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# Shear and flexural design of reinforced concrete beams strengthened with bolted side-plates

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## ABSTRACT

Existing reinforced concrete (RC) beams can be effectively strengthened by anchoring steel plates to the beams' side faces with anchor bolts, which is known as the bolted side-plating (BSP) technique. Although the BSP technique is feasible to upgrade both the shear and the flexural capacity of the RC beams, no studies have yet been completed on its strengthening design process. In this paper, analytic models are proposed for the computation of both the shear and the flexural capacities of BSP beams. Firstly the shear capacity model was developed according to the force equilibrium and deformation compatibility of the beam segment in the shear span. As for the flexural design, a new design procedure is developed to take into account the adverse local buckling of the bolted steel plates and the enhancement of the buckling restraining stiffeners.

## 1. INTRODUCTION

Comprehensive methods can be apply to the reinforced concrete (RC) beams under requirement of retrofitting, such as installing additional supports to shorten the beam span or increasing the cross section area by adding newly cast concrete with reinforcement. Then a new method, bolted-side-plating (BSP), was proposed by researchers (Roberts et al. 1989; Su, R.K.L et al. 2005). The BSP technique attaches steel plates to the side faces of RC beams by anchor bolts, which were proven to increase both the compressive and the tensile reinforcement of RC beams, thus provides a superior shear and flexural strength without an obvious ductility reduction.

Although the BSP technique is feasible to upgrade both the shear and the flexural capacity of the RC beams, the strengthening design method has yet to be found in literature. Therefore, in this paper, an analytic model is developed for the shear design of BSP beams according to the force equilibrium and deformation compatibility of the beam segment in the shear span. As for the flexural design, an analytic model is also developed to facilitate the determination of bolt spacing, the choosing of buckling restraining stiffeners, and the evaluation of the flexural bearing capacity.

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## 2. PREDICTION OF THE SHEAR CAPACITY OF BSP BEAMS

### *2.1 Hypotheses*

The following hypotheses are employed in the analytic model of the shear bearing capacity of BSP beams:

1) The shear–compression (SC) failure mode is considered, and the brittle shear−tensile (ST) failure should be avoided by proper constructional detailing.

2) The concrete in the shear−compression zone is under the shear–compression strength limit state, and the strength criterion proposed by Yoshikatsu Tsuboi is employed.

3) The interlock and friction along the main diagonal crack is ignored due to the considerable crack width, the dowel shear force of the longitudinal tensile reinforcement can be considered.

4) The compressive reinforcement and the shear stirrups are beyond yielding.

5) The longitudinal tensile reinforcement crossing the main diagonal crack may not yield.

### *2.2 Analytic model*

The beam segment between the support and the main diagonal crack is analysed as shown in Fig. 1(a).

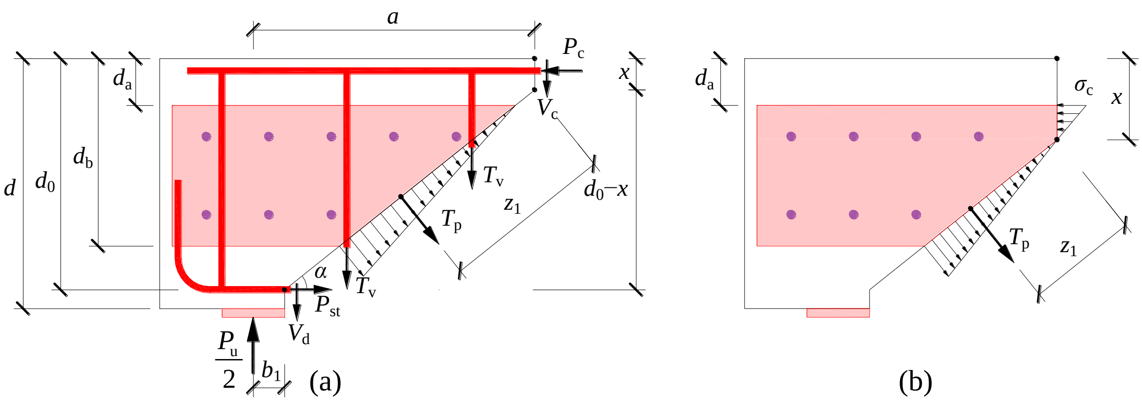


Fig. 1 Stress profile of the bolted steel plate in the shear span: (a) $x < d_a$, and (b) $x > d_a$

The force equilibriums in the vertical and horizontal directions are as follows:

$$\frac{P_u}{2} = T_p \cos\alpha + V_c + V_d + T_v \tag{1}$$

$$P_c = T_p \sin\alpha + P_{st} \tag{2}$$

$$\alpha = \arctan\left(\frac{d_0 - x}{a - b_1}\right) \tag{3}$$

$P_u$: the shear capacity of the beam.
$V_c$: the shear force of concrete.
$T_v$: the tensile force provided by the yield strength of stirrups.
$T_p$: the vertical component of the tensile force contributed from the bolted steel plates.
$V_d$: the dowel shear force of the longitudinal tensile reinforcement.

$P_c$, $P_{st}$: the compressive and tensile forces of the longitudinal compressive and tensile reinforcements.
$\alpha$: the inclination of the main diagonal crack.
$x$: the depth of the shear compression zone.
$d_0$: the effective depth of the RC beam.
$a$: the shear span.
$b_1$: the half width of the support.

By assuming a constant stress distribution as that in an RC beam, the shear force ($V_c$) and the axial compressive force ($P_c$) of concrete in the shear compression zone can be computed as:

$$V_c = \tau_c \left[ bx + \left( \alpha_E - 1 \right) A_{sc} \right] \tag{4}$$

$$P_c = \sigma_c \left( bx - A_{sc} \right) + f_{yc} A_{sc} \tag{5}$$

$$V_d = \alpha_E f_{ct} A_{st} \tag{6}$$

$\tau_c$ ,$\sigma_c$: the shear and compressive stresses of concrete in the shear compression zone, respectively.
$b$: the breadth of the RC beam section.
$A_{sc}$: the cross-sectional area of the longitudinal compressive reinforcement.
$\alpha_E$: the ratio between the Young's moduli of reinforcement steel and concrete.
$f_{ct}$: the tensile strength of concrete material Where $f_{ct}$ is the tensile strength of concrete material.

$$T_v = \frac{\left( a - b_1 \right)}{s_v} f_{yv} A_{sv} \tag{7}$$

$A_{sv}$, $s_v$, $f_{yv}$: the cross-sectional area, spacing and yield strength of the stirrups.

$$T_p = \frac{1}{2} \beta_p f_{yp} A_p \tag{8}$$

$$A_p = t_p \frac{\left( d_b - d_a \right)}{\sin \alpha} \tag{9}$$

$\beta_p$: a value of 0.8 is recommended.
$d_a$, $d_b$: the depths of the upper and lower edges of the bolted steel plates
$A_p$: the area of the inclined section of the bolted steel plates in the direction of the main diagonal crack.
$t_p$: the thickness of the bolted steel plates

In the preliminary design, the number of anchor bolts ($n_b$) can be determined by the constructional detail that the total shear capacity of the bolt group should be greater than $T_p$ ( $n_b \beta_p R_b \geq T_p$ , where $R_b$ is the shear strength of a single bolt). The lever arm between the total resultant tensile force ($T_p$) and the bottom of the shear–compression zone can be calculated as:

$$z_1 = \frac{1}{\sin \alpha} \cdot \left[ \frac{2}{3} \left( d_b - d_a \right) + \left( d_a - x \right) \right] \tag{10}$$

When $x > d_a$, the upper region of the steel plate will be in compression, and its strain is assumed to be in linear distribution due to the shear lag effect, as shown in Fig. 1(b). Therefore, $P_c$, $A_p$, and $z_1$ need to be modified as follows:

$$P_c = \sigma_c \left( bx - A_{sc} \right) + f_{yc} A_{sc} + \tfrac{1}{2} \sigma_c t_p \left( x - d_a \right)$$

$$A_{\mathrm{p}} = t_{\mathrm{p}} \frac{(d_{\mathrm{b}} - x)}{\sin\alpha}$$

$$z_1 = \frac{2}{3} \cdot \frac{(d_{\mathrm{b}} - x)}{\sin\alpha}$$

The vertical component of the deformation of the bolted steel plates ($\delta_{\mathrm{p}}$) represents the overall vertical deformation of the beam segment in the shear span, and the deformation of the tensile reinforcement ($\delta_{\mathrm{st}}$) may be assigned as the overall horizontal deformation in the shear span, as shown in Fig. 2.

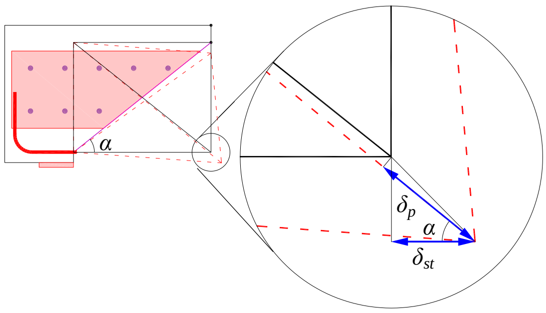


Fig. 2 Shear deformation of the shear segment

$$\delta_{\mathrm{st}} = \delta_{\mathrm{p}} \cos\alpha \quad \Rightarrow \quad \varepsilon_{\mathrm{st}} = \varepsilon_{\mathrm{p}} \cos\alpha \tag{11}$$

$\varepsilon_{\mathrm{p}}$, $\varepsilon_{\mathrm{st}}$: the strains of the bolted steel plates and the tensile reinforcement.

Substituting Eq. (8) and strain expressions of the longitudinal tensile reinforcement into Eq. (11) gives the tensile force of the longitudinal tensile reinforcement ($P_{\mathrm{st}}$) as follows:

$$P_{\mathrm{st}} = A_{\mathrm{st}} E_{\mathrm{s}} \frac{2T_{\mathrm{p}}}{\beta_{\mathrm{p}} A_{\mathrm{p}} E_{\mathrm{p}}} \cos\alpha \tag{12}$$

The moment equilibrium of the beam segment referring to the bottom point of the shear compression zone is as following:

$$\frac{P_{\mathrm{u}}}{2} a = V_{\mathrm{d}}(a - b_1) + \frac{1}{2} T_{\mathrm{v}}(a - b_1) + T_{\mathrm{p}} z_1 + P_{\mathrm{st}}(d_0 - x) + P_{\mathrm{c}} \frac{x}{2} \tag{13}$$

The shear–compression strength criterion proposed by Yoshikatsu Tsuboi is employed to describe the failure envelope curve of the concrete in the shear–compression zone:

$$\frac{\tau_{\mathrm{c}}}{f_{\mathrm{c}}} \leq [\frac{\tau_{\mathrm{c}}}{f_{\mathrm{c}}}] = \sqrt{0.0089 + 0.095 \frac{\sigma_{\mathrm{c}}}{f_{\mathrm{c}}} - 0.104 \left(\frac{\sigma_{\mathrm{c}}}{f_{\mathrm{c}}}\right)^2} \tag{14}$$

Where $\tau_{\mathrm{c}}$ is the shear stress of concrete in the shear compression zone.

## 3. FLEXURAL DESIGN

It is essential to provide suitable method to determine the required bolt spacing or choose proper size of buckling restraining stiffeners to prevent the adverse local buckling phenomenon. Moreover, it is necessary to develop practical formulas to evaluate the ultimate flexural strength of BSP beams.

### 3.1 Determination of bolt spacing

#### 3.1.1 Estimation of number of anchor bolts

The shear capacity ($S_b$) of the bolt group from the location of the maximum bending moment ($M_{max}$) to the plate end (as shown in Fig. 3(a)) should be greater than the axial yield strength of the steel plate ($T_{yp}$), as follows:

$$S_b = \frac{1}{\gamma_b} n_b R_{by} \quad \geq \quad T_{yp} = f_{yp} t_p h_p \tag{15}$$

$R_{by}$: the yield shear force of an anchor bolt,
$n_b$: the total number of anchor bolts from Mmax to the plate end,
$\gamma_b$: a coefficient taking account of the variation in the bolt shear force along the beam span, a value from 1.5 to 2.0 can be chosen.
$f_{yp}$, $t_p$ and $h_p$: the yield strength, the thickness and depth of the bolted steel plate as shown in Fig. 4(a).

Solving the inequality Eq. (15) gives:

$$n_b \geq \gamma_b \frac{f_{yp} t_p h_p}{R_{by}} \tag{16}$$

The bolt spacing ($s_b$) can be preliminarily determined according to the minimum number of anchor bolts ($n_b$). However, the bolt spacing determined according to Eq. (16) cannot guarantee that the local buckling will not occur on the upper compressive edge of the bolted steel plates.

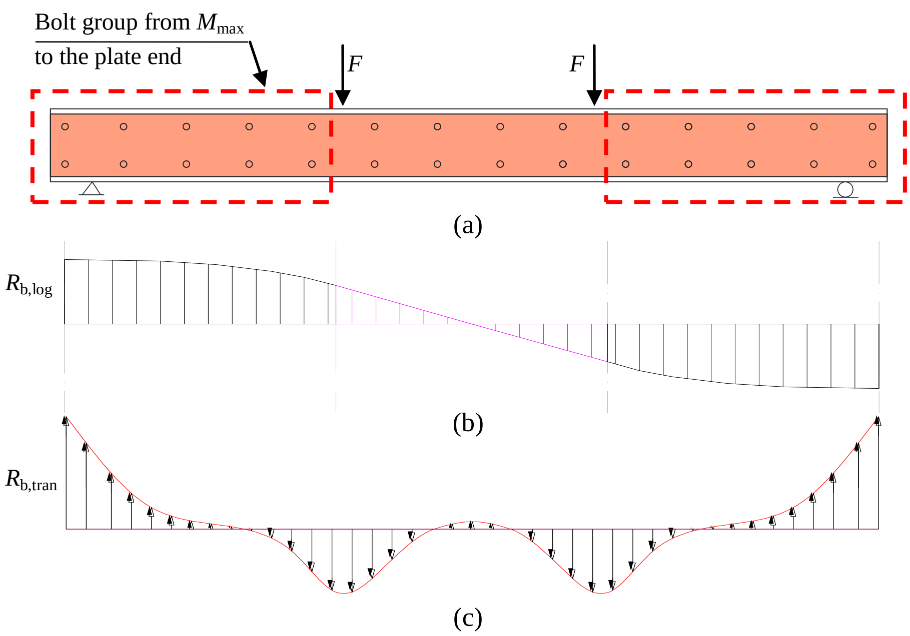


Fig. 3 The variation of bolt shear force in a BSP beam: (a) elevation view, force variation of (b) longitudinal and (c) transverse bolt shear force

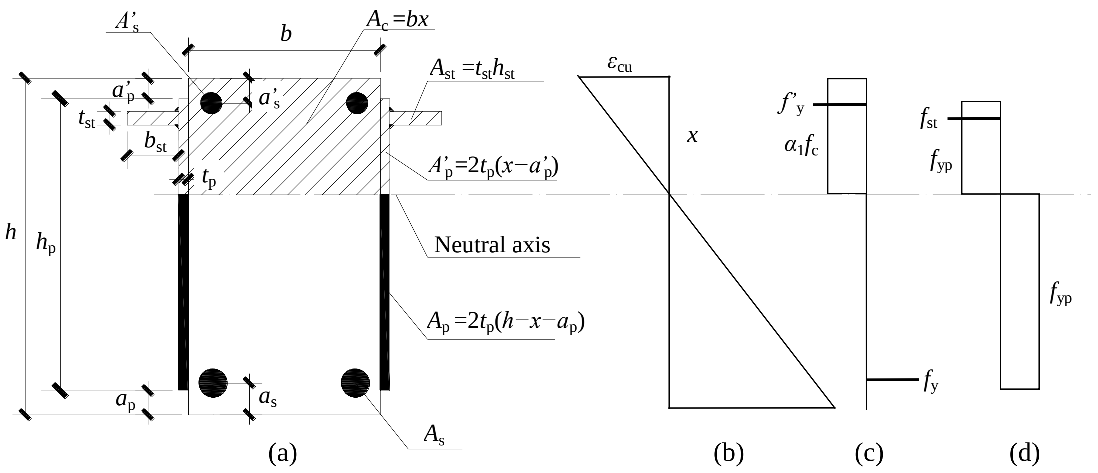


Fig. 4 Stress and strain profile of a BSP beam section: (a) cross section, (b) strain profile, stress profile of (c) the RC beam and (d) the bolted steel plates and the buckling restraining stiffeners

3.1.2 Decrease bolt spacing to prevent local buckling

In order to determine the required bolt spacing to suppress the local buckling phenomenon, the critical stress when local buckling ($\sigma_{cr}$) occurs in a steel plate under uniformly compressive stress is investigated for simplicity and conservative purpose, and it should be greater than the yield stress of the steel plate ($f_y$) as follows:

$$\sigma_{\mathrm{cr}} = k\frac{\pi^2 D}{t_{\mathrm{p}} h_{\mathrm{p}}^{\ 2}} \geq f_{\mathrm{y}} \quad \Rightarrow \quad k \geq f_{\mathrm{y}} \frac{t_{\mathrm{p}} h_{\mathrm{p}}^{\ 2}}{\pi^2 D} \tag{17}$$

Where $D$ is the bending stiffness of the steel plate and defined by:

$$D = \frac{E t_{\mathrm{p}}^{\ 3}}{12\left(1-\nu^2\right)} \tag{18}$$

$k$ is a factor whose magnitude depends on the ratio between the bolt spacing and the breadth of the steel plates ($s_b$ / $h_p$) as:

$$k = \left(\frac{m h_{\mathrm{p}}}{s_{\mathrm{b}}} + \frac{s_{\mathrm{b}}}{m h_{\mathrm{p}}}\right)^2 \tag{19}$$

The minimum value of $k$ is obtained when the plate buckles in one half-wave ($m$ = 1). While for unilateral local buckling, the minimum value of $k$ is obtained when the number of half-waves into which the plate buckles is two ($m$ = 2). Since there is inevitable gap between the bolted steel plates and the RC beam in an actual BSP beam, and the anchorage provided by the anchor bolts is far less rigid than being built in, so the local buckling is actually quasi-unilateral and it is reasonable to set $m$ as 1.5.

By defining the right hand side of the inequality Eq. (17) as a parameter $k_0$ and substituting Eq. (19) into it, the following inequality Eq. (17) is obtained:

$$\left(\frac{1.5 h_{\mathrm{p}}}{s_{\mathrm{b}}} + \frac{s_{\mathrm{b}}}{1.5 h_{\mathrm{p}}}\right)^2 \geq k_0 \,, \quad \text{where } k_0 = f_{\mathrm{y}} \frac{t_{\mathrm{p}} h_{\mathrm{p}}^{\ 2}}{\pi^2 D} \tag{20}$$

Solving Eq. (20) gives the acceptable maximum bolt spacing as:

$$s_{\mathrm{b}} \le 0.75 h_{\mathrm{p}} \cdot \left( \sqrt{k_0} - \sqrt{k_0 - 4} \right) \tag{21}$$

When the bolt spacing along the upper compressive edge of the bolted steel plates is less than the value giving in Eq. (21), the adverse local buckling effect in BSP beams can be ignored in the computation of the flexural bearing capacity.

*3.2 Employment of restraining stiffeners*

It may be a more feasible strategy to employ buckling restraining stiffeners along the upper compressive edge of the bolted steel plates to restrict the local buckling failure. In a previous numerical study (Xu et al. 2017) by the authors, the strengthening effect of buckling restraining stiffeners was studied comprehensively and recommended sectional size was proposed, so the thickness and breadth of the stiffeners can be chosen as 1.2 and 6 times of the thickness of the bolted steel plate ($t_{st} = 1.2\, t_p$, $b_{st} = 6\, t_p$).

*3.3 Evaluation of the ultimate flexural strength*

We can assume that the full cross sections of the steel plates and reinforcements can reach the yield strength when the beam section reaches the ultimate limit state (ULS), as shown in Fig. 4. Meanwhile the tensile strength of concrete can be ignored.

The depth of the compression zone ($x$) of a BSP beam section at the ULS can be calculated by solving the force equilibrium equation Eq. (22) as follows:

$$\alpha_1 f_{\mathrm{c}} b x + f_{\mathrm{y}}^{'} A_{\mathrm{s}}^{'} + 2 f_{\mathrm{yp}} t_{\mathrm{p}} \left( x - a_{\mathrm{p}}^{'} \right) + 2 f_{\mathrm{st}} t_{\mathrm{st}} b_{\mathrm{st}} = f_{\mathrm{y}} A_{\mathrm{s}} + 2 f_{\mathrm{yp}} t_{\mathrm{p}} \left( h_{\mathrm{p}} - x + a_{\mathrm{p}}^{'} \right) \tag{22}$$

$$x = \frac{f_{\mathrm{y}} A_{\mathrm{s}} + 2 f_{\mathrm{yp}} t_{\mathrm{p}} h_{\mathrm{p}} - f_{\mathrm{y}}^{'} A_{\mathrm{s}}^{'} - 2 f_{\mathrm{st}} t_{\mathrm{st}} b_{\mathrm{st}}}{\alpha_1 f_{\mathrm{c}} b + 4 f_{\mathrm{yp}} t_{\mathrm{p}}} \tag{23}$$

$$\text{where } \; x \ge 2a_{\mathrm{s}}^{'} \; \text{ and } \; x \le \xi_{\mathrm{b}} \left( h - a_{\mathrm{s}} \right)$$

$f_c$, $f_y$', $f_y$, $f_{yp}$, and $f_{st}$: the compressive strength of the concrete, the yield strength of the compressive and tensile reinforcement, the steel plates and the stiffeners, respectively.
$b$, $h$ : the breadth and depth of the RC beam.
$A_s$, $A'_s$: the areas of the tensile and compressive reinforcement.
$a_p$, $a'_p$: the distance from the bottom and upper edges of steel plates to the top and bottom faces of the RC beam.

Then the ultimate bending moment ($M_u$) can be obtained as:

$$\begin{aligned} M_{\mathrm{u}} = & \frac{1}{2} \alpha_1 f_{\mathrm{c}} b x^2 + f_{\mathrm{y}}^{'} A_{\mathrm{s}}^{'} \left( x - a_{\mathrm{s}}^{'} \right) + f_{\mathrm{yp}} t_{\mathrm{p}} \left( x - a_{\mathrm{p}}^{'} \right)^2 + 2 f_{\mathrm{st}} t_{\mathrm{st}} b_{\mathrm{st}} \left( x - a_{\mathrm{p}}^{'} \right) \\ & + f_{\mathrm{y}} A_{\mathrm{s}} \left( h_{\mathrm{p}} - x + a_{\mathrm{s}}^{'} \right) + f_{\mathrm{yp}} t_{\mathrm{p}} \left( h - x - a_{\mathrm{p}} \right)^2 \end{aligned} \tag{24}$$

Where $a_s$ and $a'_s$ represent the thicknesses of the concrete covers in the tensile and compressive region respectively.

## 4. CONCLUSION

Two analytic models are proposed to evaluate the shear and flexural bearing capacity of BSP beams. The main outcomes are summarized as follows:

(1) An analytical model is developed according to the force equilibrium and deformation compatibility of the beam segment in the shear span, it is convenient to estimate the shear capacity of BSP beams for engineers in their strengthening design practice.

(2) The proposed model can predict the flexural bearing capacity of BSP beams with satisfactory accuracy and safe margins, and the welded stiffeners is a more effective measure for buckling restraining.

## ACKNOWLEDGEMENT

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