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A modal estimation based multitype sensor placement method

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ABSTRACT

A two-stage sensor placement method, in which strain gauges are placed together with triaxial accelerometers to obtain more accurate displacement mode information of a structure, is proposed. In stage one, the selection of accelerometer locations, the redundancy coefficient is used together with the modal assurance criterion to decrease the redundancy information of the obtained displacement modes between the selected accelerometer locations. In stage two, the displacement modes of some estimated locations are estimated based on the strain modes obtained from the strain gauges. Determination of different candidate strain gauge locations affects the quality of the displacement mode estimation a lot. At last, the candidate strain gauge locations corresponding to the best estimation are selected. The effectiveness of the proposed multitype sensor placement method is demonstrated by a numerical investigation using a benchmark model.

Keywords: optimal sensor placement; triaxial accelerometer, strain gauge; candidate strain gauge locations; modal estimation

1. INTRODUCTION

In SHM systems, strain gauges and accelerometers are both widely used in practice. Generally, strain gauges are used to measure the local deformations, and accelerometers are used to obtain the global modal characteristics of the structure. It is meaningful to find an OSP method that comprehensively utilizes the two types of sensors, in which strain gauges and accelerometers are placed together instead of placed separately. (Zhang 2011) proposed an integrated optimal placement method of displacement transducers and strain gauges for response reconstruction, which uses the relationship between strains and displacements to perform response estimation.

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Furthermore, a multitype sensor placement method that uses the Kalman filter to obtain the best estimation of the responses at key locations, in which strain gauges, displacement sensors and acceleration sensors are placed together, was proposed by (Zhang 2016). The relationship between the strain mode and the displacement mode, which offers guidance regarding the integrated utilization of the two types of modal information obtained from both types of sensors, was proposed by (Yam 1996). However, few existing studies regarding strain and acceleration sensor placement methods focused on modal estimation, even though modal information is extremely important with respect to analyzing the dynamic characteristics of a structure. This paper presents a multitype sensor placement method in which strain gauges can be placed together with accelerometers to obtain the structural displacement mode information. The amount of displacement mode information contained in different candidate strain gauge locations can be calculated by using the relationship between the displacement mode and the strain mode. The estimated locations are determined via comprehensive consideration of the selected performance criteria and the practical situation. In this method, the strain modes of measured locations are utilized to estimate the displacement modes of the estimated locations without accelerometers, and the strain gauge locations are selected according to the quality of the estimation.

2. THEORY FORMULATION OF THE MODAL ESTIMATION METHOD

2.1 selection of accelerometer locations

The modal assurance criterion MAC method (Carne 1995) is used here to select the initial accelerometers, and triaxial accelerometers are taken into account here. The cosine of the angle between two mode shape vectors is used to evaluate the relationship between the two mode shape vectors. The MAC matrix can be expressed as

$$\mathbf{MAC}_{i,j} = \frac{\left(\mathbf{\Phi}_{*,i}^{T}\mathbf{\Phi}_{*,j}\right)^{2}}{\left(\mathbf{\Phi}_{*,i}^{T}\mathbf{\Phi}_{*,i}\right)\left(\mathbf{\Phi}_{*,j}^{T}\mathbf{\Phi}_{*,j}\right)} \tag{1}$$

where $\mathbf{\Phi}_{*,i}$ and $\mathbf{\Phi}_{*,j}$ are the ith and jth columns of the target mode shape matrix, respectively. A cosine value near zero shows that the two corresponding mode shape vectors can be easily distinguished from each other. After first several locations have been selected, every time, only one accelerometer location that gets the smallest value of the maximum off-diagonal MAC term is added to the existing placement.

Considering the continuity of the mode shapes, when two sensors are placed too close to each other, they usually contain similar modal information (Stephan 2012). Here, the Frobenius norm is used to evaluate the similarity of mode shapes of different locations, and the redundancy coefficient is defined as

$$\mathbf{R}_{i,j} = 1 - \frac{\left\| \boldsymbol{\Phi}_{3i} - \boldsymbol{\Phi}_{3j} \right\|_F}{\left\| \boldsymbol{\Phi}_{3i} \right\|_F + \left\| \boldsymbol{\Phi}_{3j} \right\|_F} \tag{2}$$

where $\mathbf{R}_{i,j}$ is the redundancy coefficient between the mode shapes of the ith and the jth locations. When the value of $\mathbf{R}_{i,j}$ is close to one, the two corresponding locations share almost the same modal information, which is not acceptable in the sensor placement. A near one redundancy coefficient value needs to be avoided in every sensor selection step.

2.1 Selection of strain gauge locations for modal estimation

Sometimes, after the initial accelerometer locations have been determined according to the performance criteria, the number of accelerometers can be decreased due to various practical factors. The displacement modes of these deleted locations can be estimated based on the strain modes obtained from the strain gauges. Then, the linear dynamical equation of the FE model can be written as

$$\mathbf{M}\ddot{\boldsymbol{\delta}} + \mathbf{C}\dot{\boldsymbol{\delta}} + \mathbf{K}\boldsymbol{\delta} = \mathbf{f} \tag{3}$$

where $\mathbf{M}$, $\mathbf{C}$ and $\mathbf{K}$ are the mass, damping and stiffness matrixes of the system, respectively; $\mathbf{f}$ is the input vector; $\boldsymbol{\delta}$ is the nodal displacement vector of all nodes in the global coordinate system, and a dot over the vector represents the derivative with respect to time. The relationship between strain mode and displacement mode can be obtained

$$\boldsymbol{\varepsilon} = \tilde{\mathbf{B}}\boldsymbol{\delta} = \tilde{\mathbf{B}}\boldsymbol{\Phi}\mathbf{q} = \boldsymbol{\Psi}\mathbf{q} \tag{4}$$

$$\boldsymbol{\Psi} = \tilde{\mathbf{B}}\boldsymbol{\Phi} \tag{5}$$

where $\boldsymbol{\varepsilon}$ is the strain vector of strains of candidate locations in the whole FE model; $\boldsymbol{\Psi}$ denotes the strain mode matrix of the candidate strain locations; $\tilde{\mathbf{B}}$ is the transformation matrix between the strains and the nodal displacement of all nodes, which is directly affected by the candidate strain locations; $\boldsymbol{\Phi}$ denotes the displacement mode matrix of all nodes; $\mathbf{q}$ denotes the modal coordinates.

Practically, the strain modes obtained from strain measurements may differ from the computed result from the FE model. The strain modes obtained from the measurement data can be expressed as

$$\boldsymbol{\Psi} = \tilde{\mathbf{B}}\boldsymbol{\Phi} + \mathbf{v} \tag{6}$$

where $\mathbf{v}$ is a noise matrix in which the ith column $\mathbf{v}_{(i)}$ can be seen as a stationary Gaussian noise with zero mean and a covariance $Cov(\mathbf{v}_{(i)}) = \sigma_i \mathbf{I}$. When performing the displacement mode estimation, in the displacement mode shape matrix $\mathbf{\Phi}$, only the rows corresponding to the estimated locations need to be estimated. The right side of Eq. (5) can be transformed into

$$\tilde{\mathbf{B}}\mathbf{\Phi} = \tilde{\mathbf{B}}^{r}\mathbf{\Phi}^{r} + \tilde{\mathbf{B}}^{N_d-r}\mathbf{\Phi}^{N_d-r} \tag{7}$$

where N_d is the number of rows of the displacement mode matrix $\mathbf{\Phi}$; $\mathbf{\Phi}^{r}$ denotes the r rows of $\mathbf{\Phi}$ corresponding to the estimated locations; $\mathbf{\Phi}^{N_d-r}$ denotes the remaining $N_d - r$ rows of $\mathbf{\Phi}$; $\tilde{\mathbf{B}}^{r}$ denotes the r columns of $\tilde{\mathbf{B}}$ corresponding to the estimated locations; $\tilde{\mathbf{B}}^{N_d-r}$ denotes the remaining $N_d - r$ columns of $\tilde{\mathbf{B}}$.

The selection of strain gauges involves selecting some rows in the strain mode shape matrix of all the candidate strain gauge locations; Eq. (6) can be re-written as

$$\mathbf{P}\left(\mathbf{\Psi} - \tilde{\mathbf{B}}^{N_d-r}\mathbf{\Phi}^{N_d-r}\right) = \mathbf{P}\tilde{\mathbf{B}}^{r}\mathbf{\Phi}^{r} + \mathbf{P}\mathbf{v} \tag{8}$$

where $\mathbf{P}$ is a selection matrix that consists of only zeros and ones and the number of rows of $\mathbf{P}$ is equal to the number of selected strain gauges. In Eq. (8), when the number of selected strain gauges is greater than or equal to the number of rows of the displacement mode matrix $\mathbf{\Phi}^{r}$, the least squares estimation of the displacement modes of estimated locations can be obtained

$$\hat{\mathbf{\Phi}}^{r} = \left(\tilde{\mathbf{B}}^{rT}\mathbf{P}^{T}\mathbf{P}\tilde{\mathbf{B}}^{r}\right)^{-1}\tilde{\mathbf{B}}^{rT}\mathbf{P}^{T}\mathbf{P}\left(\mathbf{\Psi} - \tilde{\mathbf{B}}^{N_d-r}\mathbf{\Phi}^{N_d-r}\right) \tag{9}$$

where $\hat{\mathbf{\Phi}}^{r}$ is the estimation of $\mathbf{\Phi}^{r}$; The estimation vector $\hat{\mathbf{\Phi}}^{r}_{(i)}$ and the covariance matrix of the estimation vector $\hat{\mathbf{\Phi}}^{r}_{(i)}$ are expressed as

$$\hat{\mathbf{\Phi}}^{r}_{(i)} = \left(\tilde{\mathbf{B}}^{rT}\mathbf{P}^{T}\mathbf{P}\tilde{\mathbf{B}}^{r}\right)^{-1}\tilde{\mathbf{B}}^{rT}\mathbf{P}^{T}\mathbf{P}\left(\mathbf{\Psi}_{(i)} - \tilde{\mathbf{B}}^{N_d-r}\mathbf{\Phi}^{N_d-r}_{(i)}\right) \tag{10}$$

$$Cov(\hat{\mathbf{\Phi}}^{r}_{(i)}) = \sigma_i^2(\tilde{\mathbf{B}}^{rT}\mathbf{P}^{T}\mathbf{P}\tilde{\mathbf{B}}^{r})^{-1} \tag{11}$$

Where the subscript (i) denotes ith column of the matrix. Considering the situation in which the diagonal terms of $Cov(\hat{\mathbf{\Phi}}^{r}_{(i)})$ represent the error of the estimation in each location of the ith modes, the trace of the covariance matrix $Cov(\hat{\mathbf{\Phi}}^{r}_{(i)})$ is used to evaluate the estimation uncertainty

$$error\left(\hat{\mathbf{\Phi}}^{r}_{(i)}\right) = \sigma_i \,\mathrm{tr}\left(\sqrt{\left(\tilde{\mathbf{B}}^{rT}\mathbf{P}^{T}\mathbf{P}\tilde{\mathbf{B}}^{r}\right)^{-1}}\right) \tag{12}$$

The uncertainty of the estimation matrix $\hat{\mathbf{\Phi}}^{r}$ can be expressed as

$$error\left(\hat{\mathbf{\Phi}}^r\right) \propto \mathrm{tr}\left(\sqrt{\left(\tilde{\mathbf{B}}^{rT}\mathbf{P}^T\mathbf{P}\tilde{\mathbf{B}}^r\right)^{-1}}\right) \tag{13}$$

Considering the existence of the prediction error, the rows of $\tilde{\mathbf{B}}^r$ containing nothing but zeros can be deleted before calculating to reduce the dimension of $\tilde{\mathbf{B}}^r$, which can improve the quality of the displacement mode estimation. Finally, the best selection of the strain gauge locations is obtained from the selection matrix corresponding to the smallest value of estimation uncertainty.

3. NUMERICAL STUDIES

To demonstrate the effectiveness of the proposed multitype sensor placement method, a bridge benchmark model (Caicedo 2006) is used for numerical investigation in this section. The first 10 mode shapes obtained from the FE model are adopted here for sensor placement investigation. The initial candidate strain gauge locations on each beam element are at 1/3 and 2/3 the longitudinal length of the beam, and the strain gauges are placed on the flange of the beam element. After the initial 8 accelerometer locations haven selected according to the MAC and the redundancy information, 3 accelerometers from the initial accelerometer locations are randomly deleted. To estimate the translation modes of the 3 estimated locations, 9 strain gauges are needed here to ensure that when estimating the translation modes, the number of selected strain gauges is greater than or equal to the number of rows of the translation modes. Similarly, the MAC is used to evaluate the displacement mode information from the sensor placement, and the change of the value of the maximum off-diagonal MAC term is used as a criterion to evaluate the performance of the proposed sensor placement. Table 1 lists the values of the maximum off-diagonal MAC terms of different selected sensor locations.

The displacement modes of the final sensor locations consist of the displacement modes of the 5 accelerometer locations and the displacement modes estimated based on the strain modes of the 9 strain gauges. When calculating the values of the off-diagonal MAC terms of different selected sensor locations, the diagonal terms are also defined as zero. Comparing the situation of 5 remaining locations with the situation of initial 8locations, the value of the maximum off-diagonal MAC term obviously increases because the 3 sensors are randomly deleted from the initial accelerometers and the remaining sensor locations cannot satisfy the MAC well. Comparing the situation of final locations with the situation of 5 remaining locations, the value of the maximum off-diagonal MAC term obviously decreases, which indicates that the strain gauges help much in improving the performance of the sensor placement. Comparing the situation of final locations with the situation initial 8locations, maximum off-diagonal MAC values are similar, which demonstrates that the obtained multitype sensor placement performs well under the MAC when compared to the initial accelerometers.

As a result, the proposed sensor placement method is effective in helping to obtain more accurate displacement mode information of the structure under MAC.

Table 1 Values of maximum off-diagonal MAC terms

Different sensor locations	8 initial locations	5 remaining locations	Final locations
Maximum off-diagonal MAC term value	0.0448	0.4624	0.0451

4. CONCLUSIONS

In this paper, the problem of the comprehensive placement of strain gauges and triaxial accelerometers is solved using a modal estimation theory. The displacement modes of the estimated locations without accelerometers can be estimated by the strain modes obtained from some selected strain gauge measurements. A bridge benchmark model is used here for a numerical investigation. The obtained sensor placement satisfies the MAC well, which demonstrates the effectiveness of the proposed sensor placement method. In this paper, the strain modes are utilized to estimate the displacement modes of the estimated locations; and the strain gauge locations need to contain as much displacement mode information as possible without being at the midpoints.

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