

$$N_u = \frac{1}{u} \frac{\partial \varphi}{\partial u} + \frac{1}{u^2} \frac{\partial^2 \varphi}{\partial v^2} = \pm \frac{EhH}{\pi} \frac{1}{u} \sum_{m=1}^{\infty} \left(\frac{d}{du} - \frac{m^2}{u} \right) \nabla_k^2(u, m) \Phi_m(u) \sin mv ,$$

$$N_v = \frac{\partial^2 \varphi}{\partial u^2} = \pm \frac{EhH}{\pi} \frac{d^2}{du^2} \sum_{m=1}^{\infty} \nabla_k^2(u, m) \Phi_m(u) \sin mv ,$$

$$S = -\frac{\partial}{\partial u} \left(\frac{1}{u} \frac{\partial \varphi}{\partial v} \right) = \frac{EhH}{\pi} \frac{1}{u} \left(\frac{d}{du} - \frac{1}{u} \right) \sum_{m=1}^{\infty} m \nabla_k^2(u, m) \Phi_m(u) \sin mv ,$$

$$Q_u = -D \frac{\partial}{\partial u} \nabla^2 u_z = D \frac{d}{du} \frac{1}{u^2} \sum_{m=1}^{\infty} \nabla^6(u, m) \Phi_m(u) \sin mv ,$$

$$Q_v = -\frac{D}{u} \frac{\partial}{\partial v} \nabla^2 u_z = \pm \frac{D}{u^3} \sum_{m=1}^{\infty} m \nabla^6(u, m) \Phi_m(u) \sin mv ,$$

$$M_u = -D \left[\frac{\partial^2 u_z}{\partial u^2} + \frac{v}{u} \left(\frac{\partial u_z}{\partial u} + \frac{1}{u} \frac{\partial^2 u_z}{\partial v^2} \right) \right] =$$

$$= D \sum_{m=1}^{\infty} \left[\frac{d^2}{du^2} + \frac{v}{u} \left(\frac{d}{du} - \frac{m^2}{u} \right) \right] \nabla^4(u, m) \Phi_m(u) \sin mv ,$$

$$M_v = -D \left[\frac{1}{u} \left(\frac{\partial u_z}{\partial u} + \frac{1}{u} \frac{\partial^2 u_z}{\partial v^2} \right) + v \frac{\partial^2 u_z}{\partial u^2} \right] =$$

$$= D \sum_{m=1}^{\infty} \left[\frac{1}{u} \left(\frac{d}{du} - \frac{m^2}{u} \right) + v \frac{d^2}{du^2} \right] \nabla^4(u, m) \Phi_m(u) \sin mv ,$$

$$M_{uv} = -(1-v) \frac{D}{u} \frac{\partial}{\partial v} \left(\frac{\partial u_z}{\partial u} - \frac{u_z}{u} \right) =$$

$$= \mp (1-v) \frac{D}{u} \left(\frac{d}{du} - \frac{1}{u} \right) \sum_{m=1}^{\infty} m \nabla^4(u, m) \Phi_m(u) \sin mv ,$$

where $\nabla_k^2(u, m) \dots = m \left(u \frac{d}{du} - 1 \right) \dots$,

$\nabla^2(u, m) \dots = \left(u^2 \frac{d^2}{du^2} + u \frac{d}{du} - m^2 \right) \dots$.

The particular solution may be found the same way and the final equations will be

$$\overline{\Phi}_m(t) = A e^{4t} \text{ for } 0 < v < \pi ,$$

$$\overline{\Phi}_m(t) = -A(-1)^{\frac{m+1}{2}} e^{4t} \quad \text{for} \quad -\frac{\pi}{2} < \nu < \frac{\pi}{2},$$

where

$$A = \frac{-4q}{D\pi m[4^6 + m^8 - 40m^6 + 528m^4 - 2560m^2 - 9p^2m^2]}.$$

According to the methodology suggested, it is possible to analyze the stress-strain state of a shallow right helicoid. Several test examples are represented in comparison with results obtained by another methodology for oblique helicoid in the case when the angle between the plane and the generator is equal to zero.

3. RESULTS AND DISCUSSION

The numeric experiments were carried out in present investigation to verify and compare the methodologies given and also to define the border between shallow and non-shallow models.

The comparison of the suggested method for oblique helicoid and finite element method demonstrates close agreement of results, obtained by the numeric-analytic method and those obtained by finite element analysis. The series of calculations were conducted for shells with different φ angle (Table 1).

Table 1. The comparison of results for oblique helicoid

φ	0	3	5	10	15
Maximum deflection along z axis method 1*, m	$8,69 \cdot 10^{-5}$	$8,45 \cdot 10^{-5}$	$8,43 \cdot 10^{-5}$	$7,99 \cdot 10^{-5}$	$4,67 \cdot 10^{-5}$
The same, method 2*, m	$8,7 \cdot 10^{-5}$	$8,6 \cdot 10^{-5}$	$8,0 \cdot 10^{-5}$	$7,4 \cdot 10^{-5}$	$6,6 \cdot 10^{-5}$
Maximum bending moment M_u , method 1*, KN·m /m	3914/ -1663	3805/ -1614	3593/ -1524	2853/ -1191	2076/ -852
The same, method 2*, KN·m /m	3711/ -1667	3689/ -1656	3639/ -1636	3428/ -1532	3108/ -1374

* method 1, -numeric- analytical, method 2 – finite element method

The finite element calculations were carried on by Lira 9.4 software application. Due to this results the boundary between shallow and non-shallow model can be determined near 10° generator obliquity angle for shells with a small pitch. This angle approximately corresponds to shell ratio of rise to plane size of 1/5.

The second comparison was made for right helicoid shell, calculated by numeric-analytic method and purely analytic method. The method for oblique helicoid is also suitable for stress-strain analysis of shells with middle surface of special and degenerated cases of helicoid: if $\varphi = 0$ then oblique helicoid degenerates into right helicoid, if simultaneously $c=0$ – into flat plate; if $c = 0$, $\varphi \neq 0$ – into conus. So we can compare results for right helicoid, calculated as a special case of oblique one, and analytically. The results obtained by the numeric-analytic method also have close agreement to those obtained by analytical method for right helicoid. The next examples illustrate this.

Let us analyze the shell of right helicoid form. The material characteristics: Young modulus $E=200000$ MPa, Poisson's ratio $\nu=0.3$. The thickness is 0.02m, the inner radius $R1=5$ m, the outer radius $R2=6.708$ m, the pitch $H= 0.314$ m (or $c=0.05$), load intensity 10 KPa. The results are presented in Table 2.

Table 2. The comparison of results obtained by numeric-analytic and analytic methods

Quantity	Method 1	Method 2
Bending moment $M_u, \frac{\text{kN m}}{\text{m}}$ First support/midspan/second support	2,57/-1,21/2,289	2,57/-1,20/2,28
bending moment $M_v, \frac{\text{kN m}}{\text{m}}$ first support/midspan/second support	0,77/-0,36/0,68	0,77/-0,37/0,68
Shear force kN first support/ second support	-9,208/8,00	-9,208/8,03

4. CONCLUSION

In this paper two methodologies of analytical and half-analytical stress-strain state analysis for thin shallow helicoidal structures in the shape of right and oblique helicoids are represented. The analytical approach which is suitable for a right helicoid structure is compared to the half-analytical methodology which is developed by authors for an oblique helicoid structure when the inclination angle of generator is equal to zero (in this case the oblique helicoid turns into the right helicoid). The results which are obtained by two approaches give the appropriate accuracy if compared with each other, as well as if compared with the finite element method results.

The analytical approach for calculation of a right helicoidal structure is realized by the means of computer software based on analytical solutions obtained, while the half-analytical approach for calculation of an oblique helicoidal structure is performed by Xcas software. The finite element analysis was carried out by the means of Lira SAPR software. The obtained results are also similar to the results obtained by analytical and numeric-analytical methods.

Both methodologies are written in the compact ways and are convenient for a practical engineering application as a means for a preliminary calculation or a deep stress-strain analysis of right and oblique helicoidal structures.

ACKNOWLEDGMENTS

This paper was financially supported by the Ministry of Education and Science of the Russian Federation on the program to improve the competitiveness of Peoples' Friendship University of Russia (RUDN University) among the world's leading research and education centers in the 2016-2020.

REFERENCES

- Bradshaw, R., Campbell, D., Gargari, M., Mirmiran, A., and Tripeny, P., (2002). "Special structures. Past, present, and future", *J. of Structural Engineering*, **6**, 691-701.
- Dekhtyar, A.S., (2013) "Load carrying capacity of helicoidal shell", *Structural Mechanics and Analysis of Constructions*, **6**, 1-6 [in Russian].
- Gbaguidi, A.G., Krivoschapko, S.N. (2012), "Two methods of analysis of thin elastic open helicoidal shells", *Int. J. of research and reviews in applied sciences*, **3**, v.12, 382-390.
- Goldenveizer, A.L., (1961), "Theory of Elastic Thin Shells", Pergamon Press, New York (translated by G. Herrmann), (Original Russian book: Gostekhteorizdat, Moscow, 1953.).
- Krivoschapko S.N., Ivanov V.N. (2015), "Encyclopedia of Analytical Surfaces", Springer, 752 p.
- Krivoschapko, S.N. and Rynkovskaya, M. (2017), "Five types of ruled helical surfaces for helical conveyers, support anchors and screws", *MATEC Web Conf.*, **95** (2017) 06002.
- Love, A.E.H., (1927), "A treatise on the mathematical theory of elasticity" I and II, Cambridge.
- Marquere, K. (1938), "Zur Theorie der gekrummter Platte prosser Formanderung", Proc. of the Fifth Int. Congress for Appl. Mech., 93-101.
- Reissner, E. (1955), "Small rotationally symmetric deformations of shallow helicoidal shells", *J. Appl. Mech.*, **22**, p. 31-34.
- Rekach, V.G., Krivoschapko, S.N., (1988), "Raschet obolochek slojnoi geometrii", Moscow, 177p.
- Rynkovskaya, M.I. (2008) Rekach's method of calculation as applied to right helicoid, *Structural Mechanics of Engineering Structures and Constructions*, **3**, 23-29. [in Russian]
- Rynkovskaya, M.I. (2015), "To the problem of determining the stress-strain state of ruled thin screw shells", *Structural Mechanics of Engineering Structures and Constructions*, **6**, 13-15. [in Russian]
- Savićević, S., (2001), "A Development of Automatized Projection of Construction Elements of Helical Shell Shape", PhD dissertation, Faculty of Mechanical Engineering, Podgorica.
- Sigrid Adriaenssens, Philippe Block, Diederick Veennendaal, Chris Williams, (2014), "Shell Structures for Arckitecture –Form finding and Optimization", Routledge, 323 p.
- Tupikova, E.M. (2016), "Investigation of V.G. Reatch's method of stress – strain analysis of the shell of long shallow oblique helicoid form", *Structural Mechanics and Analysis of Constructions*, **1**, 14-19 [in Russian]
- Tupikova, E.M. (2015), "Analysis of the thin elastic shells in the form of long oblique helicoids", *Structural Mechanics of Engineering Structures and Constructions*, **3**, 23-27. [in Russian]

The 2017 World Congress on

Advances in Structural Engineering and Mechanics (ASEM17)

28 August - 1 September, 2017, Ilsan(Seoul), Korea

Zhao, Y., Su, D., Wei, W., Dong, X., (2010).” A meshing principle for generating a cylindrical gear using an Archimedes hob with two degrees of freedom”, Proc. of the Institution of Mech. Engineers. Part C: *J. of Mech. Eng. Science*, **224**, **1**, 169-181.