

Single and multi-material topology optimization to retrofit beam-column connection using CFRP composite

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ABSTRACT

Composite materials have been commonly used to strengthen existing RC structures which were vulnerable to seismic loading during earthquakes. Among these composite materials, Carbon Fiber Reinforced Plastic (CFRP) was considered to be an effective way to retrofit current existing non-seismic designed buildings due to its beneficial features such as high strength and stiffness-to-weight ratio, simple implementation and short construction time. Many experiments were carried out to deeply investigate the CFRP effect of increasing structures stiffness and deformation capacity. This study aims to provide an optimal CFRP configuration to effectively retrofit beam-column connection, regardless of its complex application practically. Optimal CFRP configuration was obtained from a procedure of continuous material topology optimization which is extended from academic version of a 99-line Matlab code. Beam-column connection was topologically optimized under certain loadings and boundary conditions to obtain the optimal detail of material. In addition, Heaviside function and penalization factor were also considered to obtain the diverse results. Furthermore, multi-material topology optimization procedure was applied to the beam-column connection for the usage of combination of specific materials.

Keywords: Topology optimization, 99-line Matlab code, multi-material, stress concentration, beam-column joint, retrofitted.

1. INTRODUCTION

Recently, many existing building structures, which were designed and constructed according to conventional codes, are deficient in seismic detail that makes structures failed while sustaining seismic loadings during severe earthquakes. A common solution is to retrofit existing structures with extra material to strengthen structure capacity. Carbon Fiber Reinforced Plastic (CFRP), which has advanced properties among several composite materials such as high strength and stiffness-to-weight ratio, simple

implementation, excellent fatigue behavior and corrosion resistance, is popularly applied to retrofit existing structures. Many experiments were carried out previously to investigate the impact of CFRP on the beam-column connection. Le-Trung et al. [1] using different configuration arrangement of CFRP (L, T and X shape) in experiment to find which significantly strengthen the beam-column connection. Both experimental and analytical results were compared to assure CFRP effect base on its configuration. However, the adoption of CFRP configuration arrangement was a process of trials and errors that mostly depends on experience and intuition of engineers.

Topology optimization has become very popular in several fields aiming for the goal of obtaining the optimal material distribution within a prescribed design variables. Many researches have been focused on topology optimization using various approaches. Both 2D and 3D structures were deeply investigated using single and multi-material topology optimization procedures. Y. Wang et al. [2] presents a new multi-material level set topology description model for topology and shape optimization. J. Park et al. [3] proposes a multi-resolution implementation in 3D for multi-material topology optimization problem. These mentioned methods have proven their ease and effectiveness through several numerical examples.

This research aimed to find a most effective arrangement of CFRP to retrofit the beam-column connection using a procedure of continuous material topology optimization. Obtained configuration arrangement of material is expected to be beneficial in terms of structure capacity and economical aspect. Results obtained from single material and multi-material topology optimization procedure were compared and discussed.

2. TOPOLOGY OPTIMIZATION PROBLEM

2.1 Single material topology optimization

For single material topology optimization of the beam-column connection, an alternative approach so-called “power-law approach” or SIMP approach (Solid Isotropic Material with Penalization) [4] was adopted. A topology optimization problem based on the SIMP approach can be mathematically expressed as follow,

$$\begin{aligned} \text{Minimize: } & c(x) = U^T K U = \sum_{e=1}^N x_e^p u_e^T k_0 u_e \\ \text{Subject to: } & \frac{V(x)}{V_0} = f \\ & : KU = F \\ & : 0 < x_{\min} \leq x \leq 1 \end{aligned}$$

where U is the global displacement, F is the force vectors, K is the global stiffness matrix, u_e and k_e are the element displacement vector and stiffness matrix, respectively. x is the vector of design variables, x_{\min} is a vector of minimum relative densities (non-zero to avoid singularity). N is the number of elements in the design domain, p is the penalization power, $V(x)$ and V_0 is the material volume and design domain volume, respectively and f is the volume fraction. Figure 1 presents the flowchart of general topology optimization procedure.

A 99 line-Matlab code implementation proposed by O. Sigmund [4] was adopted and modified to topology optimize the beam-column connection with single material. In order to obtain clear result, Heaviside functions were also considered in the topology optimization and compared with the originals. Heaviside functions which were used in this procedure are discontinuous functions who value is toward zero for below 0.5 argument and toward one for above 0.5 argument. Figure 2 illustrates the Heaviside functions graphically. The Heaviside functions are expressed as follow,

$$F_1(x) = \frac{3}{4} \left[\frac{x-0.5}{\rho} - \frac{1}{3} \left(\frac{x-0.5}{\rho} \right)^3 \right] + \frac{1}{2}$$

$$F_2(x) = \frac{1}{2} + \frac{2}{\pi} \arctan \left(\frac{x-0.5}{\rho} \right)$$

$$F_3(x) = \frac{1}{2} \left(1 + \frac{x-0.5}{\rho} + \frac{1}{\pi} \sin \left(\pi \frac{x-0.5}{\rho} \right) \right)$$

$$F_4(x) = \frac{1}{2} \left(1 + \sin \left(\pi \frac{x-0.5}{2\rho} \right) \right)$$

Where $\rho = 0.5$

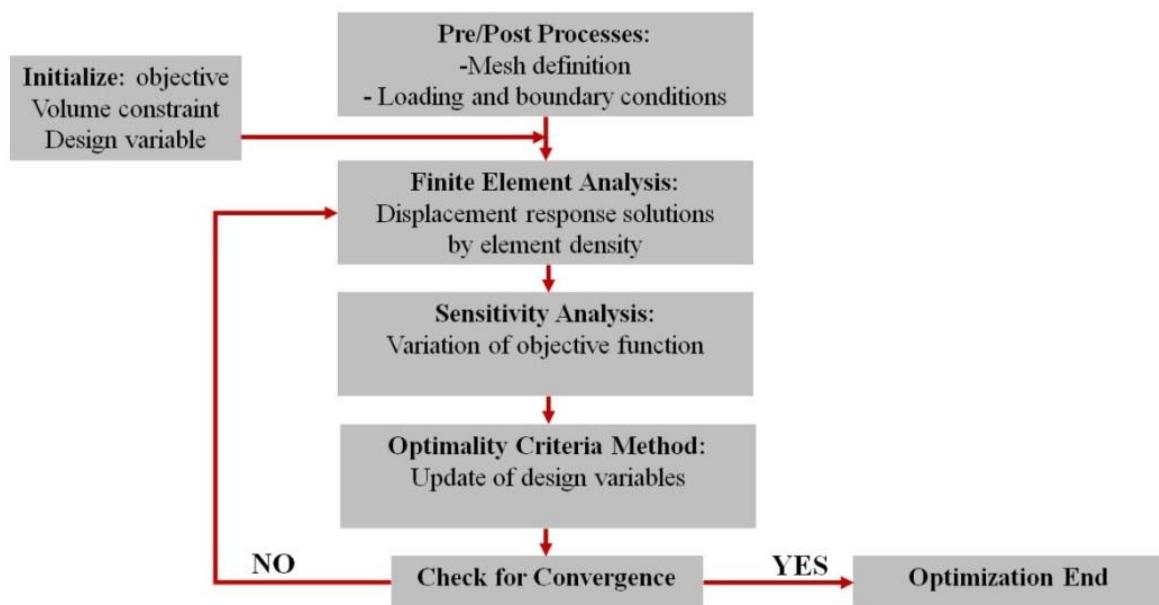


Figure 1. Flowchart of general topology optimization procedure

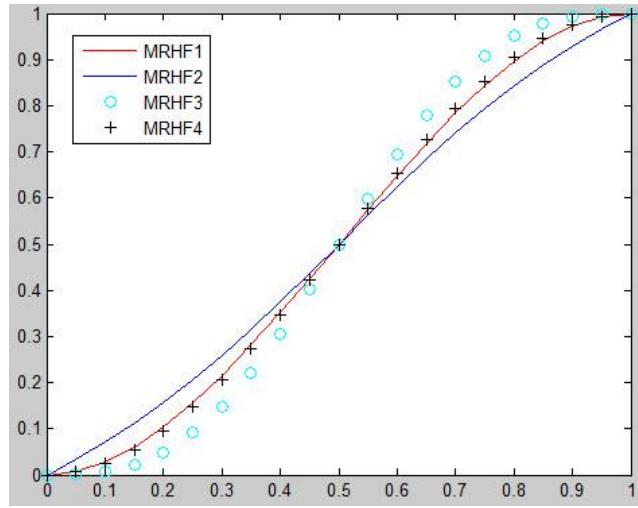


Figure 2. Heaviside function

In addition, the value penalization was also considered in this topology optimization. The SIMP approach has been criticized and argued about the existence of physical material with properties described by the power-law interpolation. However, it is proved that the power-law approach is physically permissible as long as simple conditions on the power are satisfied. Penalization with the value equal to 1 was considered or topology optimization without penalization.

2.2 Multi material topology optimization procedure

For multi-material topology optimization of the beam-column connection, alternating active-phase algorithm was adopted using modified version of 115-line Matlab implementation [5]. In Tavakoli and Mohseni [5], multi-material topology optimization problem can be expressed as follow,

$$\text{Minimize: } J^h(\alpha^h, (U^h(\alpha)^h)) \text{ where } \alpha^h \in A^h$$

$$\text{Subject to: } R^h(M(\alpha^h), (U^h(\alpha)^h)) = 0 \text{ in } \Omega^h$$

3. RESULTS FOR BEAM-COLUMN CONNECTIONS

In this section, single and multi-material topology implementations of beam-column connection are presented. For both cases, voids were put in the initial rectangular design domain to obtain the geometry of the beam-column connection. Lateral load is imposed at the center top of the column. Boundary condition of the beam-column connection is illustrated in Figure 3.

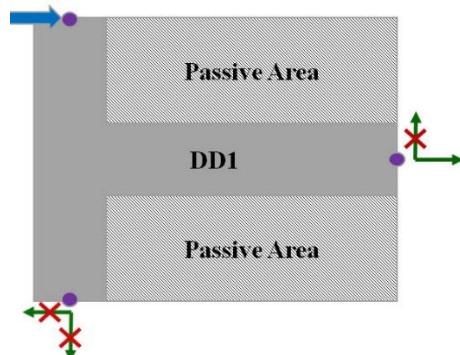


Figure 3. Design domain of the beam-column connection and boundary conditions.

3.1 Single material topology optimization results

In this single material topology optimization, Young's Modulus and Poisson's ratio are 1 and 0.3, respectively. The minimum length scale (filter size) is 1.5 and the penalization factor is 3. Many volume fraction values of 5%, 10%, 15% and 30% of the whole design domain were chosen. Detailed material distributions of these cases are depicted in Figure 5. It can be observed that with the increasing volume fraction, the rearrangement mostly occurs inside but edges of beam-column connection. The case of 10% volume fraction was chosen to apply the Heaviside functions and penalization factor of 1. Four Heaviside functions mentioned above were all

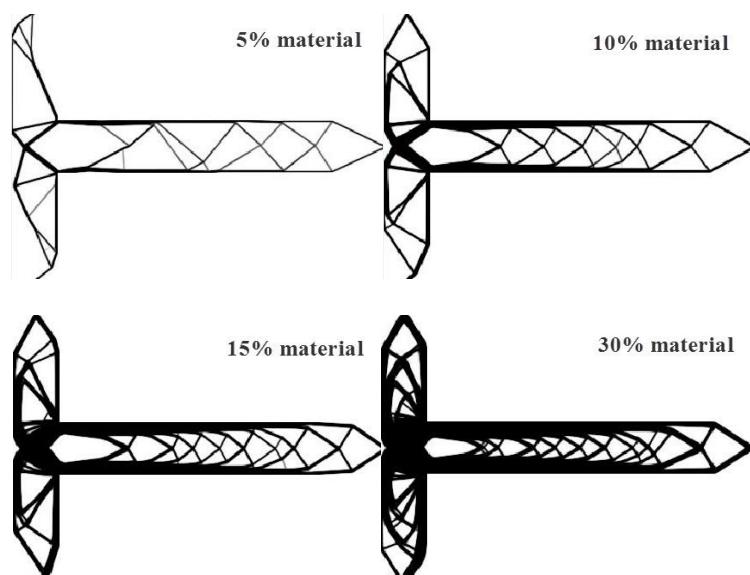


Figure 4. Detailed density arrangement of material

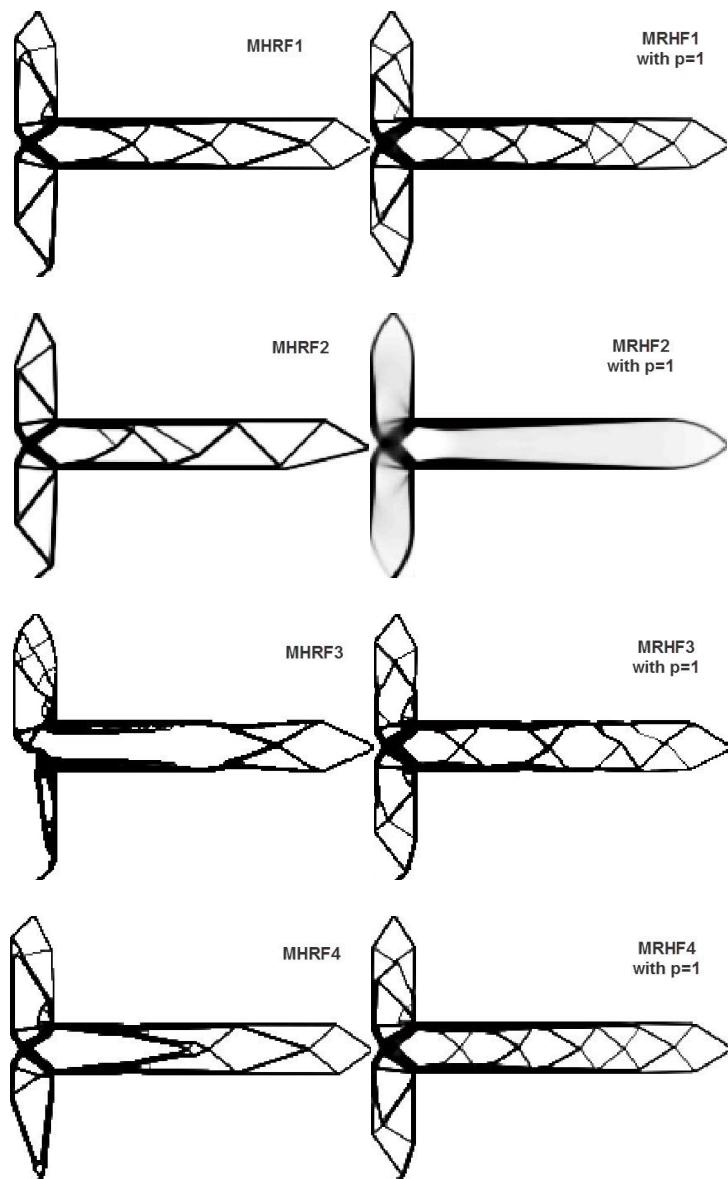


Figure 5. Detailed results after using Heaviside functions and without penalization

employed. Afterward, penalization factor was neglected ($p=1$) in these four cases. All detailed results are presented in Figure 5. Material distributions, which were filtered with the Heaviside functions, have a major change compare to the original one. Especially in cases of Heaviside function 3 and 4, material in the bottom of the column and the beam were severely relocated.

3.2 Multi material topology optimization results

In this multi-material topology optimization, concrete and CFRP were chosen to be the main and the retrofit material, respectively. Hence, elasticity modulus is assumed as $E = 8$ for CFRP and $E = 1$ for concrete, which is in approximate ratio to

their real elasticity modulus value. The void has $E = 1e-9$. The Poisson's ratio of 0.3 was employed for both material. Two case with different volume of concrete and CFRP were considered. In the first case, same volume of 10% the whole design domain was employed for both concrete and CFRP. In the second case, volume of CFRP was reduced to 5% and volume of concrete was increased to 15%. By reducing the volume of CFRP, we can find the location to effectively retrofit the beam-column connection with CFRP. In order to see the significant topological change of the beam-column connection during the solution process, several topologies at levels of iteration are shown. Figure 6 shows the topologies at selected iterations for the first case where CFRP and concrete share the same volume fraction of 10%. Convergence of the compliance (or strain energy change) of this case is also shown in Figure 7. The results of topologies and iteration history for the second case are shown in Figure 8 and 9, respectively.

Observing the change of CFRP distribution in two case, we can see the main location to put the retrofit material to the beam-column connection. Edges along the beam and the joint area, where stress mostly concentrates during loading, located the most density of stiff material (CFRP). One can also observe that the general topology of the multi-material is similar to that of the single material topology.

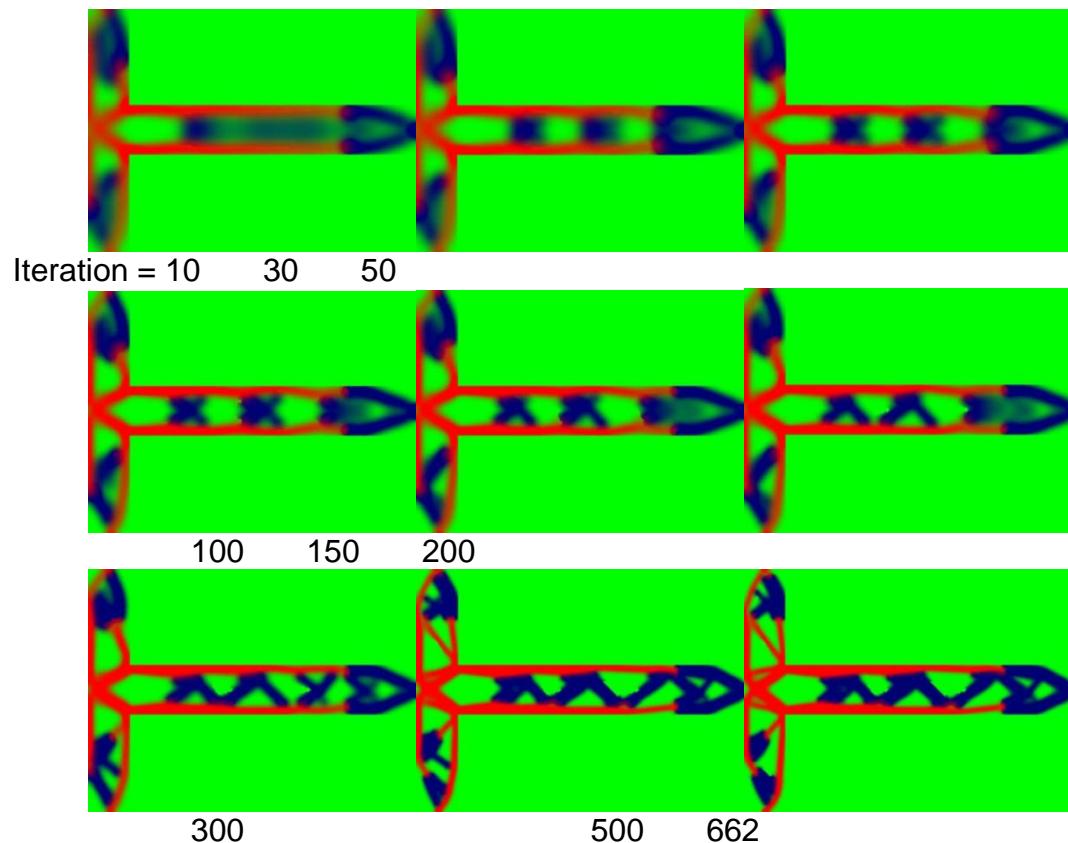


Figure 6. Results of material distributions in selected iterations

- █ $E = 8$ and $V = 0.1$ (CFRP)
- █ $E = 1$ and $V = 0.1$ (Concrete)
- █ $E = 1e-9$ and $V = 0.8$ (Void)

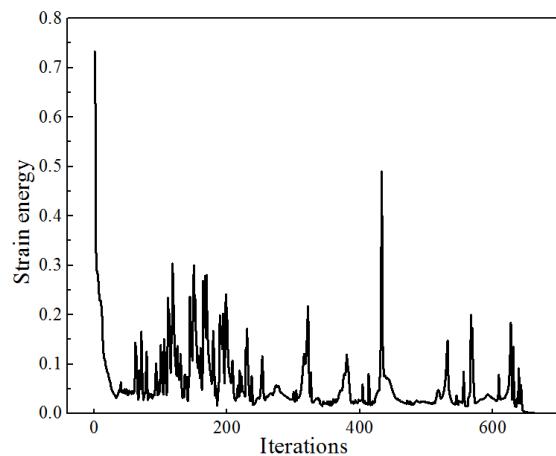


Figure 7. Iteration history of the compliance

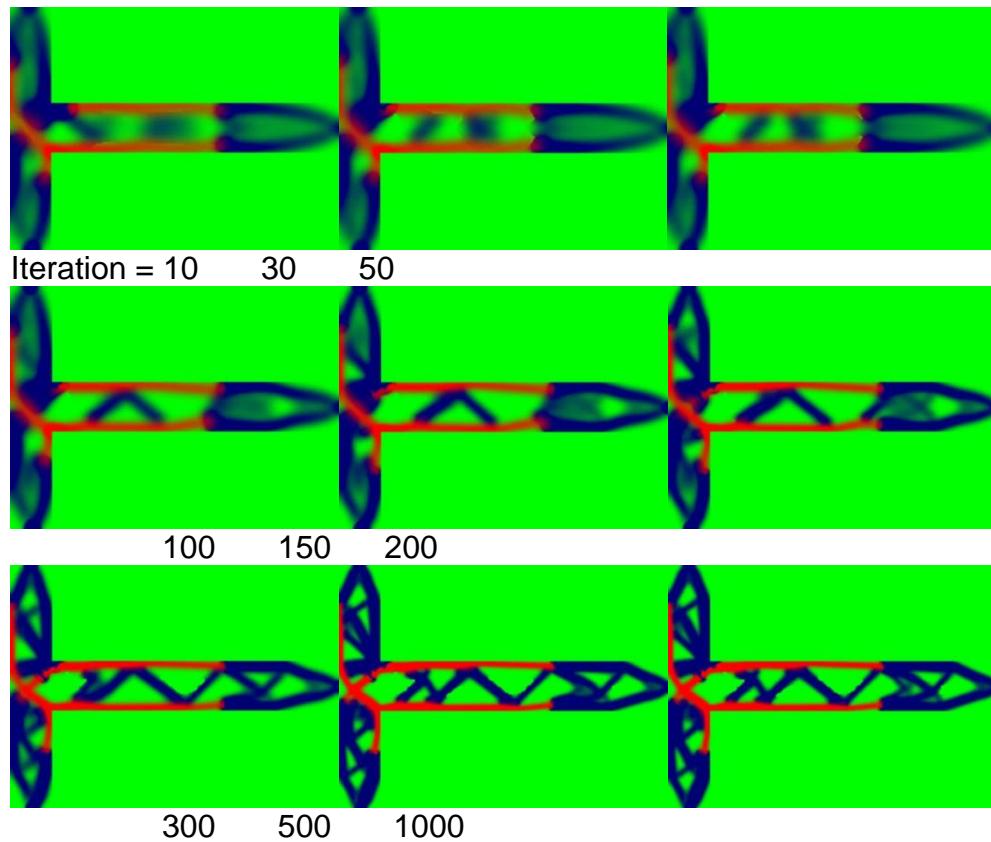


Figure 8. Results of material distributions in selected iterations

- █ $E = 8$ and $V = 0.05$ (CFRP)
- █ $E = 1$ and $V = 0.15$ (Concrete)
- █ $E = 1e-9$ and $V = 0.8$ (Void)

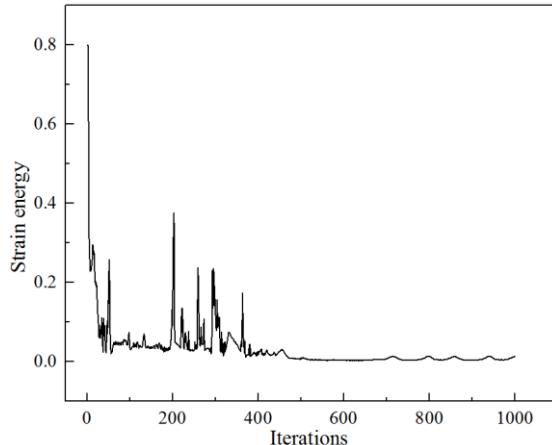


Figure 9. Iterations history of the compliance

4. CONCLUSION

This paper shows several results of single and multi-material topology optimizations for the beam-column connection. The results might be helpful in order to retrofit the beam-column connection with some foundations instead of experience and intuition. The results can also be adopted to build analytical assessment to assure for accuracy of the topology optimizations.

Acknowledgments

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