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Derivation of Paris' Law Parameters from S-N Curve Data: a Bayesian Approach

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ABSTRACT

Paris' law is one of the widely-used equations for probabilistic fatigue analysis of structures. Although the statistical information of Paris' law parameters is very critical in the analysis, it is not an easy task to determine the parameters experimentally, whereas plenty of experimental data exists for S-N curves. This paper proposes a novel method to quantify the uncertainties lying in the Paris' law parameters, by finding the best estimates of their statistical parameters from S-N curve data using a Bayesian approach. Through a series of steps, the proposed method identifies the statistical parameters (e.g., mean and standard deviation) of Paris' law parameters that will maximize the likelihood of observing the given S-N data. To validate the proposed method, it is applied to an example of H40 steel, and the corresponding analysis results show good agreement with previous experimental results.

1 INTRODUCTION

Although a variety of research has been carried out on the fatigue analysis of structures for a long time, the engineering world is still struggling to fully understand the fatigue process, which is involved with various sources of uncertainty. Previous studies are often based on S-N curves [1], whereas many other studies are based on the Paris' law [2], which accounts for the crack propagation rate based on fracture mechanics theories. Although the statistical information of Paris' law parameters is very critical in the analysis, it is not an easy task to determine Paris' law parameters experimentally, whereas plenty of experimental data exists for S-N curves.

In this paper, a new method termed the S-N Paris law (SNPL) method is proposed. Using a Bayesian approach, the SNPL method enables the derivation of the Paris' law parameters from S-N curve data. The implementation of the SNPL method is three folds: (1) dividing S-N curve data into failure and non-failure cases and providing the stress range and number of cycles as an input to the Paris' law; (2) constructing the likelihood functions of the both cases for an objective function, by use of the concept of limit-state functions; and (3) checking the response surface of the objective function and running optimization. Following these steps, the statistical parameters (e.g., mean and standard deviation) of the Paris' law parameters, which maximize the likelihood of observing the given S-N curve data, can be determined probabilistically.

2 PROPOSED METHOD

One of the most well-known approaches analyzing structural fatigue behavior is the stress-life approach, which provides the relationship between the stress range and the number of loading cycles to failure. This is often called S-N curve [1], which is a log-log plot showing the constant amplitude stress range, *S*, as a function of the number of cycles to failure, *N*. Although this method is very useful for the design of structures against fatigue failure, it does not give any information about the crack initiation or propagation.

On the other hand, the importance of understanding crack propagation led to a new field of study called fracture mechanics. Paris and Erdogan used the idea of comparing Irwin's stress intensity with the rate of crack propagation per cycle [2]. They developed a relationship between the crack propagation rate per cycle (*da/dN*) and the stress intensity factor range (ΔK), which is called the Paris' law as shown in Eq. (1).

$$\frac{da}{dN} = C\Delta K^m \tag{1}$$

where *a* is the crack length, *N* is the number of loading cycles, and *C* and *m* are the material parameters. The range of stress intensity factor ΔK can be estimated as

$$\Delta K = \Delta S \cdot Y\left(a\right)\sqrt{\pi a} \tag{2}$$

where ΔS is the range of the far-field stress and *Y(a)* is the geometry function.

The Paris' law is effective as it can characterize crack growth, which helps in assessing service life or inspection intervals for a structure [3-4]. Even though the Paris' law is useful in many structural problems, it is involved with various sources of uncertainty such as initial crack length and material parameters. Their statistical parameters can be determined experimentally, through fatigue crack growth rate experiments. However, uncertainty evolves from material inhomogeneity and also while measuring the crack growth rate and stress intensity factors during the experiment [5]. To incorporate all these uncertainties, a statistical and probabilistic approach such as the Bayesian inference can be applied.

In this paper, a new method termed the SNPL method is proposed to derive the statistical parameters of the Paris' law parameters from S-N curve data. The proposed method is based on a Bayesian approach and depicted by a flowchart in Fig. 1, and each step is explained below.

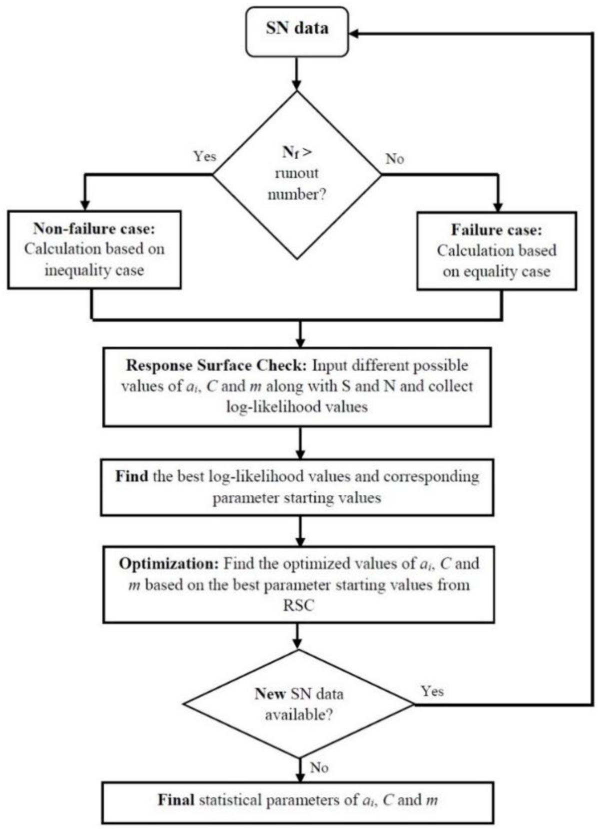

Fig. 1 SNPL Method Flowchart

Step 1: Divide S-N curve data into two cases

The obtained S-N data is divided into two cases based on number of cycles to failure, N_f. First case is non-failure case (i.e., N_f > runout number) and second case is failure case (i.e., N_f < runout number). Then, the stress range and number of cycles are provided as an input to equations based on the Paris' law for the next step.

Step 2: Construct the likelihood functions of both cases

Case 1: Inequality case (Non-failure case)

When one does not observe the failure until the runout number from an S-N test, it is considered as the inequality case, and the limit-state function g_{nofail} is given as

$$g_{nofail}(\mathbf{X}) = N_{SN} - N_{paris}(\mathbf{X}) < 0 \tag{3}$$

where **X** is the vector of random variables, N_{SN} is the number of cycles from the S-N test and N_{paris} is the number of cycles calculated as shown in Eq. (4).

$$N_{paris}(\mathbf{X}) = \frac{1}{C}\int_{a_i}^{a_c} \frac{1}{\left[\Delta S \cdot Y(a)\sqrt{\pi a}\right]^m} da \tag{4}$$

where a_c and a_i are the critical crack length and the initial crack length, respectively.

Based on the limit-state function in Eq. (3), the probability of the non-failure case is calculated as follows:

$$P_{nofail} = P\left(N_{SN} - N_{paris}(\mathbf{X}) < 0\right) \tag{5}$$

Case 2: Equality case (Failure case)

When one observes the failure within the runout number from an S-N test, it is considered as an equality case, and the limit-state function is given in Eq. (6).

$$g_{fail}(\mathbf{X}) = N_{SN} - N_{paris}(\mathbf{X}) = 0 \tag{6}$$

Based on the limit-state function in Eq. (6), the probability is calculated as follows:

$$P_{fail} = P\left(N_{SN} - N_{paris}(\mathbf{X})\right) = 0 \tag{7}$$

An infinitesimal quantity θ is introduced to the limit-state function to eliminate the equality sign. Then, Eq. (8) is obtained to proceed with the calculation, as addressed in the references [3-4].

$$P_{fail} = \left[\frac{\partial}{\partial\theta} P\left[\left(N_{SN} - N_{paris}(\mathbf{X}) + \theta\right)\right] \le 0\right]_{\theta=0} \tag{8}$$

The probabilities in Eqs (5) and (8) are multiplied to construct an objective function, which represents the probability that the given S-N test data is observed. To reduce computational errors during the optimization process, the natural logarithm is used for the SNPL method as the objective function.

Step 3: Response surface check (RSC) and optimization process

The starting point is selected through RSC followed by an optimization process based on the best results from RSC, which finds the best statistical parameters. One advantage of the SNPL method is the introduction of a Bayesian approach, which helps to update the current statistical parameters when more S-N data becomes available.

3 APPLICATION EXAMPLE

The validity of the proposed SNPL method is checked by applying it to an example of 40H steel. Through previous experiments, for this material, C and m are respectively known as 3.96×10^{-12} and 2.97, and the yield strength and the ultimate strength are respectively known as 780 MPa and 980 MPa [6]. An S-N curve with 60 samples is hypothetically generated using the yield and ultimate strengths, as shown in Fig. 2. Table 1 shows the results obtained from the SNPL method with the S-N curve data, and the results show good agreement with the previous experimental results.

Table 1. Results from SNPL method for set 2

Material	C (lognormal distribution)		m (lognormal distribution)		Correlation	a_0 (mm) (exponential distribution)
	mean	c.o.v.	mean	c.o.v.	Between C and m	mean
40H steel	4.46×10^{-12}	0.57	3.05	0.04	−0.9	1.18×10^{-1}

(c.o.v.: coefficient of variation)

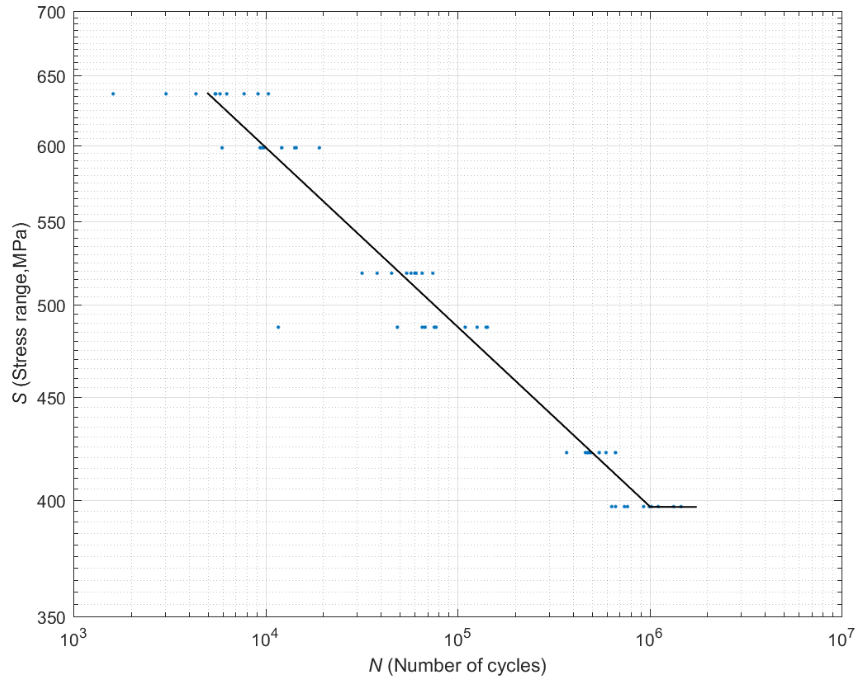

Fig. 2 S-N curve for 40H steel

4 CONCLUSION

A new method termed the SNPL method is proposed to obtain the statistical parameters of the Paris' law parameters from S-N curve data. Using a Bayesian approach, the SNPL method enables the derivation of the Paris' law parameters from S-N curve data, which maximizes the likelihood of observing the given S-N curve data. For the validation of the SNPL method, it is applied to an example of 40H steel, and the analysis results from the proposed method shows good agreement with the results from previous experiments.

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REFERENCE

[1] S. Suresh. (1998). *Fatigue of materials*. Cambridge university press.

[2] P.C. Paris and F. Erdogan. (1963). "A critical analysis of crack propagation laws," *J. Basic Eng.,* **85**(4), 528-534.

[3] Y.-J. Lee and J. Song. (2014). "System reliability updating of fatigue-induced sequential failures," *J. Struct. Eng.*, ASCE, **140**(3), 04013074.

[4] Y.-J. Lee, R.E. Kim, W. Suh, and K. Park. (2017). "Probabilistic fatigue life updating for railway bridges based on local inspection and repair," *Sensors*, **17**(4), 936.

[5] D.A. Virkler, B.M. Hillberry, and P.K. Goel. (1979). "The statistic nature of fatigue propagation," *J. Eng. Mater.-T.*, **101**, 148-53.

[6] S. Mieczyslaw and G. Lesiuk. (2009). "Algorithms for the estimation of fatigue crack growth using energy method," *Arch. Civ. Mech. Eng.*, **9**(1), 119-134.