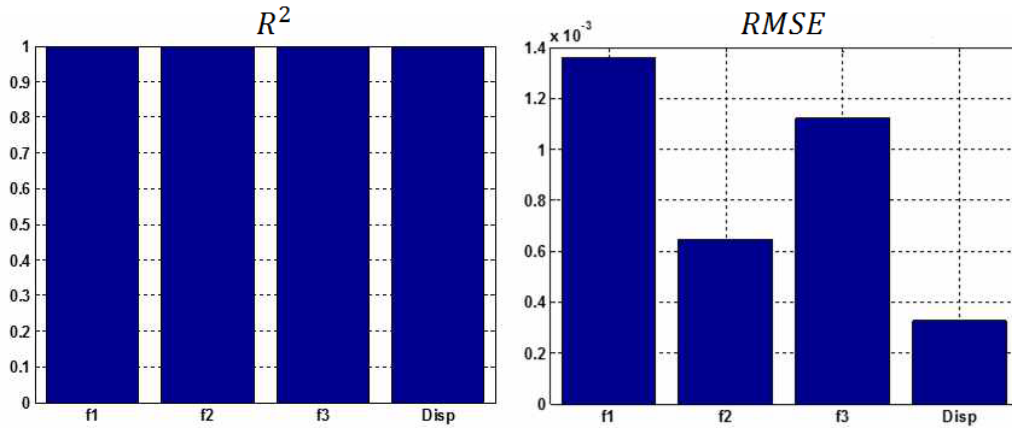


Fig. 3  $R^2$  and RMSE of the 300 sample points

In order to evaluate the simulation efficiency of the kriging surrogate model, change of object function by the generation was estimated. For each generation, 100 calculations were conducted because of its 100 population sizes. Fig. 4 shows that using the FE model, at least 50 generations are necessary to convergence of the object function. In other words, about 5000 calculations are needed to obtain the target values by using FE model. However, only 200 calculations are needed to obtain the target values by using the kriging surrogate model.

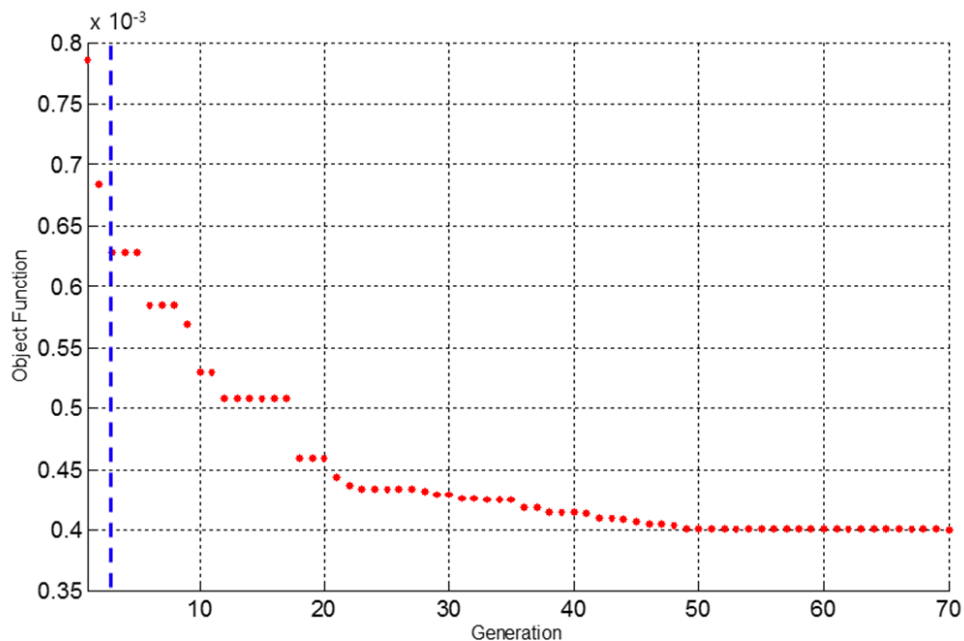


Fig. 4 Change of object function by generation

#### **4. Conclusion**

In this study, a new model updating technique based on kriging surrogate model was proposed. From the validation, the proposed method showed quite similar results, comparing with FE model analysis results. Also, it showed drastic reduction of calculation repetitions, comparing with the FE model.

#### **Acknowledgement**

This research was supported by a grant(13SCIPA01) from Smart Civil Infrastructure Research Program funded by Ministry of Land, Infrastructure and Transport(MOLIT) of Korea government and Korea Agency for Infrastructure Technology Advancement (KAIA).

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