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Extension of modal space based FE solutions into the geometrically nonlinear domain

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ABSTRACT

Modern hardware tools enable computations with rather large FE models that can easily contain up to several million degrees of freedom. Despite of that, the demands of structural dynamics call for high efficiency, particularly in the field of Multi-Body System (MBS) dynamics or real-time interactive simulations in the field of virtual reality. Model reduction represents the typical way of handling the problem. Solutions based on the modal space are quite often used. The great numerical efficiency achievable in this manner is however accompanied by the drawback that the solutions are limited to linear deformations. This paper aims at extensions of existing modal-space based solutions into moderate geometrically nonlinear domain. The extensions considered are based on stress stiffening effect and local sub-structural rotations. They keep the high numerical efficiency of the modal superposition technique. A few cases are provided to exemplify the application of the proposed methods.

1. INTRODUCTION

Consideration of structural deformations is a necessity in many fields of engineering. Accuracy is a typical requirement from a conducted structural analysis. However, modern design solutions call for an increased number of both experimental and numerical tests in order to provide adequate structural safety, robustness and reliability. With this requirement, the numerical efficiency of models gains in importance.

The Finite Element Method (FEM) has established itself as the method of choice for the computations in the field of structural analysis. It offers high accuracy, but is also numerically rather demanding. The computational power of modern hardware tools allows for numerically more expensive models containing even millions of degrees of freedom (DOFs). But computations with such models may take hours and even days, particularly if nonlinear and dynamical structural behavior is considered. Furthermore, there are fields of applications, such as virtual reality (VR) simulators, or Multi-Body

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System (MBS) dynamics), in which numerical efficiency is of similar and maybe even greater importance (for instance, in VR applications) than the accuracy. In those cases, model reduction represents the typical way of handling the problem. It should be noted that simulations in those fields involve structures that perform large motion and therewith large rotation. Obviously, savings can be made by neglecting deformation and thus reducing the problem to only 6 DOFs of rigid-body motion per object. Even such a dramatic reduction poses some complexity resulting from nonlinearity in equations of motion and constraints used to define connections between the bodies. Therefore, attempts to improve the solution strategies were also considered even for MBS systems involving rigid-bodies only. For instance, Xiang et al. (2015) introduced independent displacement modes based on Moore-Penrose generalized inverse matrix for dynamic analysis of deployable structures. However, consideration of deformable behavior makes the problem significantly more complex (Zehn 2005).

Proper and robust model reduction techniques for deformable models represent a challenging task. Typical solution in the FEM and MBS is modal reduction, with different approaches for choosing the adequate mode shapes. The main drawback of those approaches, regardless of the chosen mode shapes, is their linear character, i.e. the intrinsic applicability to linear analysis, whereby many cases of applications demand consideration of nonlinear structural behavior for acceptable accuracy. Hence, some rather simple techniques are proposed here as a possible remedy for the issue.

2. MODAL SUPERPOSITION TECHNIQUE

Modal superposition technique implies that the deformational structural behavior is represented in terms of modal degrees of freedom. A carefully chosen set of mode shapes yields the degrees of freedom, so that the nodal displacements of the FEM assemblage, $\boldsymbol{d}$, are approximated as a linear combination of the mode shapes $\boldsymbol{\theta}_i$:

$$\boldsymbol{d}(t)=\sum_{i=1}^{N_m} m_i(t)\boldsymbol{\theta}_i \tag{1}$$

where m_i are the modal coefficients and N_m is the number of selected mode shapes. In the framework of the approach, the structural linear stiffness and mass matrices, $\boldsymbol{K_L}$ and $\boldsymbol{M}$, are reduced in the following manner:

$$\boldsymbol{K_m}=\boldsymbol{\Theta}^{\mathrm{T}}\boldsymbol{K_L}\boldsymbol{\Theta} \tag{2}$$

$$\boldsymbol{M_m}=\boldsymbol{\Theta}^{\mathrm{T}}\boldsymbol{M}\boldsymbol{\Theta} \tag{3}$$

with $\boldsymbol{\Theta}=[\boldsymbol{\theta}_1\ \boldsymbol{\theta}_2\ldots]$ denoting the matrix of mode shapes. The dynamic equation now reads:

$$\boldsymbol{M_m}\ddot{\boldsymbol{m}}+\boldsymbol{C_m}\dot{\boldsymbol{m}}+\boldsymbol{K_m}\boldsymbol{m}=\boldsymbol{F_m}(t) \tag{4}$$

where $\boldsymbol{C_m}$ is the matrix of modal damping coefficients, $\boldsymbol{F_m}$ are the external forces transformed to modal space and $\boldsymbol{m}$ is the vector of modal coefficients.

In this manner a substantial reduction of the model size can be achieved. The art of performing model reduction based on this technique comes down to the proper choice of mode shapes. Typical approach used in FEM is to use the so-called normal modes (Bathe 1996), which determine the patterns of vibration of a system at specific frequencies (eigenfrequencies). On the other hand, in order to provide higher flexibility in the choice of boundary conditions, in MBS the Craig-Bampton (1968) modes are typically used. They represent a mixture of normal (vibration) modes and static modes.

Regardless of the choice of mode shapes, they characterize the structure in its initial (undeformed) configuration. Therefore, the modal superposition technique is intrinsically a linear method and applicable to small deformations. In what follows, we aim at extensions of the technique in order to cover moderate geometrically nonlinear effects.

3. EXTENSION INTO THE GEOMETRICALLY NONLINEAR DOMAIN

Over the course of deformation, the configuration (geometry) and the stress state of the structure change continuously. Both effects have their influence on the global tangent stiffness matrix. The change in geometry is reflected in the change of the linear stiffness matrix, whereas the stress state influences the geometric stiffness matrix. Now, we shall consider ideas how to include this influences in the model reduced based on the modal superposition approach.

3.1 Geometric stiffness

The commercial MBS software package SIMPACK already offers an option to include the geometric stiffness matrix in a model of a deformable body based on modal superposition. However, it is limited to certain types of elements (e.g. beams) and it is force scaled, i.e. the geometric stiffness matrix is a linear function of the force and, hence, it is applicable for forces whose effect onto the structure is of quasi-static nature. What we propose is a relatively simple, but more general approach in which the stress state in a deformable part is assumed to be directly proportional to the deformation. One may notice that this is actually the assumption of linear analysis. With this assumption, one may determine the stress state and, therewith, the geometric stiffness matrix, $\boldsymbol{K}_{\boldsymbol{G}i}$, for a mode shape and a certain value of the modal coefficient, say for the value of 1. During a simulation, for an arbitrary value of the modal coefficient, the current geometric stiffness matrix is simply scaled. The overall geometric stiffness matrix is obtained by superposing the scaled geometric stiffness matrices for single mode shapes:

$$\boldsymbol{K}_{\boldsymbol{G}} = \sum_{i=1}^{N_m} m_i \boldsymbol{K}_{\boldsymbol{G}i} \tag{5}$$

The resulting tangent stiffness matrix is given by the sum of the linear and geometric stiffness matrix. The computational effort during the simulation remains rather small as the modal $\boldsymbol{K}_{\boldsymbol{G}i}$ matrices are computed in a pre-step and this is where the actual difficulty resides. As a matter of fact, certain FEM software packages provide the option for direct extraction of the geometric stiffness matrix, whereas in some other FEM programs inventive approaches are needed. For instance, the possibility of extracting the tangent stiffness matrix can be used to resolve the problem. One may do this for a configuration obtained by deforming the initial one. Additionally, the stiffness matrix may be extracted for the configuration that has the same shape as the deformed one, but is considered to be the initial one (hence, stress free). Subtracting the latter from the formed stiffness matrix yields the geometric stiffness matrix.

A simple example of academic nature illustrates the method. A steel plate (Young's modulus of $2.1 \cdot 10^{11}$ Pa and the Poisson coefficient of 0.3), with the in-plane dimensions of 1 m and 0.8 m, and 0.008 m thick, is clamped at all four corners and exposed to a transverse force of 7.5 kN acting at its centroid, Fig. 1. Such a force gives rise to plate deformation that requires consideration of geometrically nonlinear effects for proper simulation accuracy. The results of linear and geometrically nonlinear computations performed in Abaqus with the structure discretized using 80 quadratic shell elements serve as reference solutions. The linear result in modal space is almost congruent with the linear result of the full FE model and is therefore not depicted in diagram in Fig. 1. The linear result is shown in the diagram in order to emphasize the need for consideration of geometrically nonlinear effects. It is easily recognized that, in this specific case, the addition of the geometric stiffness matrix successfully extends the applicability of the solution in modal space into the nonlinear domain. At the same time, the numerical effort is kept at a very low level.

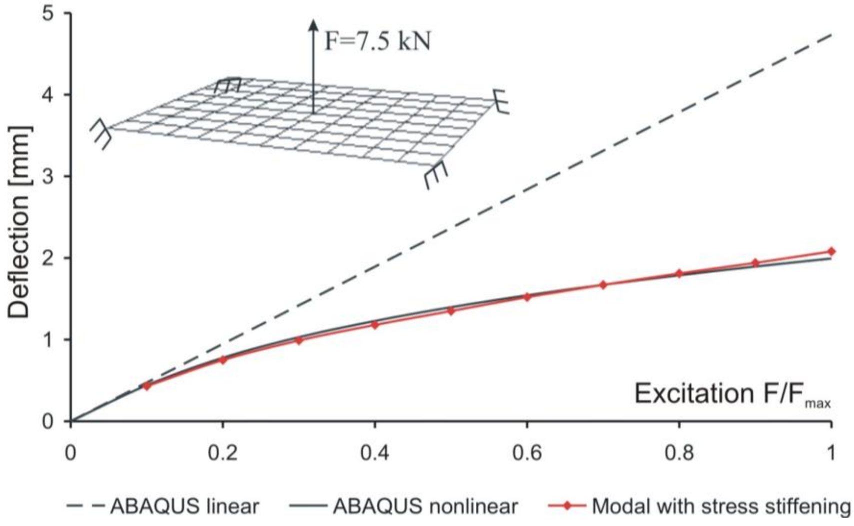

Fig. 1 Plate exposed to transverse force and diagram for the centroid transverse deflection according to different models

3.2 Displacement rotations

In many cases of structural deformation, parts of the structure exhibit large local rotations. The influence of this aspect onto the structural stiffness is captured by the change in the linear stiffness matrix of the FE assemblage. Stress stiffening could also play an additional role in such a behavior, but it cannot handle rotations. Hence, a different approach would be needed to account for local rotations.

Generally speaking, this effect demands to consider structural rotations locally. Surely, the effect is covered in geometrically nonlinear formulations with full (i.e. not reduced) FE models. However, in certain cases, substructures can be selected so that a single ‘average’ rotation can acceptably describe the rotation of a whole substructure. This depends on the structural form and the complexity of induced deformation. Beam-like structures or substructures belong to convenient candidates for this approach. The idea is to first compute the displacements using the conventional modal superposition technique. Then the average rotation of a conveniently chosen substructure is determined and, finally, the computed displacements are rotated by the same average rotation of the substructure.

A car rear axle is a good candidate to apply the method. In deformation, its crank arms may perform a rotation exceeding 10°. Therefore, the crank arms are chosen as substructures to apply the displacement rotation upon. The objective is to keep the high numerical efficiency provided by the modal superposition technique, but to improve at the same time the accuracy of predicting the geometry of the axle’s deformed state. Hence, two geometric quantities are observed during the deformation – the suspension displacement and the toe-in angle. They are very important for an accurate prediction of the car trajectory in a curve. For a chosen load case, the two quantities are depicted versus each other in Fig. 2. Again, as reference solutions, linear and geometrically nonlinear solutions obtained in Abaqus using the full FE model are used. The solution in modal space with displacement rotation is obtained using only 10 normal modes providing thus a great reduction of numerical effort. And the improvement in the correlation between the two above mentioned geometric quantities is obvious from Fig. 2.

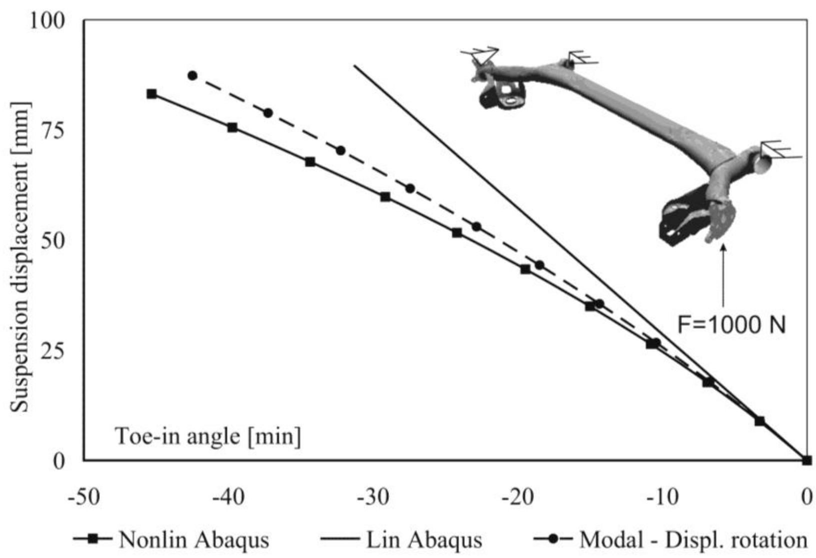

Fig. 2 Car axle – results obtained using full and reduced FE models

4. CONCLUSIONS

The efficiency of numerical simulation gains in importance each day. Therefore, various model reduction techniques are developed to offer a well-balanced compromise between the efficiency and simulation accuracy. The developed techniques are mostly limited to linear analysis. This paper addressed two relatively simple ideas to extend the applicability of model reduction based on modal superposition techniques into the realm of moderate geometric nonlinearities. How successful the techniques can be depends on the character of deformation, as they aim as specific causes of geometric nonlinearities. The considered examples obviously illustrate a successful application, but a thorough analysis of the structural deformational behavior would be worthwhile prior to model preparation in order to assess whether one of the considered techniques, or maybe even their combination, would be the adequate choice.

REFERENCES

Bathe, K.J. (1996), *Finite Element Procedures*, Prentice Hall, New Jersey, USA.

Craig, R.R. and Bampton, M.C.C. (1968), “Coupling of substructures for dynamic analyses”, *AIAA Journal*, **6**(7), 1313-1319.

Xiang, P., Wu, M. and Zhou, R.Q. (2015), “Dynamic analysis of deployable structures using independent displacement modes based on Moore-Penrose generalized inverse matrix”, Structural Engineering and Mechanics, **54**(6), 1153-1174.

Zehn, M. (2005), “MBS and FEM: A Marriage-of-Convenience or a Love Story?”, *BENCHmark Int. Magazine for Eng. Design&Analysis*, 12-15: