

























Table 2. Critical values  $\varepsilon_0$  and  $\frac{\rho}{\rho_0}$

$\nu$	0	0.1	0.2	0.3	0.4	0.5
$(\lambda_1)_*$	$\frac{0.78}{0.89}$	$\frac{0.82}{0.91}$	$\frac{0.84}{0.94}$	$\frac{0.89}{0.95}$	$\frac{0.94}{0.98}$	$\frac{1}{1}$
	$\frac{0.50}{0.71}$	$\frac{0.58}{0.76}$	$\frac{0.67}{0.82}$	$\frac{0.77}{0.87}$	$\frac{0.87}{0.94}$	$\frac{1}{1}$
$(\varepsilon_0)_*$	$\frac{-0.1958}{-0.1040}$	$\frac{-0.1638}{-0.0860}$	$\frac{-0.1472}{-0.0582}$	$\frac{-0.1040}{-0.0488}$	$\frac{-0.0582}{-0.0198}$	$\frac{0}{0}$
	$\frac{-0.375}{-0.2480}$	$\frac{-0.3318}{-0.2112}$	$\frac{-0.2756}{-0.1638}$	$\frac{-0.2036}{-0.1216}$	$\frac{-0.1216}{-0.0582}$	$\frac{0}{0}$
$\left(\frac{\rho}{\rho_0}\right)_*$	$\frac{2.1073}{1.4185}$	$\frac{1.8137}{1.3270}$	$\frac{1.6872}{1.2040}$	$\frac{1.4185}{1.1664}$	$\frac{1.2040}{1.0625}$	$\frac{1}{1}$
	$\frac{8}{2.7940}$	$\frac{5.1253}{2.2780}$	$\frac{3.3249}{1.8137}$	$\frac{2.1904}{1.5186}$	$\frac{1.5186}{1.2040}$	$\frac{1}{1}$

#### 4. NUMERICAL RESULTS AND DISCUSSIONS

In Table 2, along with the above mentioned discussion, numerical values of critical values of density corresponding to the "internal" instability and buckling of equilibrium state changes on geometric forming. They show that the buckling of equilibrium state on geometric forming leads to the "internal" instability, i.e. to the beginning of destruction process in the considered case for all values of Poisson's ratio. Similar processes of local buckling of the equilibrium state occur in the vicinity of available inclusions in the form of rods, bands, plates, etc. Therefore, the use of inequations (19)-(20) is more reasonable and correct in practice, especially in researches of the Earth's crust and the upper mantle. When the structural inhomogeneity is included to the medium by the cylindrical rod or band from more rigid media, it is known (Guz 1999; Kuliev 1987) that straight form of rod becomes instable and gets more stable curved shape in the uniform compression in quite small strains by conservative forces. In this case, the critical strength of buckling is two times less than the Eulerian's force corresponding to uniaxial compression along the rod line. This force is many times less than the pressure of partial melting and phase transitions for different substances of the Earth. Depending on geometric dimensions of the inclusion, their curvatures may lead to significant local changes (dissimilarity from uniform strain) of strains and stresses in a large scale.

Such local strain processes will influence on further change of density and other tectonophysical parameters in large geometric scales and for a geological time. In particular, processes of partial melting and phase transitions will not occur at uniform depth levels of the Earth's interior as in uniform strain. In real conditions, these processes will be implemented at different levels of the Earth's interior depending on the nature of strain distribution.

It's seen from Eqs. (10), (12)-(18) that, it's necessary to know only values of Poisson's ratio to determine critical values of elongation (shortening) and strains. This fact is of great practical importance. There are different geophysical methods allowing defining these important physico-mechanical parameters of the medium within conditions of unreachable deep interior of the Earth. It is possible to conduct assessment of various theoretical and observation results and to determine their degree of reliability using these data and inequations suggested in this paper.

Hofmeister (1993) analyzed problems of applicability of theoretical results to determine certainty of fields of Birch-Murnaghan's state equation (B-M EoS) achieved within the Eulerian's finite strain problem. Strains are modeled using equivalent repulsive interatomic potentials. As the criteria of applicability (certainty) of results, the condition of conserving the interval of change of isothermal modulus of compression is applied within the framework of which the potential structure remains stable. Based on (Webb and Jackson 1990) experimental results, it is concluded that, B-M EoS theoretical model is unreliable for some solids as orthopyroxene. Let's consider this problem from the point of view of the above received inequalities.

It is defined in the known Lin-Gun Liu's (Ringwood 1981, Zharkov 2012) experiments that as a result of uniform strain of sample from orthopyroxene (90%  $MgSiO_3 \cdot Al_2O_3$ ), phase transition from enstatite into garnet occurs in a relative decrease of the volume by 7,8%. The decrease in the volume by 8,0% causes a new phase transition from garnet into ilmenite. The further decrease in the volume by 6,9% leads to phase transition from ilmenite into perovskite. Using Eq. (4) and these experimental data, numerical calculations are conducted and their results are reflected in Fig. 1.

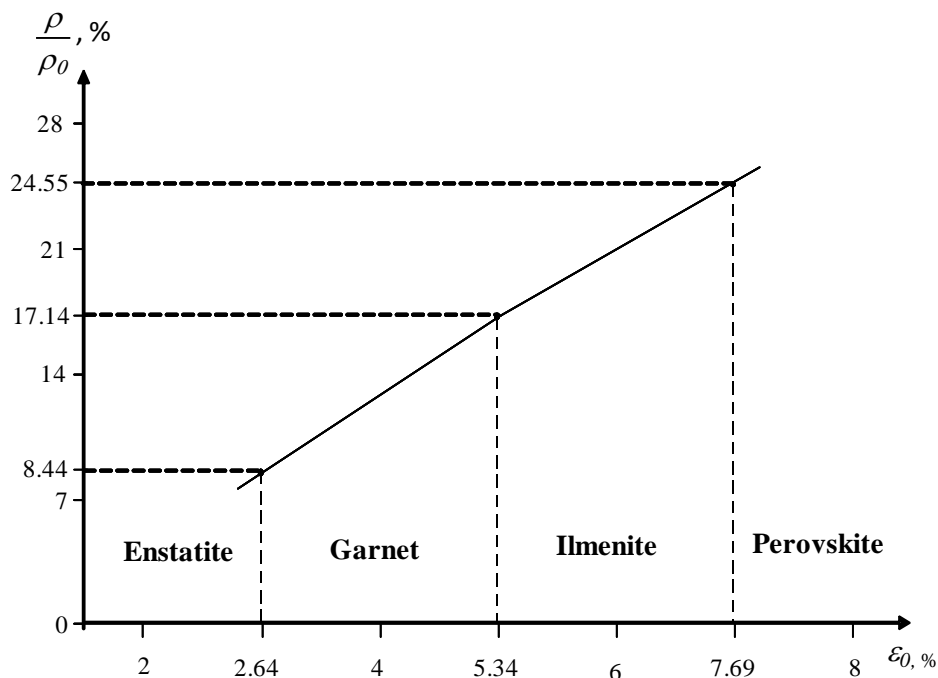


Fig. 1  $\frac{\rho}{\rho_0}$  dependence on  $\epsilon_0$  and sequence of phase transitions for orthopyroxene

Results show that enstatite undergone uniform strain of compression at its 2,64%, 5,34% and 7,69% values is sequentially changed into garnet, ilmenite and into perovskite at last. An increase in the density parameter of substance by 8,45%, 8,7% and 7,4% corresponds to these values of strains. Comparison of these results with data in Table 2 for  $\varepsilon_0^*$ , show that sequence of phase transitions of orthopyroxene causes the destruction in all values of Poisson's ratio as a result of continuous strain (in its modeling by harmonic potential). A similar conclusion is obtained for a quadratic potential except of the interval of changing the values of Poisson's ratio  $\nu > 0.38$ . Comparison of results on parameter  $(\varepsilon_0)_*$  show that the local buckling of the elastic equilibrium state can cause separate phase transitions in the vicinity of inclusions in the form of the cylindrical cavity for a range of changes of Poisson's ratio  $\nu > 0.38$  (harmonic potential) and  $\nu \geq 0.12$  (quadratic potential). In such case, the obtained results cannot be considered reliable due to violation the condition of uniformity of strain process. It is known (Prodaivoda et al. 2012), that the value of Poisson's ratio averaged in Voigt-Reuss-Hill's approximation changes within  $0,19 \leq \nu \leq 0,21$  in the interval of temperature  $25^0 C \leq T \leq 700^0 C$  for orthopyroxene. Thus, results obtained within uniform strains should be adjusted in case of presence the inhomogeneity as inclusion the form of cylindrical cavity in orthopyroxene medium. The given conclusion relates to the case when strain process is modeled by quadratic elastic potential. As a result of local instability, uniform character of distribution of compression strain is broken in the medium. Therefore, nature of density distribution and other tectonophysical parameters will differ from analogical nature corresponding to the case of uniform strain in case of uniform compression. Furthermore, it follows from results in Fig. 1 that the parameters of physico-mechanical properties of orthopyroxene are undergone significant changes in the strained state due to the occurred phase transitions. Numerical values of parameters of physico-mechanical properties of these rocks differ among themselves significantly. This example shows the difficulties in assessing the confidence intervals of state equations on criteria of fundamental moduli of the elasticity quite clearly. Apparently, conclusions on the unreliability of model B-M EoS are related with the mentioned circumstances here to describe experimental data of orthopyroxene.

The results in Table 1 show that one and the same density changes can occur in a variety of combinations between the main components of Green's strain tensor differing from uniform compression.

Thus, even in uniform strains, failure of compression uniformity, implementation on various mechanisms of instability of elastic equilibrium state (instability of equilibrium state can also be implemented at stages of elastic-plastic, elastic-viscous and other stages of strains) and phase transitions may have a significant impact on distribution of parameter of the medium density, depth of implementation of phase transitions and other parameters of theoretical models of the Earth. Hence, determination of material and other parameters of theoretical models can lead to inaccurate conclusions only on the basis of results of experimental researches on separate physico-mechanical characteristics and phase transitions in mineral associations under uniform strain.

The conditions imposed on strains are "strong". Parameters, especially elastic, theoretical models of the Earth should be defined within the suggested variation

intervals of strain. In other approaches based on conditions of saving restrictions imposed on separate parameters of state equations (depending on how successive are equations), correspondences of their results with the indicated interval of strain aren't tested and true mechanisms of strain are left out in the considered problems. Consequently, "weak" restrictions on separate parameters including to various correlation of theoretical models of the Earth's development are formulated within such approaches. It means that the required condition could be implemented on separate parameters but they will not provide unambiguity of interpretation and uncertainties in results.

## 5. CONCLUSIONS

Valid differential criteria of reliability for determining various physico-mechanical parameters of theoretical models of the Earth's development are achieved on the basis of the NLA of nonlinear theory of small and finite strains in the form of uniform strain intervals. Thus, in theoretical models, along with integral criteria, problems at all depths of the Earth should be solved considering these differential criteria on strains. Apparently, these criteria are the most universal, simple and comfortable to apply.

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