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A self-adaptive firefly algorithm for structural damage detection

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ABSTRACT

Structural damage detection (SDD) is a challenging task in the flied of structural health monitoring (SHM). As an exploring attempt to the SDD problem, a self-adaptive firefly algorithm (SAFA) based method is proposed for SDD in this study. First of all, the basic principle of FA is introduced. A self-adaptive control strategy on light absorption coefficient is proposed. After that, the SAFA is introduced into the SDD field. Combined with the character of SDD problem, the improved behavior of the best firefly and the multi-step method are used to improve the identified results. In order to assess the accuracy and the feasibility of the proposed method, 2-storey rigid frame structure is taken as an example for numerical simulation on SDD. The illustrated results show that the proposed method can accurately identify the structural damage. Some valuable conclusions are made and related issues discussed as well.

1. INTRODUCTION

Structural damage detection (SDD) based on vibration is one of the core techniques in the field of structural health monitoring (SHM) and has been widely concerned by the researchers all over the world (Yan, et al., 2007). Lots of the vibration-based structural damage detection methods have been proposed (Alvandi and Cremona, 2006). Generally, the methods can be divided into two groups, statistics-based SDD methods (Hios and Fassois, 2014) and models-based SDD methods (Fritzen, et al., 1998). The first group methods are not based on structural models. The statistics-based methods always identify the structural damage only based on the statistical characters of the dynamic response signals. Another group methods are usually implemented by finite element analysis, therefore, the identified results are based on the accuracy of the structural model (Perera, et al., 2010). The nature frequency and mode shapes are usually used to detect the damage in model-based method (Zhu and Xu, 2005). The model-based methods are generally recognized and widely used in civil engineering. These are often transformed into mathematical problem solving constrained

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optimization. However, the mathematical model of the traditional constrained optimization methods is very complex and cannot be used to solve the high dimensional and complex optimization problems. Fortunately, some swarm intelligence (SI) optimization algorithms are adopted to solve large-scale civil engineering structural optimization problems (Yu, et al., 2012), such as the PSO algorithm (Shirazi, et al., 2014), the ACO algorithm (Yu and Xu, 2011), the GAFSA algorithm (Yu and Li, 2014), etc. The performances of SI algorithms are mainly depended on the key parameters (Yang, 2014). However, for the structural optimization problems, it is very hard to select the effective key parameters for the SI algorithms.

Inspired by the flashing patterns and behavior of the fireflies, firefly algorithm (FA) is first developed by Yang and widely used (Fister, et al., 2013). In all SI algorithms, there are two important components: exploitation and exploration. Exploration means that the search space is sufficiently investigated on a rough level, while exploitation means that interesting areas are searched more intensively in order to allow for a good approximation to an optimum (Yang, et al,. 2015). In classical FA, all fireflies have two key behaviors, attraction and random walk. Therefore, the exploration components is ensured by the random behavior, while the attraction behavior enhance the exploitation component. However, the balance of exploitation and exploration is mainly depended by the key parameters of FA. Therefore, the performance of FA is mainly depended by the key parameters tuning and control. At the moment, there is no efficient method in general for parameter tuning. The methods for parameter control can be divided into three groups, fixed control method, random control method and self-adaptive control. The fixed control method means that the parameter values often fixed during iterations and the random control method means that the parameter values often vary by a random process, such as chaotic stochastic process (Gandomi, et al., 2013). Self-adaptive control method means that the parameter values will vary according to the iterations and the characters of the swarm, therefore, it has been widespread concerned by the researchers. In this study, a novel self-adaptive firefly algorithm (SAFA) is proposed and used in the SDD problem.

2. SELF-ADAPTIVE FIREFLY ALGORITHM

2.1 Basic Firefly Algorithm

Firefly algorithm is a new SI optimization algorithm inspired by nature fireflies flashing behavior. In FA, the fireflies abide by the following three rules: 1) all fireflies are unisex so that one firefly will be attracted to other fireflies regardless of their sex; 2) Attractiveness is proportional to their brightness, thus for any two flashing fireflies, the less brighter one will move towards the brighter one. The attractiveness is proportional to the brightness and they both decrease as their distance increases; 3) For a specific problem, the brightness of firefly is associated with the objective function.

The light intensity of a firefly will decrease with the increasing distance of viewer. In addition, light is also absorption by the media. Therefore, it can be defined as:

$$I\left(r\right)=I_0 e^{-\gamma r^2} \tag{1}$$

Where I_0 is the original light intensity, r is the distance between any two fireflies and

γ is the light absorption coefficient. The attractiveness is proportional to the light intensity, which is defined as:

$$\beta(r) = \beta_0 e^{-\gamma r^2} \tag{2}$$

where β_0 is the attractiveness at $r = 0$. Then the *i*-th firefly is attracted to the *j*-th firefly, and the movement is formulated by:

$$x_i^d(t+1) = x_i^d(t) + \beta_0 e^{-\gamma r^2}\left[x_j^d(\mathrm{t}) - x_i^d(\mathrm{t})\right] + \alpha \cdot L_d(rand - 0.5) \tag{3}$$

where α is the randomization parameter, L_d is the length of *d*-th dimension, $rand$ is a random number generator uniformly distributed in [0, 1]. r is the distance between *i*-th firefly and *j*-th firefly, which is defined as the Cartesian distance as:

$$r = \left\|\boldsymbol{x}_i - \boldsymbol{x}_j\right\| \tag{4}$$

2.2 Self-adaptive Firefly Algorithm

There are three key parameters to be tuned for better performance of FA. They are initial attractiveness β_0, light absorption coefficient γ and randomization parameter α. The light absorption coefficient controls the decrease of light intensity and is very important in balancing the exploration and the exploitation (Chen and Ding, 2015). Generally, the light absorption coefficient can be selected between $[0, +\infty)$ and is commonly set to be 1 as done in (Yang, 2010). However, it is difficult to select the light absorption coefficient for different problems to enhance the performance of FA, therefore, a self-adaptive control of γ is necessary.

A varying light absorption coefficient γ is designed based on both the iterations process and the maximum distances among the fireflies in this paper and can be expressed as:

$$\gamma_{t+1} = \begin{cases} average\left(\chi_1, \chi_2, \chi_3, \cdots, \chi_{t+1}\right), & t \le \dfrac{N_{Gen}}{2} \\ \gamma_t \left(\dfrac{\gamma_{min}}{\gamma_{max}}\right)^{\frac{2}{N_{Gen}}}, & t > \dfrac{N_{Gen}}{2} \end{cases} \tag{5}$$

where N_{Gen} is the max generations, $\chi_1 = \dfrac{-\ln\left(10^{-45}\right)}{s_{max}^{1\ \ 2}}$ and $\gamma_{min} = \dfrac{-\ln(0.9)}{s_{max}^{1\ \ 2}}$, $\gamma_{max} = \gamma_{\mathrm{int}(\frac{N_{Gen}}{2})}$. The other parameters can be obtained as follows:

$$\tilde{\chi} = \frac{-\ln\left(10^{-45}\right)}{s_{max}^{t+1\,2}}$$
$$\chi_{t+1} = \begin{cases} average\left(\chi_1, \chi_2, \chi_3, \cdots, \chi_t\right), & \tilde{\chi} \le 0.25\chi_1 \\ \tilde{\chi}, & others \end{cases} \quad (6)$$

where s_{max}^{t} is the maximum distance among all fireflies at *t*-th iteration. $average()$ is the function for calculating the mean value. The initial positions of swarm populations are randomly located in the searching area which is required for the self-control strategy proposed above. In order to balance the exploration and the exploitation during the latter half iteration. All fireflies has a probability P to randomly rebuild except the best firefly. The basic steps of SAFA is shown in Fig. 1.

```
---------------------------------------Begin------------------------------------------------
Define the basic parameters:   β0  and  α.
While (k < MaxGeneration)
      Update the  γ
      If   rand(1) < P        Rebuild the fireflies     End if
       For  i = 1:n
             For  j = 1:n
                    If  Ij > Ii  Move firefly  i  towards  j    End if
             End for  j
       End for  i
       Evaluate new solutions and update light intensity
       Rank the fireflies and find the current best
End While
Output result
---------------------------------------End-------------------------------------------------
```

Fig. 1 The basic steps of SAFA

3. STRUCTURAL DAMAGE DETECTION SIMULATION

Generally, for a SDD problem, only the stiffness is considered to reduce while ignoring the change of the mass, as:

$$\boldsymbol{K} = \boldsymbol{K}_0 - \sum_{i}^{N} \alpha_i \boldsymbol{K}_i \quad (7)$$

where $\boldsymbol{K}$, $\boldsymbol{K}_0$ is the global stiffness matrix of the damaged and undamaged structures, respectively. α_i is the coefficient of *i*-th element stiffness damage, $\boldsymbol{K}_i$ is the expanded stiffness matrix of the *i*-th element in the global coordinate system, N is the number of elements. The motion equation of the damaged structure can be expressed as follows:

$$\boldsymbol{K}\boldsymbol{\varphi}_j = \lambda_j \boldsymbol{M}_0 \boldsymbol{\varphi}_j \quad (8)$$

Where $\boldsymbol{M}_0$ is the mass matrix of the structure, $\boldsymbol{\varphi}_j$ is the *j*-th mode shapes of the structure, $\lambda_j = (2\pi f_j)^2$ and f_j is the ***j***-th frequency of the structure. Therefore, $(\lambda_j^\alpha, \boldsymbol{\varphi}_j^\alpha)$ can be obtained by substituting Eq. (7) into Eq. (8) with the coefficient vector of element stiffness damage $\alpha = [\alpha_1, \alpha_2, \alpha_3, \cdots, \alpha_N]$. Then the SDD problem can be transformed into the following constrained optimization problem:

$$\begin{aligned} &\min\ f(\alpha) = \sum_i^s \left[1 - MAC\left(\boldsymbol{\varphi}_i^t, \boldsymbol{\varphi}_i^a\right) + ER\left(\boldsymbol{f}_i^a, \boldsymbol{f}_i^t\right)\right], \\ &subject\ to: 0 \le \alpha_j \le 0.9,\ j = 1, 2, 3, \cdots, N \end{aligned} \tag{9}$$

Where s is the number of measured modes, $\boldsymbol{f}_i^t$ and $\boldsymbol{\varphi}_i^t$ are the ***i***-th test nature frequency and the corresponding mode shape of the structure. $\boldsymbol{f}_i^\alpha$ and $\boldsymbol{\varphi}_i^\alpha$ are the analytical nature frequency and he corresponding mode shape, respectively. The function of $MAC\left(\boldsymbol{\varphi}_i^t, \boldsymbol{\varphi}_i^a\right)$ and $ER\left(\boldsymbol{f}_i^a, \boldsymbol{f}_i^t\right)$ are defined as follows:

$$MAC\left(\boldsymbol{\varphi}_i^t, \boldsymbol{\varphi}_i^a\right) = \frac{\left|\boldsymbol{\varphi}_i^{tT} \boldsymbol{\varphi}_i^a\right|^2}{\left|\boldsymbol{\varphi}_i^{tT} \boldsymbol{\varphi}_i^t\right| \left|\boldsymbol{\varphi}_i^{aT} \boldsymbol{\varphi}_i^a\right|} \tag{10}$$

$$ER\left(\boldsymbol{f}_i^a, \boldsymbol{f}_i^t\right) = \left|\frac{\boldsymbol{f}_i^a - \boldsymbol{f}_i^t}{\boldsymbol{f}_i^t}\right| \times 100\% \tag{11}$$

4. SPECIAL ASPECTS IN IMPLEMENTATION

In FA, the best firefly will try to improve itself by flying randomly. This behavior will be improved for the SDD problem. Actually, comparing to the number of undamaged elements, the number of damaged elements is much smaller. Therefore, for the global best location of the SDD problem, most of the elements in coefficient vector $\alpha_{best} = [\alpha_1, \alpha_2, \alpha_3, \cdots, \alpha_N]$ should be 0. And the best firefly will try to improve itself by setting coefficient vector elements as 0. This can be expressed as:

$$\begin{aligned} &\alpha_i^{new} = \begin{cases} 0, & \varepsilon < 1 - \dfrac{a_i}{\max\left(a_{best}^{now}\right)} \\ a_i, & \varepsilon \ge 1 - \dfrac{a_i}{\max\left(a_{best}^{now}\right)} \end{cases} \\ &\varepsilon = rand(),\ \alpha_{best}^{now} = [\alpha_1, \alpha_2, \alpha_3, \cdots, \alpha_N],\ \max\left(a_{best}^{now}\right) \ne 0,\ i = 1, 2, 3, \cdots, N \end{aligned} \tag{12}$$

where $\varepsilon = rand()$ is the rand number distributed in [0, 1], $\alpha_{best}^{now} = [\alpha_1, \alpha_2, \alpha_3, \cdots, \alpha_N]$ is the location of the best firefly in the current iteration. If the original light intensity at location α^{new} is better than one at location α_{best}^{now}, then the best firefly will move to location α^{new}. If the $\max\left(a_{best}^{now}\right) = 0$, the best firefly will try to improve itself by flying randomly.

In order to identify the structural damage more exactly, the multi-step method will be proposed in this paper. First of all, the SAFA will be used to solve the SDD problem. Then the undamaged elements will be picked out by the threshold value ξ. This process can be expressed as:

$$element_i = \begin{cases} \text{damage}, & \alpha_i > \xi \\ undamage, & \alpha_i \le \xi \end{cases} \tag{13}$$

where the threshold value ξ can be determined by the percent of the maximum damage factor in the front step results .Then the undamaged elements will be setting as known conditions for the next step. It means that the Eq. (9) can be rewrote as:

$$\begin{aligned} &\min\ f(\alpha) = \sum_i^s \left[1 - MAC\left(\boldsymbol{\varphi}_i^t, \boldsymbol{\varphi}_i^a\right) + ER\left(\boldsymbol{f}_i^a, \boldsymbol{f}_i^t\right)\right], \\ &subject\ to: \begin{cases} 0 \le \alpha_j \le 0.9, & j \in damaged\ elements \\ \alpha_j = 0, & j \in undamaged\ elements \end{cases} \end{aligned} \tag{14}$$

The above problem can be solved by SAFA and this process can be redone as a next step.

For a special case, if the results of the first step satisfy $\alpha_i \le \xi, \quad (i = 1, 2, 3, \cdots, N)$, which show that all the elements are undamaged, then the more exactly search will be done by setting the stiffness factors as $0 \le \alpha_i \le Up, \quad (i = 1, 2, 3, \cdots, N)$. Here the $Up < 0.9$ and usually can be set to 0.1.

Fig. 2 Finite element model of 2-storey rigid frame

5. NUMERICAL SIMULATIONS

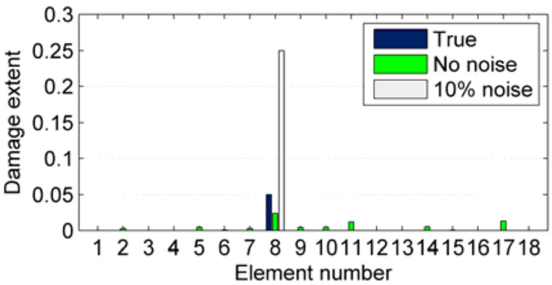
The finite element model of 2-storey rigid frame is shown in Fig. 2. Both the height and width at each storey are 1.41m and the structure is divided into 18 elements. The structure is simulation with the following parameters as shown in Table. 1. There cases are studied as: 1) 5% damage at element 17; 2) 15% damage at element 8 and 15%

damage at element 17; 3) 10% damage at element 8, 20% damage at element 11 and 15% damage at element 17. Two noise levels 0% and 10% are added to the modes shapes, respectively.

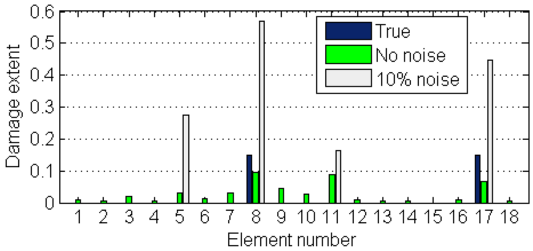
Table. 1 Material properties of 2-storey rigid frame

Type	Elastic modulus	Moment of inertia	Cross-sectional area	Density
Column	$2\times10^{11}\ N\cdot m^{-2}$	$1.26\times10^{-5}\ m^4$	$2.98\times10^{-3}\ m^2$	$8590\ kg\cdot m^{-3}$
Beam	$2\times10^{11}\ N\cdot m^{-2}$	$2.36\times10^{-5}\ m^4$	$3.2\times10^{-3}\ m^2$	$7593\ kg\cdot m^{-3}$

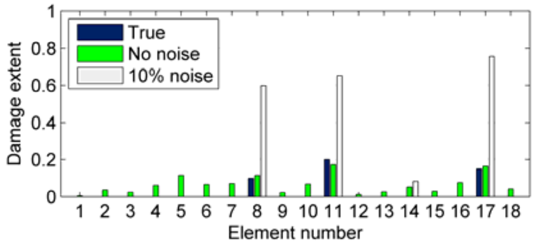
The parameters of SAFA are set as follows: initial attractiveness $\beta_0 = 1$; randomization parameter $\alpha = 0.01$, max iteration $N_{Gen} = 100$ and swarm population $N_{swarm} = 30$, the threshold value $\xi = \max(\alpha_i)\times 5\%$ and α_i is the results of the front step. The initial step identified results is taken as the mean value of 5-times running results. Five multi-steps are used to improved the identified results. The rebuild probability $P = 0$ for the first half iteration, while $P = 0.05$ for the latter half iteration. One firefly will occupied the point where all the damage factors are 0 when initializing the swarm population position for the initial step.



(a). 5%@8



(b). 15%@8 and 15%@17

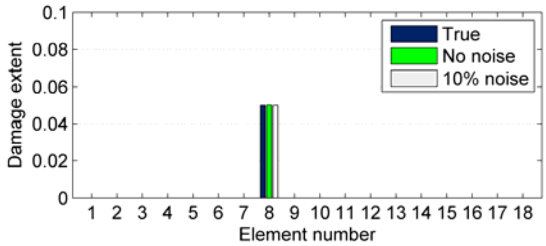


(c). 10%@8, 20%@11 and 15@17

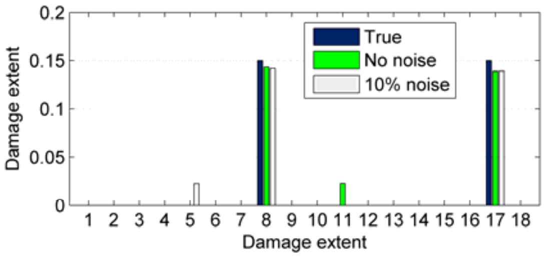
Fig. 3 SDD results only by SAFA

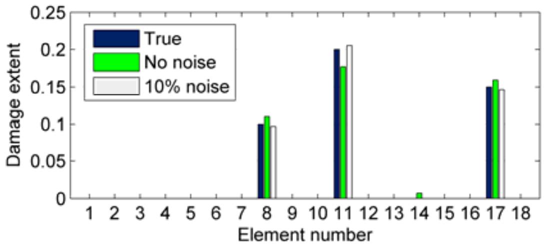
The SDD results, which are identified only by using 5-times SAFA, are shown in Fig. 3. It clearly shows that: 1) There has a large deviation between the real structure damages and detection damages for all the cases. 2) The magnitudes of the detection results at the damaged locations are always much bigger than others. Actually, the proposed SAFA has a ability of parameter self-tuning and self-control. But for high dimensional optimization problem, the SAFA is very hard to find the global best location with limited iteration and limited swarm population. That is because the searching area is too large. However, the SAFA has good ability for global searching, therefore, the SAFA can always find a better location close to the best location. So the SDD results by SAFA can always locate the damage but hard to estimate extent of damage.

The SDD results, improved by using the multi-step method, are shown in Fig. 4. It can be seen that the damage can be accurately detected for all the three cases, even though 10% noise level are considered. Actually, as the elements are estimated to be an undamaged elements, the searching area will be smaller and smaller during the multi-step processing, therefore, the SAFA can find the global best location with more possibilities.



(a). 5%@8





(b). 15%@8 and 15%@17 (c). 10%@8, 20%@11 and 15@17

Fig. 4 SDD results improved by using proposed multi-step method

6. CONCLUSIONS

A novel self-adaptive firefly algorithm based structural damage detection (SDD) method is proposed in this study. The basic principle of FA is introduced. A self-adaptive control strategy is used to improve the FA. Combined with the character of SDD problem, the new best firefly behavior is used to improve the SAFA while the multi-step method is proposed to improve the SDD results. A 2-storey rigid frame structure is taken as an example for numerical simulations on SDD. The following conclusions can be made. 1) The performance of SAFA is well with the self-adaptive control strategy based on both the distance among the fireflies and the iteration. 2) For the SDD problem, the new behavior of best firefly and the multi-step method can clearly improve the accuracy of SDD result. 3) The proposed method can accurately identify the structural damage and has a strong robustness to noises.

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