

## **Finite element model updating using multi-objective optimization with surrogate model for steel plate girder bridge**

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### **ABSTRACT**

In this paper, finite element model updating (FEMU) based on multi-objective optimization with surrogate model is investigated. Conventionally, FEMU uses single-objective optimization with finite element analysis (FEA). It needs computational burden considerably because of a lot of FEA. This issue can be addressed by replacing FEA with surrogate model. In the case of single-objective optimization, weighting factors in the cost function should be assigned properly. However, the optimal weighting factor are not known in advance. The previous studies have used the trial-and-error strategy or user's preference for this purpose. In this study, FEMU with multi-objective optimization, which can construct the Pareto optimal front through a single run without assigning the weighting factors, and surrogate model are proposed. To verify the proposed FEMU, the field test is conducted in an in-service steel plate girder bridge and the results of the proposed method are compared with those of the single-objective optimization. The comparison shows that the multi-objective optimization is superior to the single-objective optimization in calculation time as well as the relative errors between updated model and measurement.

### **1. INTRODUCTION**

Finite element model updating (FEMU) is a procedure to minimize the discrepancy between model predictions and measurements. From the method, it is possible to evaluate the overall performance of the structure. Conventionally, FEMU has used single-objective optimization with finite element analysis (FEA). In the case of single-objective optimization, residuals are formulated to objective function with weighting factors. When assigning weighting factors, the uncertainties from FE model and measurement should be considered. However, the most proper weighting factors

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are not known in advance. Weighting factors are assigned based on user's preference or trial-and-error strategy (Kim and Park 2004).

To avoid the limitation of the residual minimization, multi-objective optimization has been employed recently. Here, the Pareto-optimal front (a.k.a. Non-dominated solutions) can be obtained and weighting factors do not have to be considered. Another limitation of conventional methods is that the computational burden occurs due to FEA. This computational issue can be addressed by surrogate model. Surrogate model uses a mathematical relationship between input and output of the target structure. By constructing the function of the input and output of the target structure, changes of variables in analysis model can be immediately known. It is important to select the proper basis function which can effectively express the input and output relation of the variables to compose of the surrogate model.

In this study, multi-objective optimization is adopted to FEMU and compared with single-objective optimization. Also, surrogate model is used to enhance the efficiency of FEMU.

## 2. Theoretical Background

### 2.1 FEMU based on single-objective optimization and multi-objective optimization

Fig. 1 shows the comparison of single-objective function (SOF) and multi-objective function (MOF) approaches. Depending on the weighting factors set by subjective consideration in SOF, optimal solution can be obtained. By cross-checking several candidates, it may require multiple analysis of optimization to validate the updated model. On the other hand, MOF approach searches all models that have been replaced alternately with a single run, and with the help of taking the decision making strategy for the selection of the most preferred FE model.

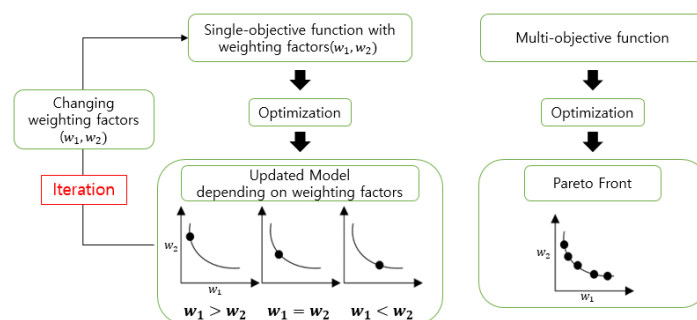


Fig. 1 Comparison SOF and MOF (Jin *et al* 2014)

### 2.2 Surrogate model

Kriging surrogate model is a prediction method which estimates the specific values by using the weighted linear combination with the known values. It is one of the surrogated models from Geostatistics. The variables required in the basis function are selected by the maximum value of the maximum likelihood function. The basis function can be expressed as

$$\psi^{ij} = \exp(-\sum_{p=1}^k \theta_p \|x_p^i - x_p^j\|) \quad (1)$$

where subscript 'p' is dimensions of a sample, superscript 'i' and 'j' indicate 'i'-th and 'j'-th samples, respectively. According to the variables involved in basis function, the accuracy and curvature of the Kriging model is determined. Correlation of the function value is defined by the distance of each samples. From this, the function of sample is expressed as the stochastic random variable which has a mean and variance value. Using the log maximum likelihood function and its mean and variance value, prediction value can be estimated. After defining the sample and covariance from the prediction point, the least square method is used to calculate the prediction value. In this study, the sequential sampling method is introduced to improve the accuracy of surrogate model. Fig. 1 shows the flow chart of construction procedure of the Kriging surrogate model. A more detail description can be found in Jin and Jung (2016).

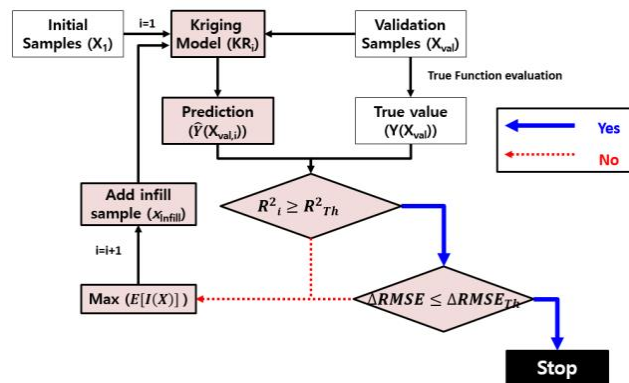


Fig. 1 Flowchart of the Kriging surrogate model with the sequential sampling strategy (Jin and Jung 2016)

### 3. Experimental Validation

In this chapter, the proposed method is verified using the field test data. The field test was conducted in a steel plate girder bridge. It is located in Chung-bu inland expressway, where many proposed and developed techniques are applied on and traffic does not allow to pass the road. The ambient vibration test was performed to obtain the natural frequencies and mode shapes of the target bridge. The target bridge is single span with 40.0 m long, 12.6 m wide and also skewed 2% as shown in Fig. 2.



(a) A steel plate girder bridge

(b) an FE model

Fig. 2 Target bridge

The dynamic responses were obtained by ambient vibration test. The measured data is used as calibration and validation value when FEMU was performed and after optimization. 15 accelerometers were used to obtain the dynamic responses and arranged as shown in Fig. 3. To obtain the data, the computer (NI PXI-1000B), multifunction DAQ (NI USB-6353), a sensor signal conditioner (PCB model 481a), and accelerometers (PBC 393B12) were used. The data was set to 100 Hz sampling frequency and investigated for about 120 minutes. Fig. 4 indicates the accelerometers data of 1<sup>st</sup> array (No.1 to No.5). From the obtained data, modal identifications, which are stochastic subspace identification (SSI), were used to figure out the modal properties.



Fig. 3 Sensor deployment

The initial FE model was modelled by ANSYS APDL as shown in Fig. 2. The initial FE model is composed of 131,909 elements with shell and beam element. Table 1 indicates the differences between the initial FE model and measurement.

Table 1 Comparison of the natural frequencies

Mode	Experimental Result (Hz)	Initial FE model	
		Value (Hz)	Error (%)
1 <sup>st</sup> bending (f1)	4.419	4.097	7.29
1 <sup>st</sup> torsion (f2)	4.787	4.404	8.00
2 <sup>nd</sup> bending (f3)	10.683	9.539	10.71
2 <sup>nd</sup> torsion (f4)	13.483	11.284	16.31

FEMU is conducted in two cases; (1) single-objective optimization, and (2) multi-objective optimization. In the case of single-objective optimization, weighting factors have 46 cases of the range from 0.05 to 0.9 depending on the cost function.

$$J = \sum_{i=1}^n \omega_i \left( \frac{f_i^{exp} - f_i^{FEM}}{f_i^{exp}} \right)^2 \quad (2)$$

where ' $i$ ' indicates ' $i$ -th natural frequencies,  $\omega_i$  indicates weighting factor of ' $i$ -th.  $f_i^{exp}$  and  $f_i^{FEM}$  indicate ' $i$ -th experiment and FE model natural frequencies, respectively. Table 2 shows the updating parameters and their upper and lower bounds.

In the case of multi-objective optimization, FEMU was performed under the same conditions as single-objective optimization as depicted in Table 3. As seen from the table, FEMU with single-objective optimization needs 46 runs to obtain the results; on the other hand, FEMU with multi-objective optimization needs only a single run to obtain the updating results. Fig. 6 shows the 3 target outputs (i.e., f1, f3 and f4) and

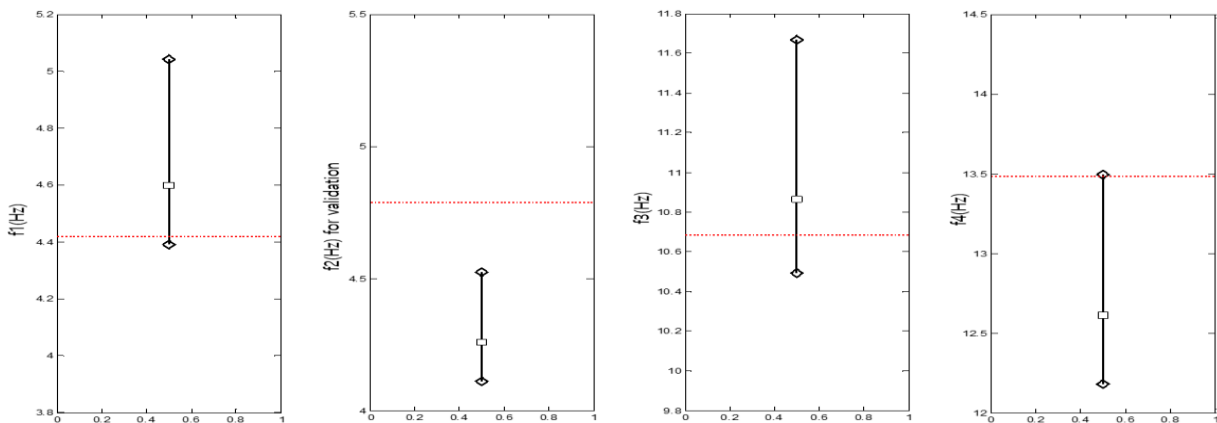
one for validation purpose (i.e.,  $f_2$ ). In the figure, the square symbol represents the mean value and two diamonds are maximum and minimum values. Also, the red dotted line indicates the measurement data. As shown in the second plot of Fig. 4(a), distribution of  $f_2$  (i.e., 1<sup>st</sup> torsion) is biased compared with the measurement data in the case of single-objective optimization. However, in the case of multi-objective optimization (see Fig. 4(b)), the updating results are well distributed covering the measurement data.

**Table 2 Updating parameters and their upper and lower bounds**

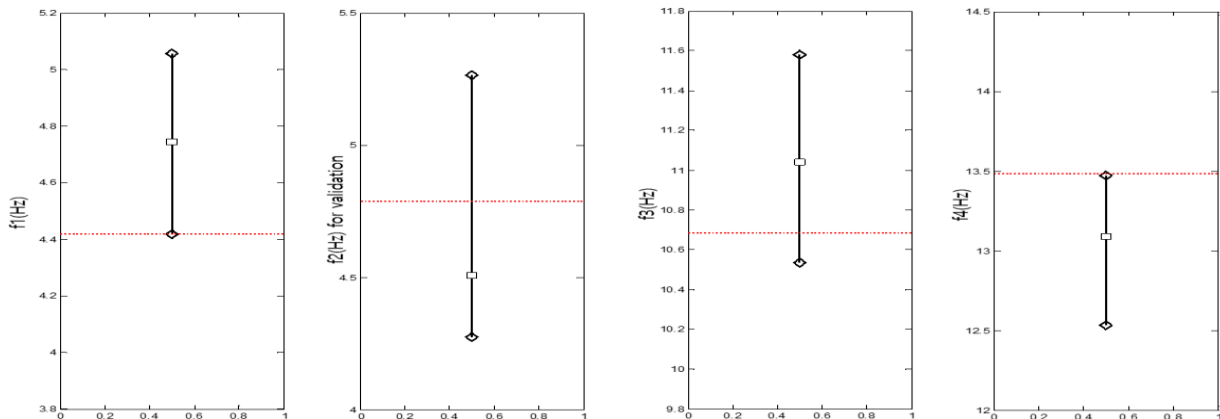
Updating parameters	Upper bound	Lower bound
Girder	1.5	0.7
Slab	1.5	0.7
Cross girder	1.5	0.7
Normal stiffness	2.5	0.1
Sticking stiffness	2.5	0.1

**Table. 3 Details of updating**

Approach	Population size	Generation	No. of run	No. of iteration
Single-objective	500	200	46	229,950
Multi-objective	500	200	1	100,000



(a) SOF result



(b) MOF result

Fig. 4 Updating results

#### 4. Conclusion

This paper investigates FEMU consisting of multi-objective optimization and surrogate model. To validate the effectiveness of the proposed method, the ambient vibration test of a steel plate girder bridge was conducted and its modal properties were used in the FEMU procedure. According to the updating results, the proposed approach can reduce the computational burden compared to the single-objective optimization. Moreover, the proposed method can obtain relatively non-biased or well-distributed updating results.

#### ACKNOWLEDGEMENTS

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