





































$$\int_0^t \cos(\omega\tau) \sin[kc(t-\tau)]d\tau = t \sin(\omega t) / 2 \quad \text{case 2: } \omega=ck \text{ and } \omega \neq 0$$

$$\int_0^t \cos(\omega\tau) \sin[kc(t-\tau)]d\tau = \frac{ck[\cos(ckt) - \cos(\omega t)]}{\omega^2 - k^2c^2} \quad \text{case 3: } \omega \text{ for other other values}$$

When  $\omega=0$ , it is found that Eq. (53) is the same as Eq. (36), that is the derived analytical solution to the axial displacement of the cantilever bar under harmonic load at special condition of  $\omega=0$  is the same as that under the Heaviside-type load. It validate from a certain viewpoint that the derivation in current research is correct.

### 4.3 Numerical treatment

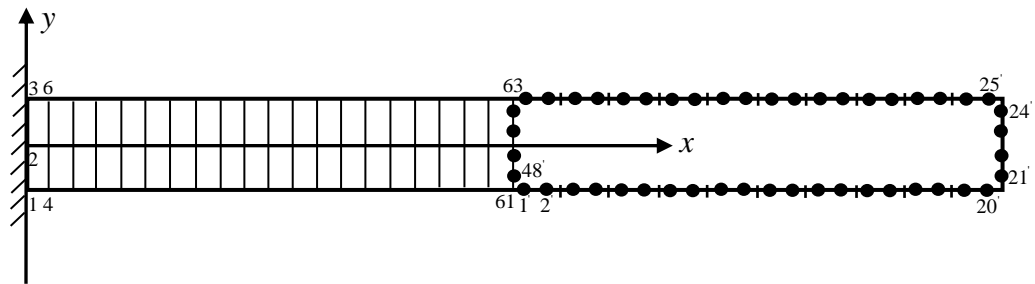


Fig. 7 Discretization of the cantilever bar

The domain of the cantilever bar is divided into two sub-domains. The sub-domain close to the fixed end is discretized with finite elements by employing plane four-node elements, while the free end sub-domain is discretized with boundary elements by employing the linear 1/4 symmetrical non-conforming boundary elements with 2 nodes. The common interface is at the middle of the bar. The FEM sub-domain is discretized into 40 finite elements with 63 nodes, while the BEM sub-domain is discretized into 24 boundary elements with 48 nodes, as shown in Fig. 7.

The FEM and BEM sub-domains are independently numbered. In the FEM sub-domain, the nodes and the finite elements are numbered in the sequence of from bottom to top and from left to right. In the BEM sub-domain, the boundary elements and nodes are numbered anti clockwise.

#### 4.4 Comparison between numerical and analytical results

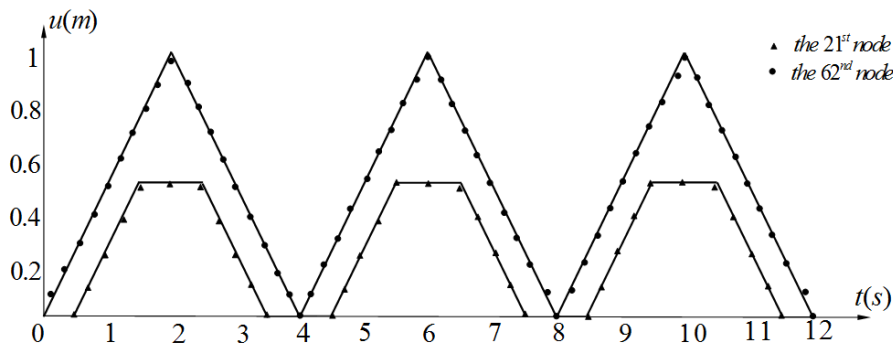


Fig. 8 Comparison between the results in terms of the axial displacement from the proposed coupled formulation and the analytical solution for the case of Heaviside-type load

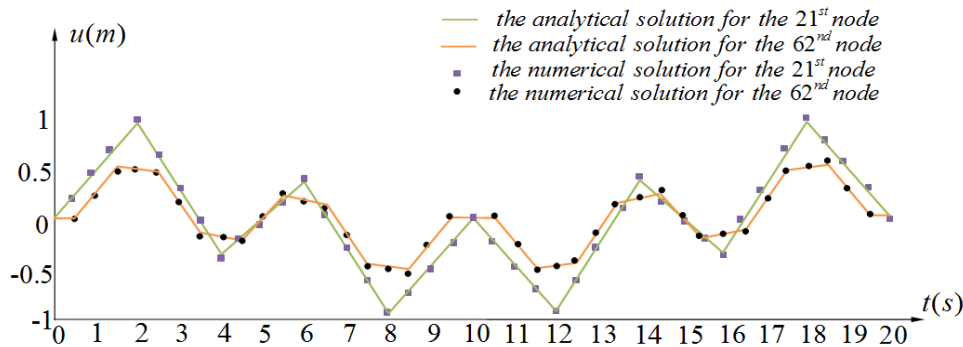


Fig. 9 Comparison between the results in terms of the axial displacement from the proposed coupled formulation and the analytical solution for the case of harmonic load

Two representative nodes, the 21<sup>st</sup> boundary element node (1, -0.0375) at the free end and the 62<sup>nd</sup> finite element node (0.5, 0) at the middle of the bar, are chosen for the comparison. Fig. 8 shows the time histories from the coupled formulation of the axial displacements of the two chosen FEM and BEM nodes for the case of the Heaviside-type load, and Fig. 9 shows those for the case of the harmonic load. For the comparison purpose, the analytical results from Eq. (36) for the two chosen nodes are included in Fig. 8, and those from Eq. (53) are included in Fig. 9. In the two figures, the sparse dots stand for the results from the coupled formulation, while the continuous lines stand for those from analytical solutions.

From Figs. 8-9, it can be seen that the results from the coupling formulation of precise integration FEM and BEM well agree with those from the analytical solutions, for different points under different loads in a long term. It indicates that the proposed force and displacement converting matrices are correct, and it also indicates that the proposed coupling formulation of precise integration FEM and BEM is correct, accurate and consistent.

## 5. Conclusions

The main findings can be summarized as follows:

- The iterative coupling formulation of the precise integration FEM and TD-BEM is proposed for dynamic problems. The coupling formulation is verified to be correct by a cantilever bar under Heaviside-type and harmonic transient loads, with good modeling accuracy and consistence.
- The force and displacement converting matrices are proposed to transfer the nodal information on different positions between FEM and BEM sub-domains. It is not required the coincident positions for the FEM and BEM nodes on the common interface between FEM and BEM sub-domains. Therefore, the coupling formulation is versatile and easily manipulated.
- Based on the analytical expression of the equation of motion in time and the discretization in space by finite elements, the precise integration FEM formulation is a semi-analytical method, with good modeling accuracy comparing with pure numerical solution. Moreover, in the proposed coupling formulation of the precise integration FEM and TD-BEM, the finite difference treatment to the time differential terms in the original FEM governing equation is evaded, where the modeling error might be accumulated. Therefore the proposed coupling formulation of the precise integration FEM and BEM is accurate.
- The analytical solution of a cantilever bar under harmonic transient load is derived. On one hand, the solution is used to verify the proposed coupling formulation of the precise integration FEM and TD-BEM. On the other hand, the good agreement between the results from the proposed coupling formulation and the derived analytical solution indicates, to a certain degree, that the derivation procedure is correct.

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