

## **Lateral strength model for preloaded rectangular RC columns strengthened with precambered steel plates**

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### **ABSTRACT**

Rectangular short reinforced concrete (RC) columns with non-ductile details subjected to high axial loads exist in many buildings, especially for those built before 1970s without consideration of the seismic design. When subjected to a rare earthquake load attack, those columns are prone to brittle axial failure with limited deformability, which could lead to partial or total collapse of the building. To address this issue, this study adopted precambered steel plate to reduce the axial load level and hence increase the deformability of the strengthened column. A theoretical model is derived for estimating the ultimate lateral load capacity of the strengthened column. In this model, the effect of pre-existing gravity loads in the RC column is taken into account. Furthermore, a coefficient is introduced to account for the partial interaction between the concrete column and steel plates. Finally, the predicted results are compared with the available experimental results to validate the theoretical model.

### **1. INTRODUCTION**

Due to the hilly terrain in HK territories, many reinforced concrete (RC) frame or wall-frame buildings are located on slopes. In those buildings, RC columns play an important role to resist earthquake loads. However, many of them were designed according to the old British Standards CP114 (BSI 1969) without any provision of ductility details prior to the implementation of the new concrete code in 2004. Those columns are prone to brittle axial failure with limited deformability when subjected to earthquake (Sezen *et al.* 2003; Doğangün 2004).

There exist three retrofitting methods, including (1) concrete jacketing technique: It can increase the strength and ductility of column but space in the building would be occupied; (2) composite jacketing technique: It makes use of fiber-reinforced polymer and is efficient for retrofitting columns with circular but not rectangular section by increasing the concrete confinement and (3) steel jacketing technique: Despite that it is an economical, efficiency and practical construction method, there exist many key

problems to be resolved, which will be described herein. Until now, most of the aforementioned techniques focused on enhancing the axial load capacity rather than the earthquake resistance of RC columns. Furthermore, Takeuti *et al.* (2008) and Giménez *et al.* (2009) studied the axial load response of concrete jacketed and steel angles-and-strips strengthened columns respectively. They found that the effect of pre-existing axial load could significantly reduce the ultimate load capacity and deformability of the strengthened columns. It is because the pre-existing loading can cause stress-lagging between the concrete column and the external steel or concrete jacket. As the increase of loads, the concrete core failed prior to the external jacket. Full material strength cannot be utilized. To address this issue, Su and Wang (2012) proposed an innovative strengthening approach, namely precambered steel plate method, to reduce the axial load level of concrete core. Su and Wang (2015) found that the lateral deformability of the precambered steel plate strengthened columns could be increased.

In this paper, previous existing models are briefly introduced and their limitations are discussed. Then a theoretical model is derived to estimate the lateral load capacity of precambered steel plate strengthened columns. Finally, the predicted and test results are compared to verify the theoretical model.

## 1. A BRIEF INTRODUCTION TO TEST RESULT OF THIS STRENGTHENING METHOD

Wang *et al.* (submitted) tested a total of six 1/3 scale specimens labelled CSC1 to CSC6 with different parameters, as shown in Table 1. Specimens labelled CSC2 to CSC6 were strengthened with precambered steel plates, as shown in Fig. 1. Prior to post-stressing, the length of the two steel plates was a bit longer than the clear height of the column. After post-stressing, a thrust action was generated on the location of both the top and base of the column and then pre-existing axial load was decompressed by the steel plates. After that, specimens were tested under reversed cyclic lateral load subjected to a pre-compression axial load.

Table 1. Summary of strengthening details

Specimen	$f_{cu}$ (MPa)	$f'_c$ (MPa)	$t_p$ (mm)	$\delta$ (mm)	$P_{pl}$ (kN)	$\phi$
CSC1	42.3	37.6	-	-	960	0.6
CSC2	41.3	34.8	6	0	960	0.6
CSC3	41.9	35.9	3	16	960	0.6
CSC4	42.7	36.5	6	8	960	0.6
CSC5	42.5	37.3	6	16	960	0.6
CSC6	42.0	36.2	6	16	1120	0.7

where  $f_{cu}$  is concrete cube compressive strength and  $f'_c$  is the concrete cylinder compressive strength;  $t_p$  is the thickness of steel plates while  $\delta$  is the precamber of steel plate;  $P_{pl}$  is the existing axial load and  $\phi$  is the axial load ratio corresponding to  $P_{pl}$ .

The lateral load-drift ratio curves are summarized in Fig. 2. The drift ratio was obtained by dividing lateral displacement by effective column height equal to the sum of column clear height and half of the top beam depth. As shown in Fig. 2, the strength and deformability were improved significantly by this method. From CSC2 to CSC6, shear capacity increased by 54.5%, 32.5%, 66.1% and 60.6%, respectively. All strengthened columns failed due to the formation of a plastic hinge at the base of columns.

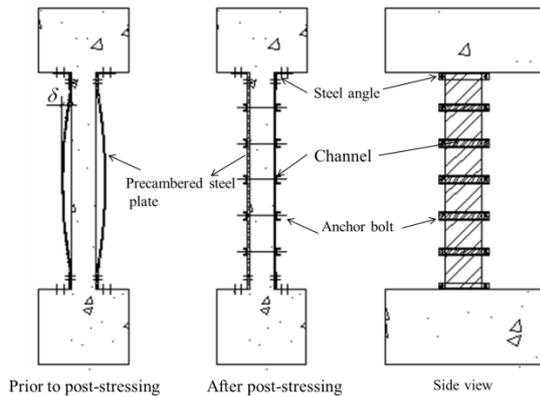


Fig. 1 Layout of a strengthened specimen

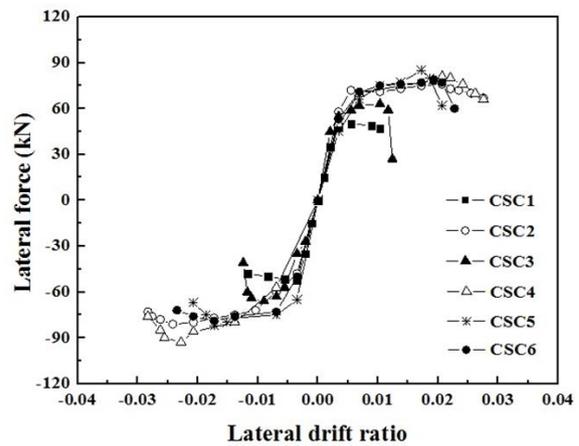


Fig. 2 Envelope curves of lateral load versus drift ratio of columns

## 2. BRIEF REVIEW OF PREVIOUS MOMENT CAPACITY MODELS

According to the ACI 318-14 (ACI 2014), nominal moment strength,  $M_u$ , is derived from the following two equilibrium equations.

$$P_u = P_c + P_s \quad (1)$$

$$M_u = M_c + M_s \quad (2)$$

where  $P_u$  is the axial load;  $P_c$  and  $P_s$  are the axial load resisted by concrete and longitudinal reinforcement;  $M_u$  is the ultimate moment capacity;  $M_c$  and  $M_s$  are the moment resisted by concrete and longitudinal reinforcement.

This model cannot be applied directly to obtain the moment capacity of the strengthened column as strain-lagging between steel plates and RC column and slip between RC column and steel plates have not been considered. In this paper, this model will be modified to estimate the ultimate moment capacity in the following sections.

Zhang and Liu (2007) proposed a moment-axial load curvature analysis for the determination of lateral load capacity of square tube confined RC columns. In their method, lateral deformation profile of column and the corresponding curvature of different sections were assumed to follow certain sine functions as follows:

$$y = \delta \sin \frac{\pi x}{L} \quad (3)$$

$$\phi = y'' = -\frac{\delta\pi^2}{L^2} \sin \frac{\pi x}{L} \quad (4)$$

in which,  $L$  is the height of the column and  $\delta$  is the lateral deformation at the loading point;  $x$  and  $y$  are the location along axial direction of column and the corresponding lateral displacement and  $\phi$  is the curvature of section.

Following these assumptions, the curvature at plastic hinge and moment capacity of the composite column could be determined. However, it is unreasonable to assume that the lateral deformation profile of column follows a semi-sine curve. The actual deformation profile is far more complex as the lateral deflections consist of bending and shear deformations and the concrete stress-strain relationship is nonlinear. In addition, the slip between steel tube and RC column has not been considered in the formulation. Such deficiencies in the model would lead to inaccurate estimation of the lateral load capacity.

### 3. PROPOSED LATERAL STRENGTH MODEL

The following assumptions are made prior to deriving the theoretical ultimate shear capacity ( $V$ ) of the strengthened RC column:

- (1) Column section remains plane after deformation.
- (2) Equivalent rectangular stress block is adopted.
- (3) Ultimate compressive concrete strain is taken as 0.003.
- (4) Steel plate and reinforcement are assumed as perfect elastic-plastic material.
- (5) At the ultimate limit state, concrete compressive strain reached its ultimate value.
- (6) Partial interaction between steel plate and concrete is taken into account.

During the post-compression stage prior to the application of the lateral loads, the initial strains ( $\epsilon_{su}$  and Wang 2012) in the steel plate ( $\epsilon_{p,ps}$ ) and RC column ( $\epsilon_{rc,ps}$ ) as illustrated in Fig. 3 can be expressed as Eq. (5) and Eq. (6).

$$\epsilon_{p,ps} = \frac{(\pi\delta)^2}{2} \frac{E_c A_c L_{rc,pl}}{E_p A_p L_{rc,pl} + 2E_c A_c L_p} \quad (5)$$

$$\epsilon_{rc,ps} = \frac{P_{pl}}{E_c A_c} - \frac{(\pi\delta)^2}{2} \frac{E_c A_c L_{rc,pl}}{E_p A_p L_{rc,pl} + 2E_c A_c L_p} \quad (6)$$

where  $\delta$  is the initial preamber displacement at the mid-height of the steel plate and  $L_{rc,pl}$  is the clear height of the RC column under axial preload ( $P_{pl}$ ).  $E_c$  and  $E_p$  represent the Young's modulus of the RC column and steel plate respectively.  $A_c$  and  $A_p$  are the cross-sectional area of the RC column and steel plate respectively and  $L_p$  is the non-deformation length of the steel plate.

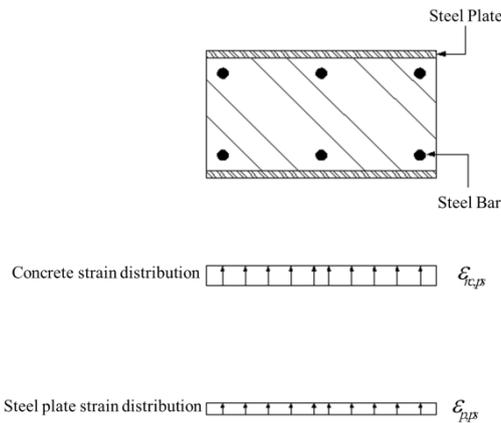


Fig. 3. Strain distributions at the post-compression stage

At the ultimate limit state, assuming that the plane section remains plane after loading and adopting an equivalent rectangular stress block of concrete for the evaluation of the ultimate flexural capacity, the strain and stress distributions of the RC column and steel plate can be presented in Fig. 4 and Fig. 5 respectively.

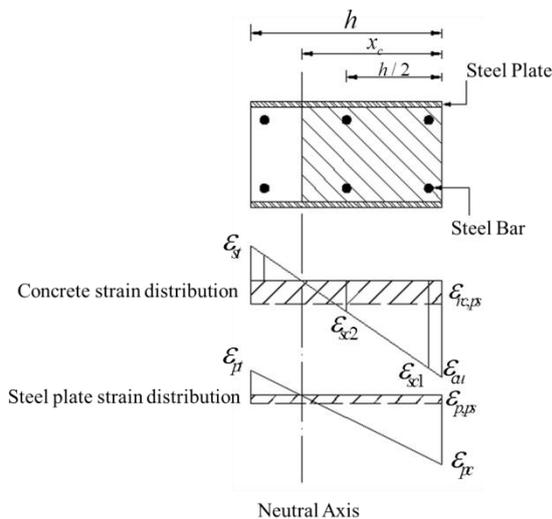


Fig. 4. Strain distribution at the ultimate limit state

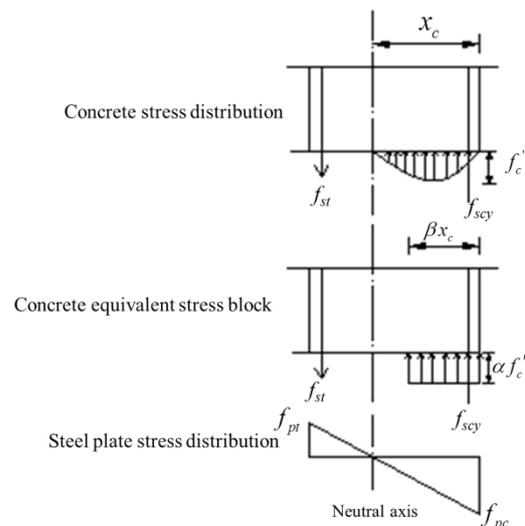


Fig. 5. Stress distribution at the ultimate limit state

Fig. 5 shows the equivalent stress block of concrete, in which  $f'_c$  is the cylinder compressive strength of the concrete,  $\alpha$  and  $\beta$  are the stress block factors (Collins and Mitchell, 1991) which can be expressed as Eqs. (7) and (8).

$$\alpha = \left[ \frac{\varepsilon_{cu}}{\varepsilon_{c0}} - \frac{1}{3} \left( \frac{\varepsilon_{cu}}{\varepsilon_{c0}} \right)^2 \right] / \beta \quad (7)$$

$$\beta = \left( 4 - \frac{\varepsilon_{cu}}{\varepsilon_{c0}} \right) / \left( 6 - \frac{2\varepsilon_{cu}}{\varepsilon_{c0}} \right) \quad (8)$$

where  $\varepsilon_{cu}$  is the ultimate compressive strain of concrete and  $\varepsilon_{c0}$  is the compressive strain at the peak compressive strength of concrete.

Fig. 5 shows the strain distribution of concrete, vertical steel bar and steel plate, in which  $h$  is the depth of the concrete section and  $x_c$  is the depth of the compressive zone of the concrete section.  $\varepsilon_{sc1}$ ,  $\varepsilon_{sc2}$  and  $\varepsilon_{st}$  are the compressive and tensile strains of the longitudinal reinforcement bars while  $\varepsilon_{pc}$  and  $\varepsilon_{pt}$  are the compressive and tensile strains of the steel plate, respectively. Assuming that the plane section remains plane after loading, the strains in the reinforcement and steel plate are derived and shown in Eqs. (9)-(13).

$$\varepsilon_{sc2} = \varepsilon_{cu} \left(1 - 0.5 \frac{\beta}{\xi}\right) \quad (9)$$

$$\varepsilon_{st} = \varepsilon_{cu} \left(\frac{\beta}{\xi} - 1\right) \quad (10)$$

$$\varepsilon_{pt} = \varepsilon_{pc} \left(\frac{\beta}{\xi} - 1\right) \quad (11)$$

$$\xi = \frac{\beta x_c}{h_0} \quad (12)$$

$$\varepsilon_{pc} = \begin{cases} \varepsilon_{cu} - \varepsilon_{rc,ps} + \varepsilon_{p,ps} & (\varepsilon_{cu} - \varepsilon_{rc,ps} + \varepsilon_{p,ps} < \varepsilon_{pcy}) \\ \varepsilon_{pcy} & (\varepsilon_{cu} - \varepsilon_{rc,ps} + \varepsilon_{p,ps} \geq \varepsilon_{pcy}) \end{cases} \quad (13)$$

where  $h_0$  is the effective depth of the concrete section,  $\xi$  is the relative compressive depth of the concrete section and  $\varepsilon_{pcy}$  is the yield strain of the steel plate.

Similar to the conventional sectional analysis of the RC column, the depth of the compressive zone ( $x_c$ ) can be calculated by invoking the force equilibrium in Eqs. (14)-(20).

$$P_{pl} = P_c + \lambda P_p \quad (14)$$

$$P_c = \alpha f'_c b h_0 \xi + 2 f_{scy} A_{se1} + 2 f_{sc2} A_{se2} - 2 f_{st} A_{st} \quad (15)$$

$$P_p = f_{pc} t_p \xi h_0 / \beta - f_{pt} t_p (h - \xi h_0 / \beta) \quad (16)$$

$$f_{sc2} = \begin{cases} f_{scy} & (|\varepsilon_{sc2}| \geq f_{scy} / E_s) \\ E_s \varepsilon_{sc2} & (|\varepsilon_{sc2}| < f_{scy} / E_s) \end{cases} \quad (17)$$

$$f_{st} = \begin{cases} f_{sty} & (|\varepsilon_{st}| \geq f_{sty} / E_s) \\ E_s \varepsilon_{st} & (|\varepsilon_{st}| \leq f_{sty} / E_s) \end{cases} \quad (18)$$

$$f_{pt} = \begin{cases} f_{pty} & (|\varepsilon_{pt}| \geq f_{pty} / E_p) \\ E_p \varepsilon_{pt} & (|\varepsilon_{pt}| < f_{pty} / E_p) \end{cases} \quad (19)$$

$$f_{pc} = E_p \varepsilon_{pc} \quad (20)$$

where  $P_p$  is the axial force in the steel plate,  $P_c$  is the axial force in the concrete,  $b$  is the breadth of column,  $f_{sc2}$  and  $f_{st}$  are the compressive and tensile stresses of the longitudinal reinforcement bars,  $A_{sc1}$ ,  $A_{sc2}$  and  $A_{st}$  are sectional areas of the longitudinal reinforcement,  $f_{pt}$  and  $t_p$  are the stress and thickness of the steel plate,  $f_{scy}$  and  $f_{sty}$  are the compressive and tensile strengths of the longitudinal reinforcement,  $f_{pty}$  is the tensile strength of the steel plate and  $\lambda$  is a reduction factor for the flexural strength of the steel plate, taking into account the partial interaction between the steel plate and concrete, which is taken as 0.8.

Then the ultimate bending moment which can be obtained by taking moment with the strong axis is expressed in Eqs. (21-23) .

$$M_u = M_c + \lambda M_p \quad (21)$$

$$M_c = \alpha f'_c b h_0 \xi (0.5h - 0.5\xi h_0) + 2f_{scy} A_{sc1} (h/2 - a_s) + 2f_{st} A_{st} (h/2 - a_s) \quad (22)$$

$$M_p = f_{pc} t_p x_c \left(\frac{h}{2} - \frac{x_c}{3}\right) + f_{pt} t_p (h - x_c) \left(\frac{h}{2} - \frac{x_c}{3}\right) \quad (23)$$

where  $M_p$  is the moment resisted by the steel plate and  $M_c$  is the ultimate moment resisted by the concrete.

The ultimate bending moment ( $M_u$ ) is related to the externally applied load in Eq. (24), with consideration of p-delta effects.

$$M_u = P_{pl} H \theta + V H \quad (24)$$

where  $P_{pl}$  and  $V$  are the preloaded and shear forces that are acting on the column, respectively.  $\theta$  is the drift ratio,  $H$  is the height of the applied lateral load measuring from the base of the column and  $a_s$  represents the thickness of concrete cover in the tensile region of the section.

Fig. 6 shows the comparisons between the test results and predicted values from different models. The capacity obtained from Zhang and Liu (2007) underestimated the load-carrying capacity of section as the accurate curvature of the composite column cannot be derived from the assumed lateral deformation profile. After the formation of plastic hinge, the curvature increases rapidly in the region of plastic hinge so that it cannot be estimated by a simply assumed deformation profile. The predicted capacities from the ACI 318-14 (ACI, 2014) overestimated the test values since the slip between the steel plates and RC column and the strain-lagging effect are not considered. The proposed model takes into account the effects of slip and strain-lagging between steel plates and RC column yields the most accurate prediction for the lateral load capacity of the strengthened columns.

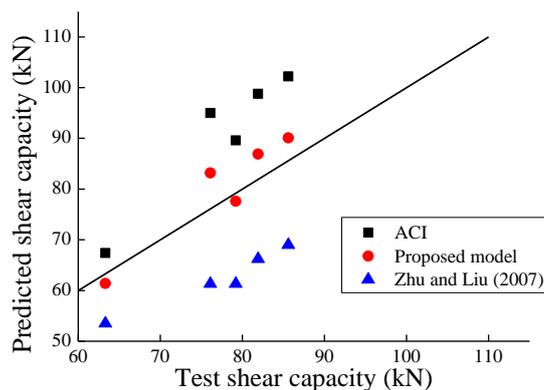


Fig. 6 Comparison of the predicted and test results

#### 4. CONCLUSIONS

The innovative precambered steel plate strengthening method which has alleviated the strain-lagging problem between the concrete core and the steel plates can significantly increase the lateral load carrying capacity of RC columns. A new lateral strength model for precambered steel plate strengthening method which has incorporated the pre-loading effects and partial interaction between the steel plates and concrete core was introduced in this paper. By comparing the predicted and test results, the proposed model is found to be more accurate than the other available models.

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