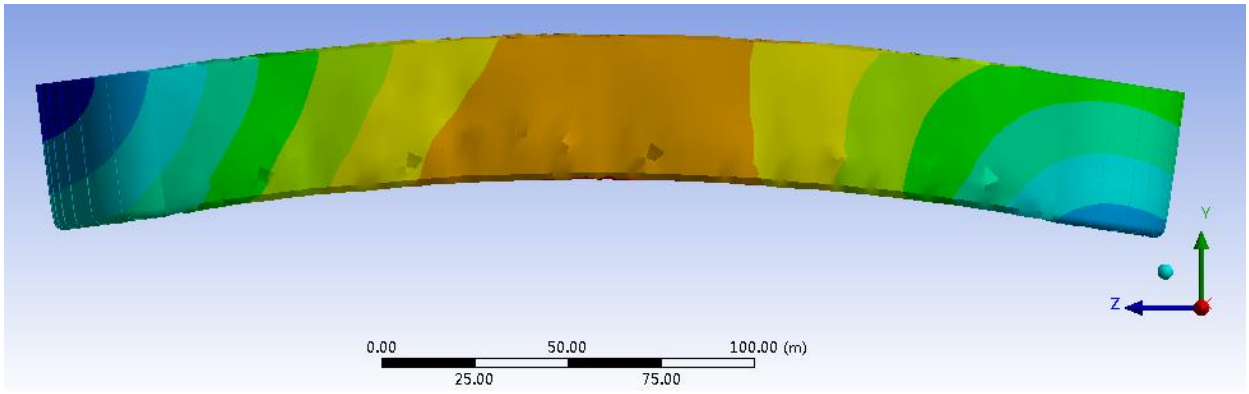
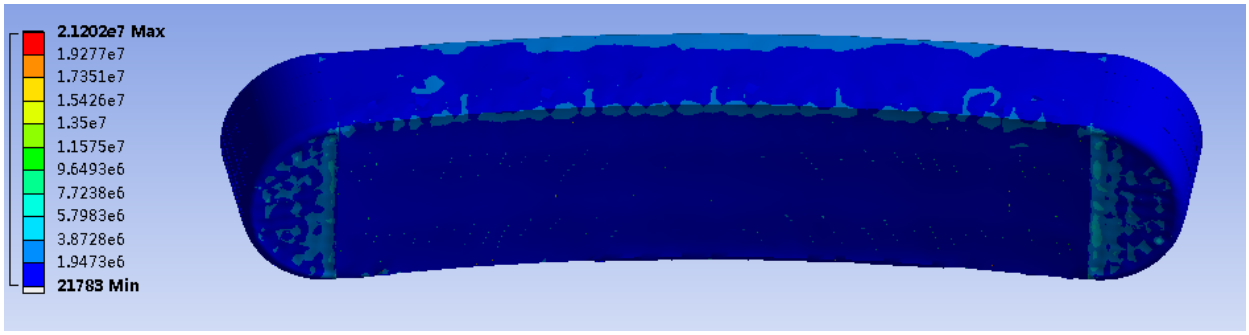
With further decease in weight of the stored oil, the FPSO will go into hogging. The extent of hogging is the highest under the ballast draft condition. In fact, this condition just happens when most of the stored oil is exported from the FPSO to the shuttle tankers. Since the oil & gas production equipment is working all the time on the main deck, it is not possible that no oil exists in the cargo oil tankers. The total deformation and equivalent stress under ballast draft condition are shown in Fig. 6.

The total deformation will be the biggest under the full load or ballast draft condition, and the results depend on the weight distribution and the design of the FPSO.

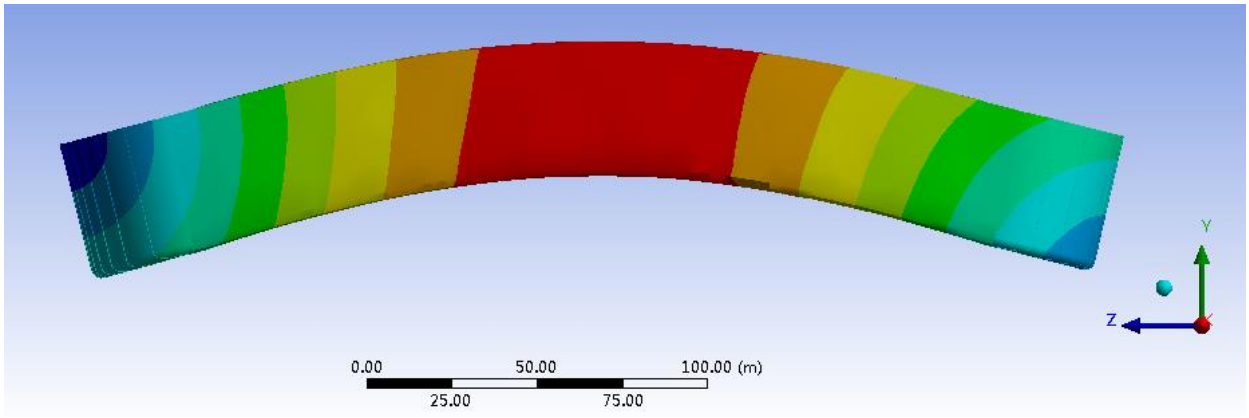


(a) Total deformation diagram under half load condition (scale 4.1e003)

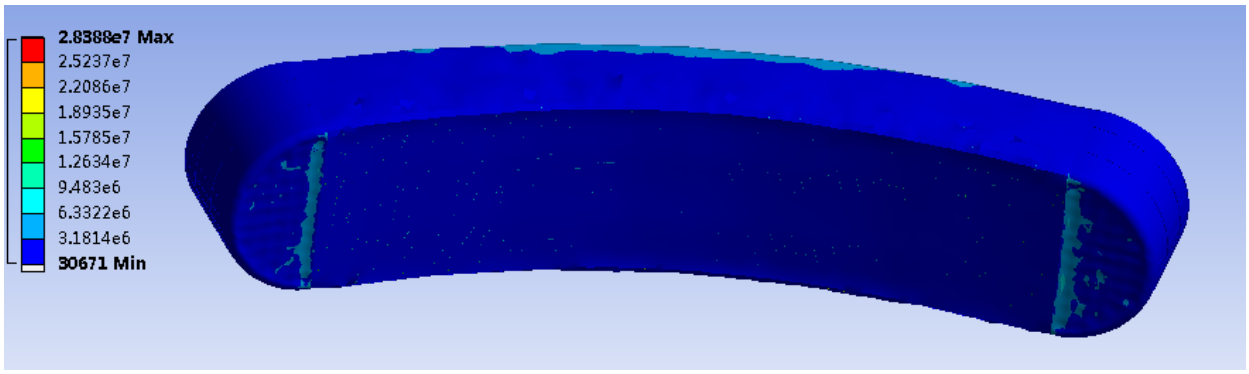


(b) Equivalent stress diagram under half load condition (scale 4.1e003)

Fig.5 Simulation results under half load condition



(a) Total deformation diagram under ballast draft condition (scale 4.1e003)



(b) Equivalent stress diagram under ballast draft condition (scale 4.1e003)

Fig.6 Simulation results under ballast draft condition

4.2 Dynamic simulation

As stated earlier, the VWBM is caused by wave that hits the FPSO. Since the waves are random events, meaning that future behavior of wave cannot be precisely predicted (Wang 2014), it is reasonable to describe the wave using statistical or probabilistic approaches (Cui, Chen and He 2017). Several wave spectra have been proposed, such as the Neumann spectrum, Bretschneider spectrum, P-M spectrum and JONSWAP spectrum (Goggins and Finnegan 2014). In this paper, the JONSWAP spectrum, which is formulated according to the marine condition in North Sea, is adopted to simulate the dynamic load of FPSO (Yurovskaya, Dulov, Chapron and Kudryavtsev 2013). The detailed parameters of JONSWAP spectrum are shown in Table 3. As shown in Table 3, the energy of wave is concentrated at the frequency interval from 0.1Hz to 1.2 Hz. The dynamic simulation is conducted in ANSYS Workbench 15.0 using the 3-D model of FPSO mentioned above. The simulation results are shown in Fig. 7.

Table 3. JONSWAP spectrum

ω	$S(\omega)$	ω	$S(\omega)$
0.50	3.59E-11	1.35	4.20E-01
0.55	2.40E-07	1.40	3.58E-01
0.60	5.55E-05	1.45	3.15E-01
0.65	1.78E-03	1.50	2.79E-01
0.70	1.71E-02	1.60	2.20E-01
0.75	7.59E-02	1.70	1.72E-01
0.80	2.05E-01	1.80	1.35E-01
0.85	3.98E-01	1.90	1.07E-01
0.90	6.32E-01	2.00	8.48E-02
0.95	9.38E-01	2.50	2.97E-02
1.00	1.48E+00	3.00	1.22E-02
1.05	2.40E+00	4.00	2.95E-03
1.10	3.00E+00	5.00	9.71E-04
1.15	2.40E+00	6.00	3.91E-04
1.20	1.41E+00	7.00	1.81E-04
1.25	8.11E-01	8.00	9.29E-05
1.30	5.40E-01		

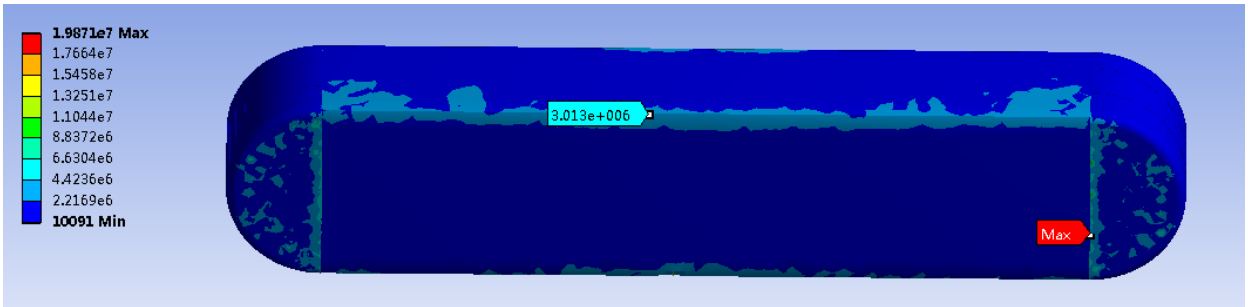


Fig. 7 Dynamic simulation results

As shown in Fig. 7, the total deformation and the equivalent stress caused by wave are much less than that caused by the SWBM. But during the whole service life (over 20 years), the FPSO may be subjected to the wave load more than 10^8 times. When predicting the fatigue life of FPSO, all the static load and wave load play important roles.

4.3 Combination of the effect of SWBM and VWBM

The equivalent stress caused by static load σ_{static} is related to the self-weight of FPSO W_{self}, the weight of stored oil W_{oil}, and the weight distribution, as follows.

$$\sigma_{static} = F\left(W_{self}, W_{oil}, f(weight, length)\right) \tag{6}$$

where $f(weight, length)$ is the function of weight distribution of the FPSO.

The equivalent stress caused by wave load σ_{wave} is related to the wave spectrum $S(\omega)$, as follows.

$$\sigma_{wave} = |T(x)|^2 S(\omega) \tag{7}$$

where $T(x)$ is the transfer function of the wave spectrum for the stress.

The total equivalent stress of the FPSO can be calculated as:

$$\sigma_T = \sigma_{static} + \sigma_{wave} = F\left(W_{self}, W_{oil}, f(weight, length)\right) + |T(x)|^2 S(\omega) \tag{8}$$

Based on the S-N curve, it is easy to calculate the cycle times N_{static} and N_{wave} under σ_{static} and σ_{wave}. Further, the cumulative fatigue damage (CFD) theory, which is the traditional theoretical framework for fatigue life prediction (Pinto, Pujol, and Cimini 2014) is adopted for predicting the fatigue life. According to the CFD and Miner ruler, which hypothesizes that the cumulative damage follows a simple linear trend (Aid, Amrouche, Bouiadjra, Benguediab and Mesmacque 2011), the fatigue damage accumulation *D* can be calculated as follow:

$$D = \left(\frac{n_{i-static}}{N_{static}}\right)^{\lambda_i} + \left(\frac{n_{i-wave}}{N_{wave}}\right)^{\lambda_j} \tag{9}$$

where $n_{i-static}$ is the alternating times of static load, n_{i-wave} is the alternating times of wave load, λ_i and λ_j are parameters related to the stress range. Failure is expected to occur if $D \geq 1$.

5 Conclusions

In this paper, the total deformation and equivalent stress caused by still water and vertical wave-induced bending moments are calculated for a specific FPSO using finite element analysis. The static simulations are conducted under full load, half load and ballast draft conditions. The FPSO is sagging under the full load condition and hogging under the ballast draft condition. The total deformation will be the biggest under the full load or ballast draft condition, and the results depend on the weight distribution and the design of the FPSO. Dynamic simulation of FPSO is carried out based on a specific wave spectrum. Further, based on cumulative fatigue damage theory and Miner ruler, an empirical formula is proposed to calculate fatigue damage accumulation of FPSO so as to predict the fatigue life of FPSO.

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