

Table 2. Critical values ε_0 and $\frac{\rho}{\rho_0}$

ν	0	0.1	0.2	0.3	0.4	0.5
$(\lambda_1)_*$	$\frac{0.78}{0.89}$	$\frac{0.82}{0.91}$	$\frac{0.84}{0.94}$	$\frac{0.89}{0.95}$	$\frac{0.94}{0.98}$	$\frac{1}{1}$
λ_1^*	$\frac{0.50}{0.71}$	$\frac{0.58}{0.76}$	$\frac{0.67}{0.82}$	$\frac{0.77}{0.87}$	$\frac{0.87}{0.94}$	$\frac{1}{1}$
$(\varepsilon_0)_*$	$\frac{-0.1958}{-0.1040}$	$\frac{-0.1638}{-0.0860}$	$\frac{-0.1472}{-0.0582}$	$\frac{-0.1040}{-0.0488}$	$\frac{-0.0582}{-0.0198}$	$\frac{0}{0}$
ε_0^*	$\frac{-0.375}{-0.2480}$	$\frac{-0.3318}{-0.2112}$	$\frac{-0.2756}{-0.1638}$	$\frac{-0.2036}{-0.1216}$	$\frac{-0.1216}{-0.0582}$	$\frac{0}{0}$
$\left(\frac{\rho}{\rho_0}\right)_*$	$\frac{2.1073}{1.4185}$	$\frac{1.8137}{1.3270}$	$\frac{1.6872}{1.2040}$	$\frac{1.4185}{1.1664}$	$\frac{1.2040}{1.0625}$	$\frac{1}{1}$
$\left(\frac{\rho}{\rho_0}\right)^*$	$\frac{8}{2.7940}$	$\frac{5.1253}{2.2780}$	$\frac{3.3249}{1.8137}$	$\frac{2.1904}{1.5186}$	$\frac{1.5186}{1.2040}$	$\frac{1}{1}$

4. NUMERICAL RESULTS AND DISCUSSIONS

In Table 2, along with the above mentioned discussion, numerical values of critical values of density corresponding to the "internal" instability and buckling of equilibrium state changes on geometric forming. They show that the buckling of equilibrium state on geometric forming leads to the "internal" instability, i.e. to the beginning of destruction process in the considered case for all values of Poisson's ratio. Similar processes of local buckling of the equilibrium state occur in the vicinity of available inclusions in the form of rods, bands, plates, etc. Therefore, the use of inequations (19)-(20) is more reasonable and correct in practice, especially in researches of the Earth's crust and the upper mantle. When the structural inhomogeneity is included to the medium by the cylindrical rod or band from more rigid media, it is known (Guz 1999; Kuliev 1987) that straight form of rod becomes instable and gets more stable curved shape in the uniform compression in quite small strains by conservative forces. In this case, the critical strength of buckling is two times less than the Eulerian's force corresponding to uniaxial compression along the rod line. This force is many times less than the pressure of partial melting and phase transitions for different substances of the Earth. Depending on geometric dimensions of the inclusion, their curvatures may lead to significant local changes (dissimilarity from uniform strain) of strains and stresses in a large scale.

Such local strain processes will influence on further change of density and other tectonophysical parameters in large geometric scales and for a geological time. In particular, processes of partial melting and phase transitions will not occur at uniform depth levels of the Earth's interior as in uniform strain. In real conditions, these processes will be implemented at different levels of the Earth's interior depending on the nature of strain distribution.

It's seen from Eqs. (10), (12)-(18) that, it's necessary to know only values of Poisson's ratio to determine critical values of elongation (shortening) and strains. This fact is of great practical importance. There are different geophysical methods allowing defining these important physico-mechanical parameters of the medium within conditions of unreachable deep interior of the Earth. It is possible to conduct assessment of various theoretical and observation results and to determine their degree of reliability using these data and inequations suggested in this paper.

Hofmeister (1993) analyzed problems of applicability of theoretical results to determine certainty of fields of Birch-Murnaghan's state equation (B-M EoS) achieved within the Eulerian's finite strain problem. Strains are modeled using equivalent repulsive interatomic potentials. As the criteria of applicability (certainty) of results, the condition of conserving the interval of change of isothermal modulus of compression is applied within the framework of which the potential structure remains stable. Based on (Webb and Jackson 1990) experimental results, it is concluded that, B-M EoS theoretical model is unreliable for some solids as orthopyroxene. Let's consider this problem from the point of view of the above received inequalities.

It is defined in the known Lin-Gun Liu's (Ringwood 1981, Zharkov 2012) experiments that as a result of uniform strain of sample from orthopyroxene (90% $MgSiO_3 \cdot Al_2O_3$), phase transition from enstatite into garnet occurs in a relative decrease of the volume by 7,8%. The decrease in the volume by 8,0% causes a new phase transition from garnet into ilmenite. The further decrease in the volume by 6,9% leads to phase transition from ilmenite into perovskite. Using Eq. (4) and these experimental data, numerical calculations are conducted and their results are reflected in Fig. 1.

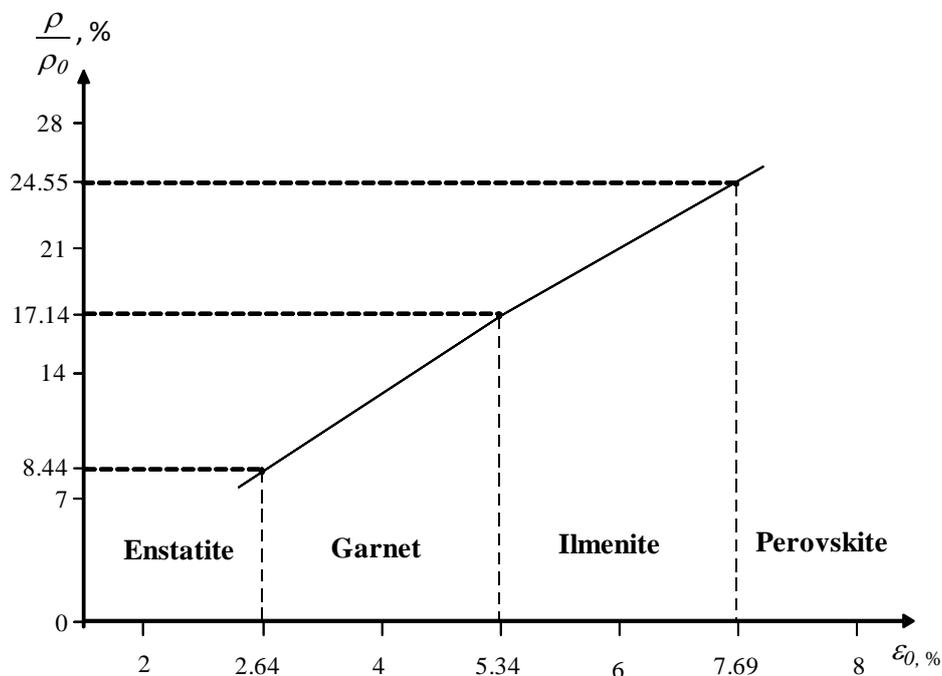


Fig. 1 $\frac{\rho}{\rho_0}$ dependence on ϵ_0 and sequence of phase transitions for orthopyroxene

Results show that enstatite undergone uniform strain of compression at its 2,64%, 5,34% and 7,69% values is sequentially changed into garnet, ilmenite and into perovskite at last. An increase in the density parameter of substance by 8,45%, 8,7% and 7,4% corresponds to these values of strains. Comparison of these results with data in Table 2 for ε_0^* , show that sequence of phase transitions of orthopyroxene causes the destruction in all values of Poisson's ratio as a result of continuous strain (in its modeling by harmonic potential). A similar conclusion is obtained for a quadratic potential except of the interval of changing the values of Poisson's ratio $\nu > 0.38$. Comparison of results on parameter $(\varepsilon_0)_*$ show that the local buckling of the elastic equilibrium state can cause separate phase transitions in the vicinity of inclusions in the form of the cylindrical cavity for a range of changes of Poisson's ratio $\nu > 0.38$ (harmonic potential) and $\nu \geq 0.12$ (quadratic potential). In such case, the obtained results cannot be considered reliable due to violation the condition of uniformity of strain process. It is known (Prodaivoda et al. 2012), that the value of Poisson's ratio averaged in Voigt-Reuss-Hill's approximation changes within $0,19 \leq \nu \leq 0,21$ in the interval of temperature $25^0 C \leq T \leq 700^0 C$ for orthopyroxene. Thus, results obtained within uniform strains should be adjusted in case of presence the inhomogeneity as inclusion the form of cylindrical cavity in orthopyroxene medium. The given conclusion relates to the case when strain process is modeled by quadratic elastic potential. As a result of local instability, uniform character of distribution of compression strain is broken in the medium. Therefore, nature of density distribution and other tectonophysical parameters will differ from analogical nature corresponding to the case of uniform strain in case of uniform compression. Furthermore, it follows from results in Fig. 1 that the parameters of physico-mechanical properties of orthopyroxene are undergone significant changes in the strained state due to the occurred phase transitions. Numerical values of parameters of physico-mechanical properties of these rocks differ among themselves significantly. This example shows the difficulties in assessing the confidence intervals of state equations on criteria of fundamental moduli of the elasticity quite clearly. Apparently, conclusions on the unreliability of model B-M EoS are related with the mentioned circumstances here to describe experimental data of orthopyroxene.

The results in Table 1 show that one and the same density changes can occur in a variety of combinations between the main components of Green's strain tensor differing from uniform compression.

Thus, even in uniform strains, failure of compression uniformity, implementation on various mechanisms of instability of elastic equilibrium state (instability of equilibrium state can also be implemented at stages of elastic-plastic, elastic-viscous and other stages of strains) and phase transitions may have a significant impact on distribution of parameter of the medium density, depth of implementation of phase transitions and other parameters of theoretical models of the Earth. Hence, determination of material and other parameters of theoretical models can lead to inaccurate conclusions only on the basis of results of experimental researches on separate physico-mechanical characteristics and phase transitions in mineral associations under uniform strain.

The conditions imposed on strains are "strong". Parameters, especially elastic, theoretical models of the Earth should be defined within the suggested variation

intervals of strain. In other approaches based on conditions of saving restrictions imposed on separate parameters of state equations (depending on how successive are equations), correspondences of their results with the indicated interval of strain aren't tested and true mechanisms of strain are left out in the considered problems. Consequently, "weak" restrictions on separate parameters including to various correlation of theoretical models of the Earth's development are formulated within such approaches. It means that the required condition could be implemented on separate parameters but they will not provide unambiguity of interpretation and uncertainties in results.

5. CONCLUSIONS

Valid differential criteria of reliability for determining various physico-mechanical parameters of theoretical models of the Earth's development are achieved on the basis of the NLA of nonlinear theory of small and finite strains in the form of uniform strain intervals. Thus, in theoretical models, along with integral criteria, problems at all depths of the Earth should be solved considering these differential criteria on strains. Apparently, these criteria are the most universal, simple and comfortable to apply.

ACKNOWLEDGEMENT

This work has been carried out by the full financial support of the NAS of Azerbaijan within the program "Complex of researches of theoretical and experimental interdisciplinary problems of geomechanics", under the resolution of the Presidium of the NAS of Azerbaijan #5/3 dated on February 11, 2015.

REFERENCES

- Adushkin, V.V. and Vityazev, A.V. (2007), "The origin and evolution of the Earth: a modern view", *The Bulletin of the Russian Academy of Sciences*, **77**(5), 396-402.
- Altshuler, L.V., Krupnikov, K.K., Fortov, V.E. and Funtikov, A.I. (2004), "The beginning of physics of megabar pressures", *The Bulletin of the Russian Academy of Sciences*, **74**(11), 1011-1022.
- Anderson, D. (2007), *New Theory of the Earth*, Cambridge University Press, New York, USA.
- Anderson, O.L. (1995), *Equations of state of solids for geophysics and ceramic science*, Oxford University Press. New York, Oxford.
- Akbarov, S.D., Guliyev, H.H. and Yahnioglu, N. (2016), "Natural vibration of the three-layered solid sphere with middle layer made of FGM: three-dimensional approach", *Structural Engineering and Mechanics*, **57**(2), 239-263.
- Akbarov, S.D., Guliyev, H.H. and Yahnioglu, N. (2017), "Three-dimensional analysis of the natural vibration of the three-layered hollow sphere with middle layer made of FGM", *Structural Engineering and Mechanics*, **61**(5), 563-576.

- Akbarov, S.D. and Guz, A.N. (2000), *Mechanics of curved composites*, Kluwer Academic Publishers, Dordrecht.
- Akbarov, S.D. (2013), *Stability Loss and Buckling Delamination: Three-Dimensional Linearized Approach for Elastic and Viscoelastic Composites*, Springer, Berlin, Germany.
- Biot, M.A. (1965), *Mechanics of Incremental Deformation*, Willey, New York, USA.
- Birch, F. (1952), "Elasticity and constitution of the Earth's interior", *J. Geophys. Res.*, (57), 227-286.
- Bullen, K.E. (1963), *An Introduction to the theory of seismology*, Third edition. Cambridge University Press, Cambridge.
- Bullen, K.E. (1975), *The earth's density*, Chapman and Hall, London.
- Dziewonski, A.M. and Anderson, D.L. (1981). "Preliminary reference Earth model", *Phys. Earth Planet. Inter.*, **25**(4), 297-356.
- Gufeld, I.L. (2013), "The degassing at depths and structure of lithosphere and upper mantle", *Electronic journal "Glubinnaya neft"*, **1**(2), 172-189.
- Guz, A.N. (1977), *Basis of the theory of stability of mine workings*, Naukova Dumka, Kiev.
- Guz, A.N. (1989), *Fracture mechanics of composite materials under compression*, Naukova Dumka, Kiev.
- Guz, A.N. (1999), *Fundamentals of the three-dimensional theory of stability of deformable bodies*, Springer, Berlin.
- Guliyev, H.H. (2010), "A new theoretical conception concerning the tectonic processes of the Earth", *New Concepts in Global Tectonics Newsletter*, (56), 50-74.
- Guliyev, H.H. (2011), "Fundamental role of deformations in internal dynamics of the Earth", *New Concepts in Global Tectonics Newsletter*, (61), 33-50.
- Guliyev, H.H. (2013), "Deformations, corresponding to processes of consolidation, deconsolidation and phase transitions in internal structures of the Earth", *Geophysical Journal*, **35**(3), 166-176.
- Hofmeister, A.M. (1993), "Interatomic potentials calculated from equations of state: Limitations of finite strain to moderate K", *Geophys. Res. Lett.*, **20**(7), 635-638.
<https://ds.iris.edu/spud/earthmodel>
- Kalinin, V.A. (2000), *Properties of geomaterials and physics of the Earth*, The Selected Works, IPE RAS, Moscow.
- Kuliev, H.H. (1987), "The stability of the bars under nonuniform compression by the dead and tracer loads", *Proceedings of the Academy of Sciences of Azerbaijan SSR, Ser. of phys. techn. and mat. sciences*, (5), 43-48.
- Kuliev, H.H. (1988), *Basis of mathematical theory of stability of the wells*, Elm, Baku.
- Knopoff, L. (1963), "Solids: Equations of state of solids at moderately high pressures", (1st Volume). In: *High Pressure Physics and Chemistry* (Edited by R. S. Bradley), Academic Press, New York.
- Liu, J. and Lin, J.-F. (2014), "Abnormal acoustic wave velocities in basaltic and (Fe,Al)-bearing silicate glasses at high pressures", *Geophys. Res. Lett.*, (41), 8832 -8839.
- Li, X. and Tao, M. (2015), "The influence of initial stress on wave propagation and dynamic elastic coefficients", *Geomechanics and Engineering An International Journal*, **8**(3), 377-390.

- Mao, Z., Lin, J.-F., Jacobsen, S.D., Duffy, T.S., Chang, Y.-Y., Smyth, J.R., Frost, D.J., Hauri, E.H. and Prakapenka, V.B. (2012), "Sound velocities of hydrous ringwoodite to 16 GPa and 673 K", *Earth and Planetary Science Letters*, **331-332**, 112-119.
- Molodenskii, S.M. and Molodenskaya, M.S. (2009), "On mechanical Q parameters of the lower mantle inferred from data on the Earth's free oscillations and nutation", *Proceedings, Physics of the Solid Earth*, **45**(9), 744-752.
- Molodenskii, S.M. and Molodenskaya, M.S. (2015), "Attenuation of free spheroidal oscillations of the Earth after the M = 9 earthquake in Sumatra and super-deep earthquake in the Sea of Okhotsk: I. The admissible Q-factor range for the fundamental mode and overtones of the free spheroidal oscillations", *Izvestiya, Physics of the Solid Earth*, **51**(6), 821-839.
- Molodenskii, S.M. and Molodenskii, M.S. (2015), "Attenuation of free spheroidal oscillations of the Earth after the M = 9 earthquake in Sumatra and super-deep earthquake in the Sea of Okhotsk: II. Interpretation of the observed Q-factor", *Izvestiya, Physics of the Solid Earth*, **51**(6), 840-858.
- Navrotsky, A. (1994), *Physics and chemistry of Earth materials*, Cambridge University press, Cambridge.
- Prodaivoda, G.T., Vyzhva, S.A. and Vershilo, I.V. (2012). *Mathematical modeling of effective geophysical parameters*, Publishing-polygraph center "Kiev University", Kiev.
- Ringwood, A.E. (1981), *The structure and the petrology of the Earth's mantle*, Nedra, Moscow.
- Ritsema, R. (2005), "Global seismic structure maps", In: Plates, plumes, and paradigms, (Edited by Foulger, G.R., Natland, J.H., Presnall, D.C. and Anderson D.L.), *Geological Society of America, Special paper*, **388**, 11-18.
- Tateno, S., Hirose, K., Ohishi, Y. and Tatsumi, Y. (2010), "The Structure of Iron in Earth's Inner Core", *Science*, **330**, 359-361.
- Trampert, J., Deschamps, F., Resovsky, J. and Yuen, D. (2004), "Probabilistic tomography maps chemical heterogeneities throughout the lower mantle", *Science*, **306** (5697), 853-856.
- Truestell, C. (1972), *A first course in rational continuum mechanics*, The Johns Hopkins University, Baltimore.
- Van Der Hilst, R.D. and Karason, H. (1999), "Compositional Heterogeneity in the Bottom 1000 kilometers of the Earth's Mantle: Toward a Hybrid Convection Model", *Science*, (283), 1885-1888.
- Webb, S.L. and Jackson, L. (1990), "Polyhedral rationalization of variation among the single crystal elastic moduli for upper-mantle silicates, garnet, olivine and orthopyroxene", *Am. Miner*, (75), 731-738.
- www.sciencedirect.com/science/referenceworks/9780444538031
- Zharkov, V.N. (2012), *Physics of the Earth's interior*, Nauka i obrazovanie, Moscow.